Chapter - 1

SET LANGUAGE

Exercise - 1.1

1. Which of the following are Sets?

(i) The Collection of Prime numbers upto 100.

Sol :- Set.

(ii) The collection of rich people in India.

Sol :- Not Set.

(iii) The collection of all rivers in India

Sol :- Set.

(iv) The Collection of good Hockey Players.

Sol :- Not Set.

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(2) List the set of letters of the following words in Roster form.

(i) INDIA $= \{I, N, D, A\}$

(ii) PARALLELOGRAM $= \{P, A, R, L, E, O, G, M\}$

(iii) MISSISSIPPI $= \{M, I, S, P\}$

(iv) CZECHOSLOVAKIA $= \{C, Z, E, H, O, S, L, V, A, K, I\}$

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(3) Consider the following Sets

$A = \{0, 3, 5, 8\}$, $B = \{2, 4, 6, 10\}$ and

$C = \{12, 14, 18, 20\}$

(a) State whether True or False :

(i) $18 \in C$ $\longrightarrow$ True

(ii) $6 \notin A$ $\longrightarrow$ True

(iii) $14 \notin C$ $\longrightarrow$ False

(iv) $10 \in B$ $\longrightarrow$ True.

(v) $5 \in B \longrightarrow$ False

(vi) $0 \in B \longrightarrow$ False.

(b) Fill in the blanks:-

(i) $3 \in \underline{\quad A \quad}$

(ii) $14 \in \underline{\quad C \quad}$

(iii) $18 \underline{\quad \notin \quad} B$

(iv) $4 \underline{\quad \in \quad} B$

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(4) Represent the following sets in Roster form:-

(i) A = The set of all even natural numbers less than 20.

Sol:- $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

(ii) $B = \{y : y = \frac{1}{2n}, n \in N, n \leq 5\}$

Sol:- $n = 1, 2, 3, 4, 5$

$n=1;\quad y=\frac{1}{2(1)}=\frac{1}{2}$

$n=2;\quad y=\frac{1}{2(2)}=\frac{1}{4}$

$n=3;\quad y=\frac{1}{2(3)}=\frac{1}{6}$

$n=4;\quad y=\frac{1}{2(4)}=\frac{1}{8}$

$n=5;\quad y=\frac{1}{2(5)}=\frac{1}{10}$

$\therefore B=\left\{\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10}\right\}$

(iii) $C=\{x: x \text{ is perfect cube}, 27<x<216\}$

Sol:- $C=\{64, 125\}$

(iv) $D=\{x: x\in Z;\ -5<x\leq 2\}$

Sol:- $D=\{-4,-3,-2,-1,0,1,2\}$

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(5) Represent the following sets in set builder form.

(i) B = The set of all cricket players in India who scored double centuries in One Day Internationals.

Sol:- $B = \{x : x$ is all Indian cricket players who scored double centuries in One Day International$\}$

(ii) $C = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots\right\}$

Sol:- $C = \left\{x : x = \frac{n}{n+1} ; n \in N\right\}$

(iii) D = The set of all tamil months in a year.

Sol:- $D = \{x : x$ is all tamil months in a year$\}$

(iv) E = The Set of odd whole numbers less than 9.

Sol:- E = {x : x is odd whole numbers less than 9}.

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⑥ Represent the following sets in descriptive form.

(i) P = {January, June, July}

Sol:- P = The set of Months starting with 'J'.

(ii) Q = {7, 11, 13, 17, 19, 23, 29}

Sol:- Q = The set of all Prime numbers between 5 and 31.

(iii) $R = \{x : x \in N, x < 5\}$

Sol:- R = The set of all natural number less than 5.

(iv) $S = \{x : x$ is a consonants in English alphabets$\}$

Sol:- S = The Set of all Consonants in English alphabets.

Exercise - 1.2

① Find the cardinal number of the following sets.

(i) $M = \{p, q, r, s, t, u\}$

Sol:- $\boxed{n(M) = 6}$

(ii) $P = \{x : x = 3n+2, n \in W \text{ and } x < 15\}$

Sol:- $n \in W \Rightarrow n = 0, 1, 2, 3, \ldots$

$n = 0$; $x = 3(0)+2 = 0+2 = 2$

$n = 1$; $x = 3(1)+2 = 3+2 = 5$

$n = 2;\quad x = 3(2)+2 = 6+2 = 8$

$n = 3;\quad x = 3(3)+2 = 9+2 = 11$

$n = 4;\quad x = 3(4)+2 = 12+2 = 14$

$$\boxed{x < 15}$$

$\therefore P = \{2, 5, 8, 11, 14\}$

$$\boxed{n(P) = 5}$$

(iii) $Q = \{y : y = \frac{4}{3n},\ n \in N \text{ and } 2 < n \leq 5\}$

Sol :- $n = 3, 4, 5$

$n = 3;\quad y = \frac{4}{3(3)} = \frac{4}{9}$

$n = 4;\quad y = \frac{4}{3(4)} = \frac{4}{12}$

$n = 5;\quad y = \frac{4}{3(5)} = \frac{4}{15}$

$Q = \{\frac{4}{9}, \frac{4}{12}, \frac{4}{15}\}$

$\therefore \boxed{n(Q) = 3}$

(iv) $R = \{x : x$ is an integers, $x \in Z$ and $-5 \leq x < 5\}$

Sol :- $R = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$$\boxed{n(R) = 10}$$

(v) S = The set of all leap years between 1882 and 1906.

Sol :- A leap year comes four years once.

$\therefore S = \{1884, 1888, 1892, 1896, 1900, 1904\}$

$$\begin{array}{r} 47 \\ 4\overline{)1882} \\ 16 \\ \hline 28 \\ 28 \\ \hline 2 \end{array}$$

$\therefore \boxed{n(S) = 6}$

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(2) Identify the following sets as finite or Infinite.

(i) X = The set of all districts in Tamilnadu.

Sol :- Finite Sets

(ii) Y = The set of all straight lines passing through a point.

Sol :- Infinite set

(iii) $A = \{x : x \in Z \text{ and } x < 5\}$

Sol :- $A = \{\ldots\ldots, -2, -1, 0, 1, 2, 3, 4\}$

Infinite set.

(iv) $B = \{x : x^2 - 5x + 6 = 0, x \in N\}$

Sol :- $x^2 - 5x + 6 = 0$

6
-3 -2

$(x-3)(x-2) = 0$

$x - 3 = 0$ | $x - 2 = 0$

$\boxed{x = 3}$ | $\boxed{x = 2}$

$\therefore \boxed{x = 3, 2}$

Finite set

③ Which of the following sets are equivalent or unequal or equal sets?

(i) A = The set of vowels in English alphabets.

B = The set of all letters in the word "VOWEL".

Sol $A = \{a, e, i, o, u\}$

$B = \{v, o, w, e, l\}$

$\because n(A) = n(B) = 5$

$\therefore$ Equivalent sets

(ii) $C = \{2, 3, 4, 5\}$; $D = \{x : x \in W, 1 < x < 5\}$

Sol:- $C = \{2, 3, 4, 5\}$

$D = \{2, 3, 4\}$

Unequal sets.

(iii) $X = \{x : x$ is the letter in the word "LIFE"$\}$

$Y = \{F, I, L, E\}$

Sol :- $X = \{L, I, F, E\}$

$Y = \{F, I, L, E\}$

Equal sets

(iv) $G = \{x : x$ is a prime number and $3 < x < 23\}$

$H = \{x : x$ is a divisor of $18\}$

Sol :- $G = \{5, 7, 11, 13, 17, 19\}$

$H = \{1, 2, 3, 6, 9, 18\}$

$n(G) = n(H) = 6$

Equivalent sets

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(4) Identify the following sets as null sets or Singleton sets.

(i) $A = \{x : x \in N, 1 < x < 2\}$

Sol $A = \{\ \}$ $\Rightarrow$ Null sets

(ii) B = The Set of all even natural numbers which are not divisible by 2.

Sol :- B = { } ⇒ Null sets

(iii) C = {0}

Sol :- Singleton Sets

(iv) D = The Set of all triangles having four sides

Sol :- Null Sets.

⑤ State which pairs of sets are disjoint or overlapping ?

(i) A = {f, i, a, s} ; B = {a, n, f, h, s}

Sol

Overlapping

[element f, a, s are common in set A & B.]

(ii) $C = \{x : x \text{ is a prime number}, x > 2\}$

$D = \{x : x \text{ is an even prime number}\}$

<u>Sol</u> :- $C = \{3, 5, 7, \ldots\ldots\}$

$D = \{2\}$

Disjoint

[No elements are Common on in Set C & D].

(iii) $E = \{x : x \text{ is a factor of } 24\}$

$F = \{x : x \text{ is a multiple of } 3, x < 30\}$

<u>Sol</u> :- $E = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$F = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

Overlapping

[elements 3, 6, 12, 24 are Common in Set E, F}.

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(6). If S = {square, rectangle, circle, rhombus, triangle},
list the elements of the following subsets of S.

(i) The set of shapes which have 4 equal sides.

Sol:- {square, rhombus}

(ii) The set of shapes which have radius.

Sol:- {Circle}.

(iii) The set of shapes in which the sum of all interior angles is 180°.

Sol:- {Triangle}

(iv) The set of shapes which have 5 sides.

Sol:- { }

(7.) If $A = \{a, \{a, b\}\}$, write all the Subsets of A.

Sol:- Subsets of A are $\{\ \}, \{a\}, \{a, b\}, \{a, \{a, b\}\}$

(8.) Write down the power set of the following sets:-

(i) $A = \{a, b\}$

Sol:- $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

(ii) $B = \{1, 2, 3\}$

Sol:- $P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}\}$

(iii) $D = \{p, q, r, s\}$

Sol:- $P(D) = \{\phi, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{q, r\},$
$\{r, s\}, \{s, p\}, \{s, q\}, \{p, r\},$
$\{p, q, r\}, \{p, r, s\}, \{p, q, s\}$
$\{s, q, r\}, \{p, q, r, s\}\}$

(iv) $E = \phi$

Sol:- $P(E) = \{ \}$

(9) Find the number of Subsets and the number of proper Subsets of the following sets.

(i) $W = \{red, blue, yellow\}$

Sol:- $n(W) = 3$

Number of Subsets $= n[P(W)] = 2^3 = 8$

Number of proper Subsets $= n[P(W)] - 1$

$= 2^3 - 1$

$= 8 - 1$

$= 7$

(ii) $X = \{x^2 : x \in N ; x^2 \leq 100\}$

$x = 1, 2, 3, \ldots$

$x^2 = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$X = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$\therefore n(X) = 10$

Number of Subsets $= n[P(X)] = 2^{10} = 1024$

Number of Proper Subsets $= n[P(X)] - 1$

$= 2^{10} - 1$

$= 1024 - 1$

$= 1023$

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(10) (i) If $n(A) = 4$; find $n[P(A)]$

Sol :- $n(A) = 4$

$n[P(A)] = 2^{n(A)}$

$= 2^4$

$\boxed{n[P(A)] = 16}$

(ii) If $n(A) = 0$; find $n[P(A)]$

Sol :- $n(A) = 0$

$n[P(A)] = 2^{n(A)}$

$= 2^0$

$\boxed{n[P(A)] = 1}$

(iii) If $n[P(A)] = 256$, find $n(A)$.

Sol:- $n[P(A)] = 256$

$$n[P(A)] = 2^8$$

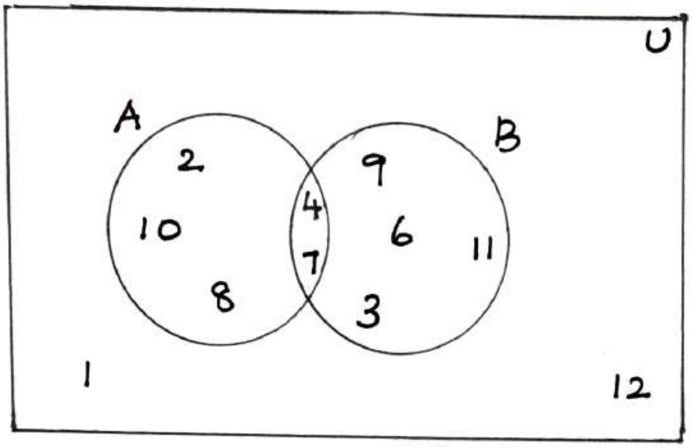
We know $\boxed{n[P(A)] = 2^{n(A)}}$

$$\therefore \boxed{n(A) = 8}$$

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Exercise - 1.3

(1) Using the given Venn diagram, write the elements of



(i) $A = \{2, 10, 8, 4, 7\}$

(ii) $B = \{4, 7, 9, 6, 3, 11\}$

(iii) $A \cup B = \{2, 10, 8, 4, 7, 9, 6, 3, 11\}$

(iv) $A \cap B = \{4, 7\}$

(v) $A - B = \{2, 10, 8\}$

(vi) $B - A = \{9, 6, 3, 11\}$

(vii) $A' = \{9, 6, 3, 11, 1, 12\}$

(viii) $B' = \{2, 10, 8, 1, 12\}$

(ix) $U = \{2, 10, 8, 4, 7, 9, 6, 3\ 11, 1, 12\}$

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(2) Find $A \cup B$, $A \cap B$, $A - B$ and $B - A$ for the following sets.

(i) $A = \{2, 6, 10, 14\}$, $B = \{2, 5, 14, 16\}$

Sol:- * $A \cup B = \{2, 6, 10, 14\} \cup \{2, 5, 14, 16\}$

$A \cup B = \{2, 6, 10, 14, 5, 16\}$

* $A \cap B = \{2, 6, 10, 14\} \cap \{2, 5, 14, 16\}$

$A \cap B = \{2, 14\}$

* $A-B = \{\cancel{2}, 6, 10, \cancel{14}\} - \{\cancel{2}, 5, \cancel{14}, 16\}$

$A-B = \{6, 10\}$

* $B-A = \{\cancel{2}, 5, \cancel{14}, 16\} - \{\cancel{2}, 6, 10, \cancel{14}\}$

$B-A = \{5, 16\}$

(ii) $A = \{a, b, c, e, u\}$ and $B = \{a, e, i, o, u\}$

Sol :-

* $A \cup B = \{a, b, c, e, u\} \cup \{a, e, i, o, u\}$

$A \cup B = \{a, b, c, e, u, i, o\}$

* $A \cap B = \{a, b, c, e, u\} \cap \{a, e, i, o, u\}$

$A \cap B = \{a, e, u\}$

* $A-B = \{\cancel{a}, b, c, \cancel{e}, \cancel{u}\} - \{\cancel{a}, \cancel{e}, i, o, \cancel{u}\}$

$A-B = \{b, c\}$

* $B-A = \{\cancel{a}, \cancel{e}, i, o, \cancel{u}\} - \{\cancel{a}, b, c, \cancel{e}, \cancel{u}\}$

$B-A = \{i, o\}$

(iii) $A = \{x : x \in N, x \leq 10\}$ and $B = \{x : x \in W, x < 6\}$

Sol: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B = \{0, 1, 2, 3, 4, 5\}$

★ $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cup \{0, 1, 2, 3, 4, 5\}$

$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

★ $A \cap B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cap \{0, 1, 2, 3, 4, 5\}$

$A \cap B = \{1, 2, 3, 4, 5\}$

★ $A - B = \{\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, 7, 8, 9, 10\} - \{0, \cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}\}$

$A - B = \{6, 7, 8, 9, 10\}$

★ $B - A = \{0, \cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}\} - \{\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, 6, 7, 8, 9, 10\}$

$B - A = \{0\}$

(iv) A = The Set of all letters in the Word "Mathematics"

B = The Set of all Letters in the Word "Geometry"

<u>Sol</u>:- $A = \{m, a, t, h, e, i, c, s\}$

$B = \{g, e, o, m, t, r, y\}$.

* $A \cup B = \{m, a, t, h, e, i, c, s\} \cup \{g, e, o, m, t, r, y\}$

  $A \cup B = \{m, a, t, h, e, i, c, s, g, o, r, y\}$

* $A \cap B = \{m, a, t, h, e, i, c, s\} \cap \{g, e, o, m, t, r, y\}$

  $A \cap B = \{m, t, e\}$

* $A - B = \{\cancel{m}, a, \cancel{t}, h, \cancel{e}, i, c, s\} - \{g, \cancel{e}, o, \cancel{m}, \cancel{t}, r, y\}$

  $A - B = \{a, h, i, c, s\}$

* $B - A = \{g, \cancel{e}, o, \cancel{m}, \cancel{t}, r, y\} - \{\cancel{m}, a, \cancel{t}, h, \cancel{e}, i, c, s\}$

  $B - A = \{g, o, r, y\}$

(3) If $U = \{a, b, c, d, e, f, g, h\}$, $A = \{b, d, f, h\}$ and $B = \{a, d, e, h\}$, find the following Sets.

(i) A'

Sol $\boxed{A' = U - A}$

$$A' = \{a, \cancel{b}, c, \cancel{d}, e, \cancel{f}, g, \cancel{h}\} - \{\cancel{b}, \cancel{d}, \cancel{f}, \cancel{h}\}$$

$$A' = \{a, c, e, g\}$$

(ii) B'

Sol $\boxed{B' = U - B}$

$$B' = \{\cancel{a}, b, c, \cancel{d}, \cancel{e}, f, g, \cancel{h}\} - \{\cancel{a}, \cancel{d}, \cancel{e}, \cancel{h}\}$$

$$B' = \{b, c, f, g\}$$

(iii) $A' \cup B'$

Sol:- $A' \cup B' = \{a, c, e, g\} \cup \{b, c, f, g\}$

$$A' \cup B' = \{a, c, e, g, b, f\}$$

[A' and B' from (i) and (ii)]

(iv) $A' \cap B'$

Sol:- $A' \cap B' = \{a, c, e, g\} \cap \{b, c, f, g\}$

$A' \cap B' = \{c, g\}$.

(v) $(A \cup B)'$

Sol:- $\boxed{(A \cup B)' = U - (A \cup B)}$

$(A \cup B) = \{b, d, f, h\} \cup \{a, d, e, h\}$

$(A \cup B) = \{b, d, f, h, a, e\}$

$(A \cup B)' = \{\cancel{a}, \cancel{b}, c, \cancel{d}, \cancel{e}, \cancel{f}, g, \cancel{h}\} - \{\cancel{b}, \cancel{d}, \cancel{f}, \cancel{h}, \cancel{a}, \cancel{e}\}$

$(A \cup B)' = \{c, g\}$

(vi) $(A \cap B)'$

Sol $\boxed{(A \cap B)' = U - (A \cap B)}$

$\because A \cap B = \{b, d, f, h\} \cap \{a, d, e, h\}$

$A \cap B = \{d, h\}$

$(A \cap B)' = \{a, b, c, \cancel{d}, e, f, g, \cancel{h}\} - \{\cancel{d}, \cancel{h}\}$

$(A \cap B)' = \{a, b, c, e, f, g\}$

(vii) $(A')'$

Sol:- $\boxed{(A')' = U - A'}$

[The Value of A' is in (i)]

$(A')' = \{\cancel{a}, b, \cancel{c}, d, \cancel{e}, f, \cancel{g}, h\} - \{\cancel{a}, \cancel{c}, \cancel{e}, \cancel{g}\}$

$(A')' = \{b, d, f, h\}$

ie, $\boxed{(A')' = A}$

(viii) $(B')'$

Sol:- $\boxed{(B')' = U - B'}$

[The Value of B' is in (ii)]

$(B')' = \{a, \cancel{b}, \cancel{c}, d, e, \cancel{f}, \cancel{g}, h\} - \{\cancel{b}, \cancel{c}, \cancel{f}, \cancel{g}\}$

$(B')' = \{a, d, e, h\}$

ie, $\boxed{(B')' = B}$

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(4) Let $U = \{0,1,2,3,4,5,6,7\}$, $A = \{1,3,5,7\}$ and $B = \{0,2,3,5,7\}$, find the following sets.

(i) A'

Sol:- $A' = \{0,\cancel{1},2,\cancel{3},4,\cancel{5},6,\cancel{7}\} - \{\cancel{1},\cancel{3},\cancel{5},\cancel{7}\}$

$A' = \{0,2,4,6\}$

(ii) B'

Sol $B' = \{\cancel{0},1,\cancel{2},\cancel{3},4,\cancel{5},6,\cancel{7}\} - \{\cancel{0},\cancel{2},\cancel{3},\cancel{5},\cancel{7}\}$

$B' = \{1,4,6\}$

(iii) $A' \cup B'$

Sol:- $A' \cup B' = \{0,2,4,6\} \cup \{1,4,6\}$

$A' \cup B' = \{0,2,4,6,1\}$

(iv) $A' \cap B'$

Sol:- $A' \cap B' = \{0,2,4,6\} \cap \{1,4,6\}$

$A' \cap B' = \{4,6\}$

(v) $(A \cup B)'$

Sol $A \cup B = \{1, 3, 5, 7\} \cup \{0, 2, 3, 5, 7\}$

$A \cup B = \{1, 3, 5, 7, 0, 2\}$

$(A \cup B)' = \{\cancel{0}, \cancel{1}, \cancel{2}, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}\} - \{\cancel{1}, \cancel{3}, \cancel{5}, \cancel{7}, \cancel{0}, \cancel{2}\}$

$(A \cup B)' = \{4, 6\}$

(vi) $(A \cap B)'$

Sol $A \cap B = \{1, 3, 5, 7\} \cap \{0, 2, 3, 5, 7\}$

$A \cap B = \{3, 5, 7\}$

$(A \cap B)' = \{0, 1, 2, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}\} - \{\cancel{3}, \cancel{5}, \cancel{7}\}$

$(A \cap B)' = \{0, 1, 2, 4, 6\}$

(vii) $(A')'$

Sol:- $(A')' = U - A'$

$= \{\cancel{0}, 1, \cancel{2}, 3, \cancel{4}, 5, \cancel{6}, 7\} - \{\cancel{0}, \cancel{2}, \cancel{4}, \cancel{6}\}$

$(A')' = \{1, 3, 5, 7\}$

(viii) $(B')'$

Sol:- $(B')' = U - B'$

$= \{0, \cancel{1}, 2, 3, \cancel{4}, 5, \cancel{6}, 7\} - \{\cancel{1}, \cancel{4}, \cancel{6}\}$

$(B')' = \{0, 2, 3, 5, 7\}$

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(5) Find the Symmetric difference between the following Sets.

(i) $P = \{2, 3, 5, 7, 11\}$ and $Q = \{1, 3, 5, 11\}$

Sol

$$\boxed{P \Delta Q = (P - Q) \cup (Q - P)}$$

$P - Q = \{2, \cancel{3}, \cancel{5}, 7, \cancel{11}\} - \{1, \cancel{3}, \cancel{5}, \cancel{11}\}$

$P - Q = \{2, 7\}$

$Q - P = \{1, \cancel{3}, \cancel{5}, \cancel{11}\} - \{2, \cancel{3}, \cancel{5}, 7, \cancel{11}\}$

$Q - P = \{1\}$

$P \Delta Q = \{2, 7\} \cup \{1\}$

$$\boxed{P \Delta Q = \{2, 7, 1\}}$$

(ii) $R = \{l, m, n, o, p\}$ and $S = \{j, l, n, q\}$

Sol:- $\boxed{R \Delta S = (R-S) \cup (S-R)}$

$R - S = \{\cancel{l}, m, \cancel{n}, o, p\} - \{j, \cancel{l}, \cancel{n}, q\}$

$R - S = \{m, o, p\}$

$S - R = \{j, \cancel{l}, \cancel{n}, q\} - \{\cancel{l}, m, \cancel{n}, o, p\}$

$S - R = \{j, q\}$

$R \Delta S = \{m, o, p\} \cup \{j, q\}$

$\boxed{R \Delta S = \{m, o, p, j, q\}}$

(iii) $X = \{5, 6, 7\}$ and $Y = \{5, 7, 9, 10\}$

Sol $\boxed{X \Delta Y = (X - Y) \cup (Y - X)}$

$X - Y = \{\cancel{5}, 6, \cancel{7}\} - \{\cancel{5}, \cancel{7}, 9, 10\}$

$X - Y = \{6\}$

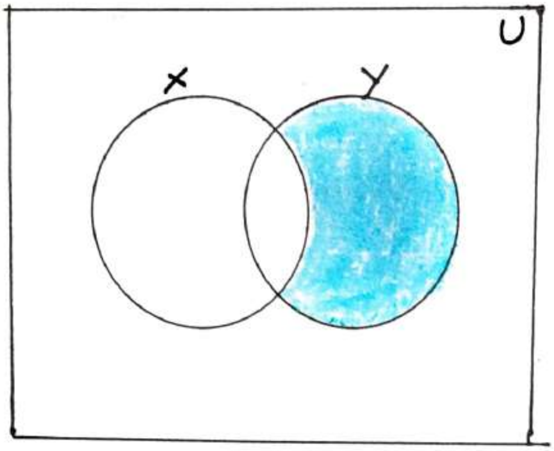
$Y - X = \{\cancel{5}, \cancel{7}, 9, 10\} - \{\cancel{5}, 6, \cancel{7}\}$

$Y - X = \{9, 10\}$

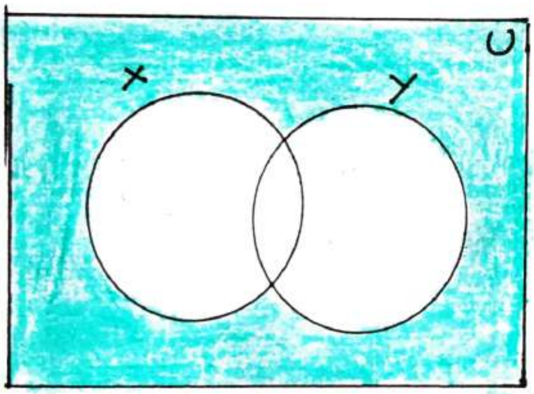
$X \Delta Y = \{6\} \cup \{9, 10\}$

$\boxed{X \Delta Y = \{6, 9, 10\}}$

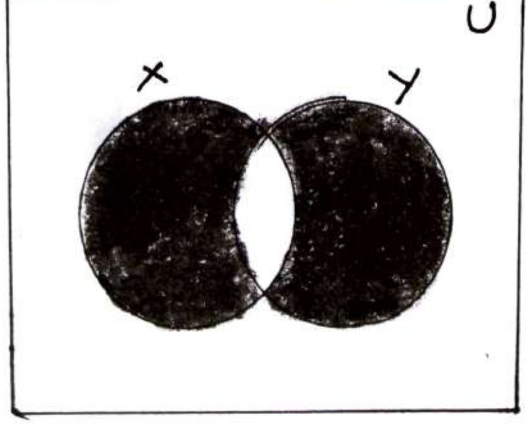
(6) Using the set symbols, write down the expressions for the shaded region in the following.

(i) 

$\Rightarrow Y - X$

(ii) 

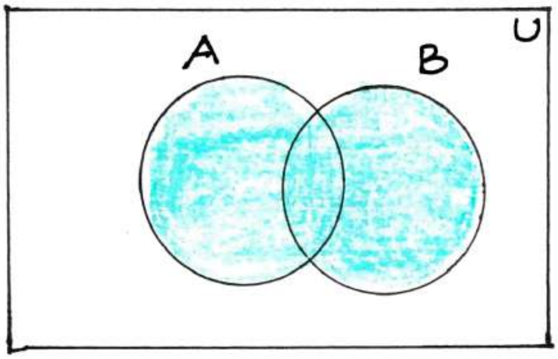
$\Rightarrow (X \cup Y)'$

(iii) 

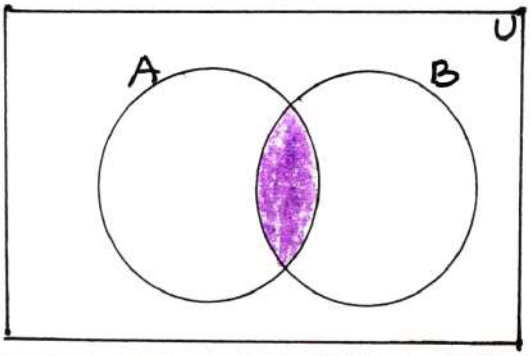
$\Rightarrow (X \cap Y)'$

(7) Let A and be two Overlapping Sets and Universal Set be U. Draw appropriate Venn diagram for each of the following.

(i) $A \cup B$.

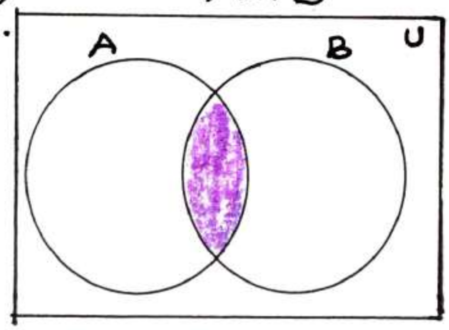


(ii) $A \cap B$

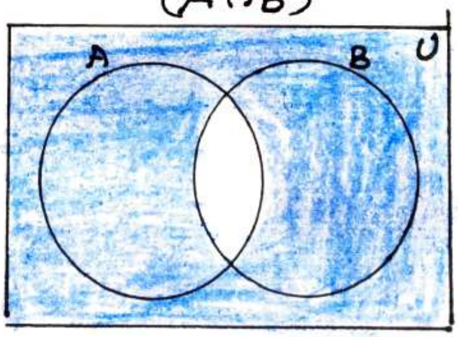


(iii) $(A \cap B)'$

$A \cap B$

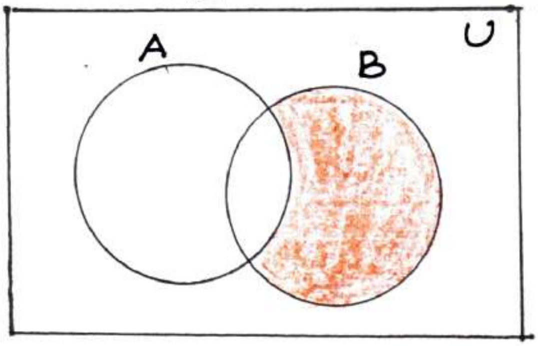


$(A \cap B)'$

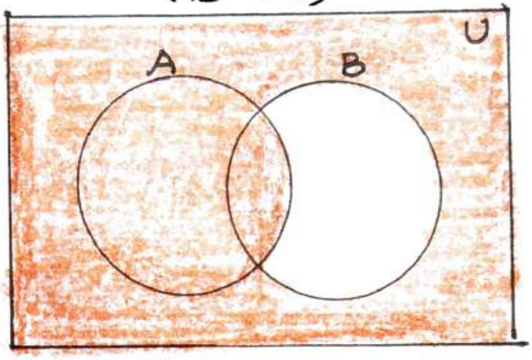


(iv) $(B-A)'$

$B-A$

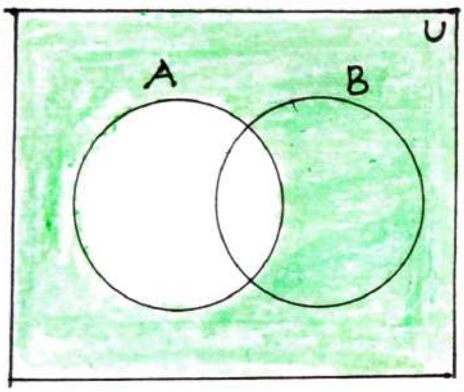


$(B-A)'$

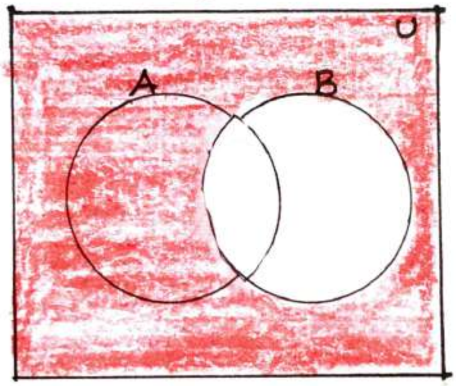


(v) $A' \cup B'$

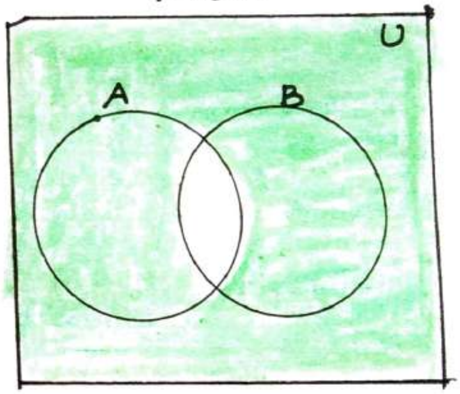
A'



B'

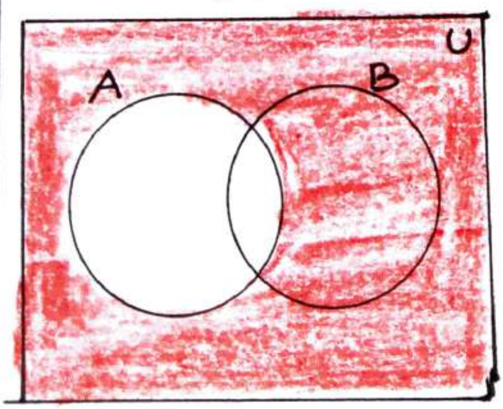


$A' \cup B'$

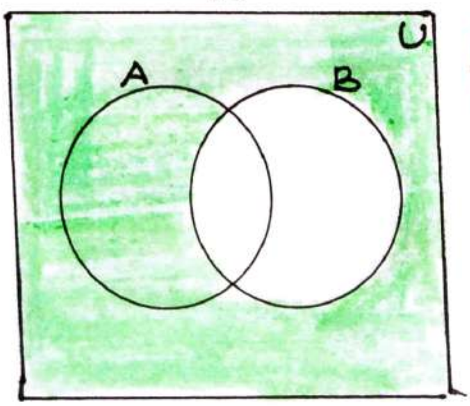


(vi) $A' \cap B'$

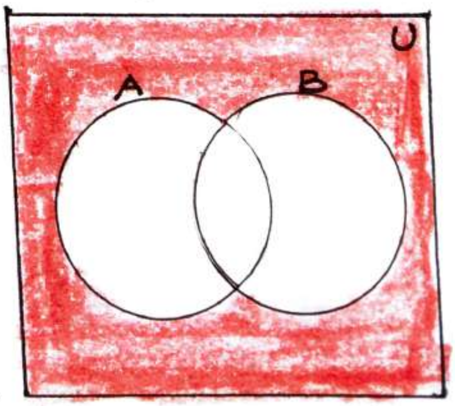
A'



B'



$A' \cap B'$



(vii) What do you Observe from the Venn diagram (iii) and (v) ?

Sol :-

Venn diagram (iii) and (v) are equal.

ie, $(A \cap B)' = A' \cup B'$

Exercise - 1.4

1. If $P = \{1, 2, 5, 7, 9\}$, $Q = \{2, 3, 5, 9, 11\}$ $R = \{3, 4, 5, 7, 9\}$ and $S = \{2, 3, 4, 5, 8\}$ then find.

(i) $(P \cup Q) \cup R$

Sol $P \cup Q = \{1, 2, 5, 7, 9\} \cup \{2, 3, 5, 9, 11\}$

$P \cup Q = \{1, 2, 5, 7, 9, 3, 11\}$

$(P \cup Q) \cup R = \{1, 2, 5, 7, 9, 3, 11\} \cup \{3, 4, 5, 7, 9\}$

$(P \cup Q) \cup R = \{1, 2, 5, 7, 9, 3, 11, 4\}$

(ii) $(P \cap Q) \cap S$

Sol $(P \cap Q) = \{1, 2, 5, 7, 9\} \cap \{2, 3, 5, 9, 11\}$

$P \cap Q = \{2, 5, 9\}$

$(P \cap Q) \cap S = \{2, 5, 9\} \cap \{2, 3, 4, 5, 8\}$

$(P \cap Q) \cap S = \{2, 5\}$

(iv) $(Q \cap S) \cap R$

Sol :-

$Q \cap S = \{2, 3, 5, 9, 11\} \cap \{2, 3, 4, 5, 8\}$

$Q \cap S = \{2, 3, 5\}$

$(Q \cap S) \cap R = \{2, 3, 5\} \cap \{3, 4, 5, 7, 9\}$

$(Q \cap S) \cap R = \{3, 5\}$

---

2. Test for the commutative Property of Union and intersection of the sets :-

$P = \{x : x$ is a real number between 2 and 7$\}$

$Q = \{x : x$ is a rational number between 2 and 7$\}$

Sol: Real Number = Rational No + Irrational No

P = The Set of all Rational No and irrational No between 2 and 7

Q = The Set of Only rational No between 2 and 7

(i) Commutative [union]

$(P \cup Q) = (Q \cup P)$

$P \cup Q$ = The Set of all rational No and irrational No between 2 and 7

$Q \cup P$ = The Set of all rational No and irrational No between 2 and 7

$\therefore \boxed{P \cup Q = Q \cup P}$

(ii) Commutative [intersection]

$P \cap Q = Q \cap P$

$P \cap Q$ = The Set of Only rational No between 2 & 7

$Q \cap P$ = The Set of Only rational No between 2 & 7

$\therefore \boxed{P \cap Q = Q \cap P}$

(3) If $A = \{p, q, r, s\}$, $B = \{m, n, q, s, t\}$ and $C = \{m, n, p, q, s\}$, then Verify the associative Property of Union of sets.

Sol :-

Associative Property [union]

$$A \cup (B \cup C) = (A \cup B) \cup C$$

LHS :- $A \cup (B \cup C)$

$$B \cup C = \{m, n, q, s, t\} \cup \{m, n, p, q, s\}$$
$$= \{m, n, q, s, t, p\}$$

$$A \cup (B \cup C) = \{p, q, r, s\} \cup \{m, n, q, s, t, p\}$$

$$\boxed{A \cup (B \cup C) = \{p, q, r, s, m, n, t\}}$$

RHS :- $(A \cup B) \cup C$

$$(A \cup B) = \{p, q, r, s\} \cup \{m, n, q, s, t\}$$
$$= \{p, q, r, s, m, n, t\}$$

$$(A \cup B) \cup C = \{p, q, r, s, m, n, t\} \cup \{m, n, p, q, s\}$$

$$\boxed{(A \cup B) \cup C = \{p, q, r, s, m, n, t\}}$$

$$\therefore A \cup (B \cup C) = (A \cup B) \cup C$$

(4) Verify the associative property of intersection of sets. for $A = \{-11, \sqrt{2}, \sqrt{5}, 7\}$ $B = \{\sqrt{3}, \sqrt{5}, 6, 13\}$ and $C = \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$

<u>Sol</u>:- Associative Property [intersection]

$$A \cap (B \cap C) = (A \cap B) \cap C$$

LHS:- $A \cap (B \cap C)$

$B \cap C = \{\sqrt{3}, \sqrt{5}, 6, 13\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$

$= \{\sqrt{3}, \sqrt{5}\}$

$A \cap (B \cap C) = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}\}$

$\boxed{A \cap (B \cap C) = \{\sqrt{5}\}}$

RHS:- $(A \cap B) \cap C$

$A \cap B = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}, 6, 13\}$

$= \{\sqrt{5}\}$

$(A \cap B) \cap C = \{\sqrt{5}\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$

$\boxed{(A \cap B) \cap C = \{\sqrt{5}\}}$

$\therefore A \cap (B \cap C) = (A \cap B) \cap C$

(5) If $A = \{x : x = 2^n, n \in W \text{ and } n < 4\}$

$B = \{x : x = 2n, n \in N \text{ and } n \leq 4\}$ and

$C = \{0, 1, 2, 5, 6\}$, then Verify the associative Property of intersection of Sets.

Sol :-

For A :- $n \in W, n < 4 \Rightarrow \boxed{n = 0, 1, 2, 3}$

$n = 0 \Rightarrow x = 2^0 = 1$

$n = 1 \Rightarrow x = 2^1 = 2$

$n = 2 \Rightarrow x = 2^2 = 4$

$n = 3 \Rightarrow x = 2^3 = 8$

$\therefore \boxed{A = \{1, 2, 4, 8\}}$

For B :- $n \in N, n \leq 4 \Rightarrow \boxed{n = 1, 2, 3, 4}$

$n = 1 \Rightarrow x = 2(1) = 2$

$n = 2 \Rightarrow x = 2(2) = 4$

$n = 3 \Rightarrow x = 2(3) = 6$

$n = 4 \Rightarrow x = 2(4) = 8$

$\therefore B = \{2, 4, 6, 8\}$

Associative property [intersection]

$$A \cap (B \cap C) = (A \cap B) \cap C$$

LHS :- $A \cap (B \cap C)$

$B \cap C = \{2, 4, 6, 8\} \cap \{0, 1, 2, 5, 6\}$

$= \{2, 6\}$

$A \cap (B \cap C) = \{1, 2, 4, 8\} \cap \{2, 6\}$

$\boxed{A \cap (B \cap C) = \{2\}}$

RHS :- $(A \cap B) \cap C$

$A \cap B = \{1, 2, 4, 8\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 8\}$

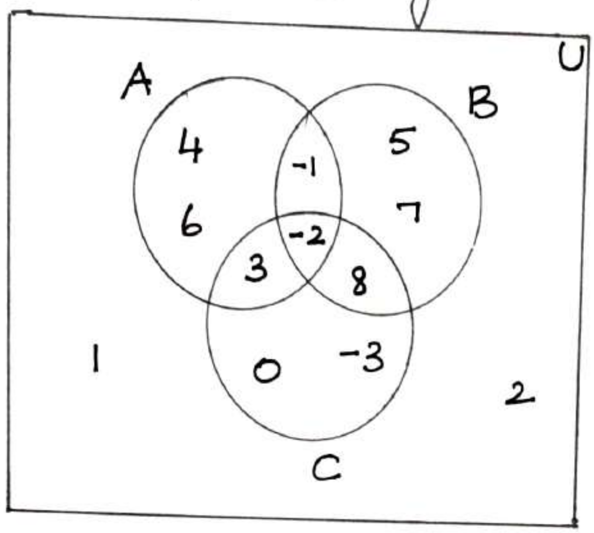
$(A \cap B) \cap C = \{2, 4, 8\} \cap \{0, 1, 2, 5, 6\}$

$\boxed{(A \cap B) \cap C = \{2\}}$

$\therefore A \cap (B \cap C) = (A \cap B) \cap C$

Exercise - 1.5

(1) Using the adjacent Venn diagram, Find the following sets.



(i) $A - B$

Sol :- $A - B = \{4, 6, 3\}$

(ii) $B - C$

Sol :- $B - C = \{-1, 5, 7\}$

(iii) $A' \cup B'$

Sol :- $A' = \{5, 7, 8, 0, -3, 1, 2\}$

[The Values apart from set A is A']

$B' = \{4, 6, 3, 0, -3, 1, 2\}$

[The Values apart from B is B']

$\therefore A' \cup B' = \{5, 7, 8, 0, -3, 1, 2, 4, 6, 3\}$

(iv) $A' \cap B' = \{5, 7, 8, 0, -3, 1, 2\} \cap \{4, 6, 3, 0, -3, 1, 2\}$

$A' \cap B' = \{0, -3, 1, 2\}$

(v) $(B \cup C)' = \{4, 6, 1, 2\}$

The Values apart from $B \cup C$ is $(B \cup C)'$

(vi) $A - (B \cup C) = \{4, 6\}$

[The Remaining Values in $\underline{\underline{A}}$ after Subtracting $(B \cup C)$]

(vii) $A - (B \cap C) = \{4, 6, 3, -1\}$

[The Remaining Values in A after Subtracting $(B \cap C)$]

(2) If $K = \{a, b, d, e, f\}$, $L = \{b, c, d, g\}$ and $M = \{a, b, c, d, h\}$ then find the following:

(i) $K \cup (L \cap M)$

Sol $(L \cap M) = \{b, c, d, g\} \cap \{a, b, c, d, h\}$

$= \{b, c, d\}$

$K \cup (L \cap M) = \{a, b, d, e, f\} \cup \{b, c, d\}$

$K \cup (L \cap M) = \{a, b, d, e, f, c\}$

(ii) $K \cap (L \cup M)$

Sol:- $L \cup M = \{b, c, d, g\} \cup \{a, b, c, d, h\}$

$= \{b, c, d, g, a, h\}$

$K \cap (L \cup M) = \{a, b, d, e, f\} \cap \{b, c, d, g, a, h\}$

$K \cap (L \cup M) = \{a, b, d\}$

(iii) $(K \cup L) \cap (K \cup M)$

Sol:- $K \cup L = \{a, b, d, e, f\} \cup \{b, c, d, g\}$

$= \{a, b, d, e, f, c, g\}$

$K \cup M = \{a, b, d, e, f\} \cup \{a, b, c, d, h\}$

$= \{a, b, d, e, f, c, h\}$

$(K \cup L) \cap (K \cup M) = \{a, b, d, e, f, c, g\} \cap \{a, b, d, e, f, c, h\}$

$\boxed{(K \cup L) \cap (K \cup M) = \{a, b, d, e, f, c\}}$

(iv) $(K \cap L) \cup (K \cap M)$

Sol:- $K \cap L = \{a, b, d, e, f\} \cap \{b, c, d, g\}$

$K \cap L = \{b, d\}$

$K \cap M = \{a, b, d, e, f\} \cap \{a, b, c, d, h\}$

$K \cap M = \{a, b, d\}$

$(K \cap L) \cup (K \cap M) = \{b, d\} \cup \{a, b, d\}$

$(K \cap L) \cup (K \cap M) = \{b, d, a\}$

Distributive law Satisfies

ie, $K \cup (L \cap M) = (K \cup L) \cap (K \cup M)$

$K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$

(3) If $A = \{x : x \in Z, -2 < x \leq 4\}$
$B = \{x : x \in W, x \leq 5\}$, $C = \{-4, -1, 0, 2, 3, 4\}$, then
Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sol :-

$A = \{-1, 0, 1, 2, 3, 4\}$

$B = \{0, 1, 2, 3, 4, 5\}$

$C = \{-4, -1, 0, 2, 3, 4\}$

LHS :- $A \cup (B \cap C)$

$B \cap C = \{0, 1, 2, 3, 4, 5\} \cap \{-4, -1, 0, 2, 3, 4\}$

$= \{0, 2, 3, 4\}$

$A \cup (B \cap C) = \{-1, 0, 1, 2, 3, 4\} \cup \{0, 2, 3, 4\}$

$A \cup (B \cap C) = \{-1, 0, 1, 2, 3, 4\}$

R.H.S :- $(A \cup B) \cap (A \cup C)$

$A \cup B = \{-1, 0, 1, 2, 3, 4\} \cup \{0, 1, 2, 3, 4, 5\}$

$= \{-1, 0, 1, 2, 3, 4, 5\}$

$A \cup C = \{-1, 0, 1, 2, 3, 4\} \cup \{-4, -1, 0, 2, 3, 4\}$

$= \{-1, 0, 1, 2, 3, 4, -4\}$

$(A \cup B) \cap (A \cup C) = \{-1, 0, 1, 2, 3, 4\}$

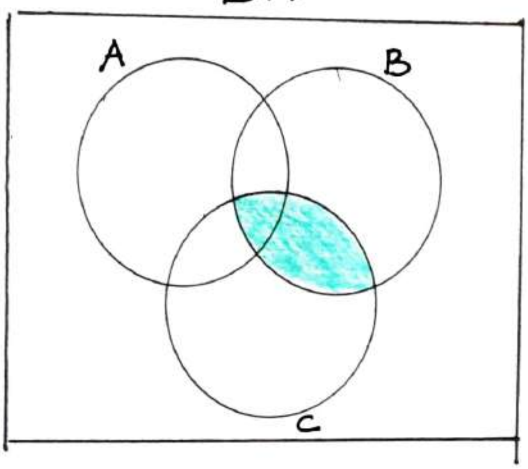
$\therefore$ LHS = RHS.

(4) Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using Venn diagram.

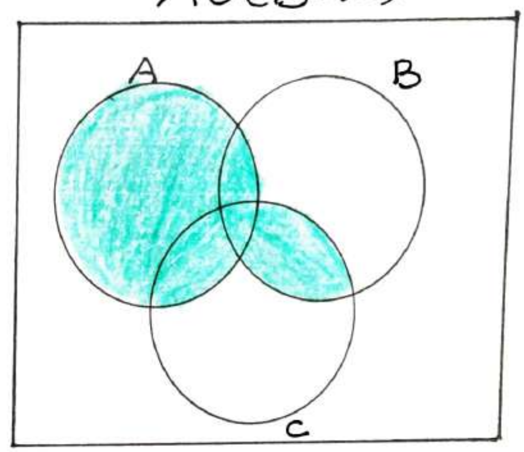
<u>Sol</u>:- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

<u>L.H.S</u> :- $A \cup (B \cap C)$

$B \cap C$

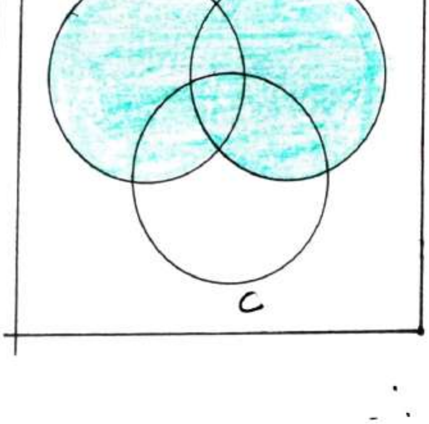


$A \cup (B \cap C)$

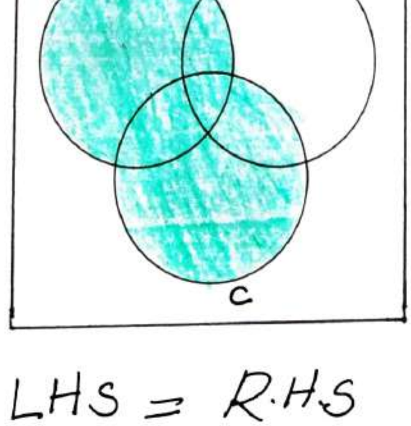


<u>RHS</u> :- $(A \cup B) \cap (A \cup C)$

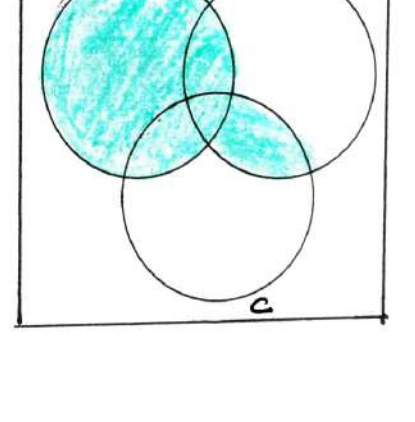
$A \cup B$



$A \cup C$



$(A \cup B) \cap (A \cup C)$



$\therefore$ LHS = R.H.S

(5) If $A = \{b, c, e, g, h\}$, $B = \{a, c, d, g, i\}$ and $C = \{a, d, e, g, h\}$, then show that $A-(B \cap C) = (A-B) \cup (A-C)$

Sol :- $A-(B \cap C) = (A-B) \cup (A-C)$

L.H.S :- $A-(B \cap C)$

$$B \cap C = \{a, c, d, g, i\} \cap \{a, d, e, g, h\}$$

$$= \{a, d, g\}$$

$$A-(B \cap C) = \{b, c, e, \cancel{g}, h\} - \{a, d, \cancel{g}\}$$

$$A-(B \cap C) = \{b, c, e, h\}$$

RHS :- $(A-B) \cup (A-C)$

$$A-B = \{b, \cancel{c}, e, \cancel{g}, h\} - \{a, \cancel{c}, d, \cancel{g}, i\}$$

$$= \{b, e, h\}$$

$$A-C = \{b, c, \cancel{e}, \cancel{g}, \cancel{h}\} - \{a, d, \cancel{e}, \cancel{g}, \cancel{h}\}$$

$$= \{b, c\}$$

$$(A-B) \cup (A-C) = \{b, e, h\} \cup \{b, c\}$$

$$(A-B) \cup (A-C) = \{b, e, h, c\}$$

$\therefore$ LHS = RHS.

(6) If $A = \{x : x = 6n, n \in W \text{ and } n < 6\}$
$B = \{x : x = 2n, n \in N \text{ and } 2 < n \leq 9\}$ and
$C = \{x : x = 3n, n \in N \text{ and } 4 \leq n < 10\}$ then
Show that $A - (B \cap C) = (A - B) \cup (A - C)$

Sol:- For A ; $n \in W, n < 6 \Rightarrow \boxed{n = 0, 1, 2, 3, 4, 5}$

$n = 0 \Rightarrow x = 6(0) = 0$

$n = 1 \Rightarrow x = 6(1) = 6$

$n = 2 \Rightarrow x = 6(2) = 12$

$n = 3 \Rightarrow x = 6(3) = 18$

$n = 4 \Rightarrow x = 6(4) = 24$

$n = 5 \Rightarrow x = 6(5) = 30$

$\therefore \boxed{A = \{0, 6, 12, 18, 24, 30\}}$

For B ; $n \in N, 2 < n \leq 9$
$\Rightarrow \boxed{n = 3, 4, 5, 6, 7, 8, 9}$

$n = 3 \Rightarrow x = 2(3) = 6$

$n = 4 \Rightarrow x = 2(4) = 8$

$n = 5 \Rightarrow x = 2(5) = 10$

$n = 6 \Rightarrow x = 2(6) = 12$

$n = 7 \Rightarrow x = 2(7) = 14$

$n = 8 \Rightarrow x = 2(8) = 16$

$n = 9 \Rightarrow x = 2(9) = 18$

$$\boxed{B = \{6, 8, 10, 12, 14, 16, 18\}}$$

For C

$n \in N,\ 4 \leq n < 10$

$\Rightarrow n = 4, 5, 6, 7, 8, 9$

$n = 4 \Rightarrow x = 3(4) = 12$

$n = 5 \Rightarrow x = 3(5) = 15$

$n = 6 \Rightarrow x = 3(6) = 18$

$n = 7 \Rightarrow x = 3(7) = 21$

$n = 8 \Rightarrow x = 3(8) = 24$

$n = 9 \Rightarrow x = 3(9) = 27$

$$\therefore \boxed{C = \{12, 15, 18, 21, 24, 27\}}$$

$A = \{0, 6, 12, 18, 24, 30\}$

$B = \{6, 8, 10, 12, 14, 16, 18\}$

$C = \{12, 15, 18, 21, 24, 27\}$

$A - (B \cap C) = (A - B) \cup (A - C)$

L.H.S :- $A - (B \cap C)$

$B \cap C = \{6, 8, 10, 12, 14, 16, 18\} \cap \{12, 15, 18, 21, 24, 27\}$

$= \{12, 18\}$

$A - (B \cap C) = \{0, 6, \cancel{12}, \cancel{18}, 24, 30\} - \{\cancel{12}, \cancel{18}\}$

$A - (B \cap C) = \{0, 6, 24, 30\}$

R.H.S :- $(A - B) \cup (A - C)$

$A - B = \{0, \cancel{6}, \cancel{12}, \cancel{18}, 24, 30\} - \{\cancel{6}, 8, 10, \cancel{12}, 14, 16, \cancel{18}\}$

$= \{0, 24, 30\}$

$A - C = \{0, 6, \cancel{12}, \cancel{18}, \cancel{24}, 30\} - \{\cancel{12}, 15, \cancel{18}, 21, \cancel{24}, 27\}$

$= \{0, 6, 30\}$

$(A - B) \cup (A - C) = \{0, 24, 30\} \cup \{0, 6, 30\}$

$(A - B) \cup (A - C) = \{0, 24, 30, 6\}$

L.H.S = RHS

(7) If $A = \{-2, 0, 1, 3, 5\}$, $B = \{-1, 0, 2, 5, 6\}$, and $C = \{-1, 2, 5, 6, 7\}$, then show that

$$A - (B \cup C) = (A - B) \cap (A - C)$$

Sol :- $A - (B \cup C) = (A - B) \cap (A - C)$

LHS :- $A - (B \cup C)$

$$B \cup C = \{-1, 0, 2, 5, 6\} \cup \{-1, 2, 5, 6, 7\}$$

$$= \{-1, 0, 2, 5, 6, 7\}$$

$$A - (B \cup C) = \{-2, \cancel{0}, 1, 3, \cancel{5}\} - \{-1, \cancel{0}, 2, \cancel{5}, 6, 7\}$$

$$A - (B \cup C) = \{-2, 1, 3\}$$

R.H.S :- $(A - B) \cap (A - C)$

$$A - B = \{-2, \cancel{0}, 1, 3, \cancel{5}\} - \{-1, \cancel{0}, 2, \cancel{5}, 6\}$$

$$= \{-2, 1, 3\}$$

$$A - C = \{-2, 0, 1, 3, \cancel{5}\} - \{-1, 2, \cancel{5}, 6, 7\}$$

$$= \{-2, 0, 1, 3\}$$

$$(A - B) \cap (A - C) = \{-2, 1, 3\} \cap \{-2, 0, 1, 3\}$$

$$(A - B) \cap (A - C) = \{-2, -1, 3\}$$

$\therefore$ LHS = RHS.

(8). If $A = \left\{ y : y = \frac{a+1}{2},\ a \in W \text{ and } a \le 5 \right\}$.

$B = \left\{ y : y = \frac{2n-1}{2},\ n \in W \text{ and } n < 5 \right\}$ and

$C = \left\{ -1, -\frac{1}{2}, 1, \frac{3}{2}, 2 \right\}$, then show that

$A - (B \cup C) = (A - B) \cap (A - C)$

Sol:- For A

$$y = \frac{a+1}{2} \ ; \ a \in W,\ a \le 5 \Rightarrow \boxed{a = 0, 1, 2, 3, 4, 5}$$

$a = 0;\ y = \frac{0+1}{2} = \frac{1}{2}$

$a = 1;\ y = \frac{1+1}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$

$a = 2;\ y = \frac{2+1}{2} = \frac{3}{2}$

$a = 3;\ y = \frac{3+1}{2} = \frac{\cancel{4}}{\cancel{2}} = 2$

$a = 4;\ y = \frac{4+1}{2} = \frac{5}{2}$

$a = 5;\ y = \frac{5+1}{2} = \frac{6}{2} = 3$

$\therefore \boxed{A = \left\{ \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3 \right\}}$

For B

$$B = \left\{ y : y = \frac{2n-1}{2};\ n \in W \text{ and } n < 5 \right\}$$

$$y = \frac{2n-1}{2};\ n \in W,\ n < 5 \Rightarrow \boxed{n = 0, 1, 2, 3, 4}$$

$$n = 0,\quad y = \frac{2(0)-1}{2} = \frac{0-1}{2} = -\frac{1}{2}$$

$$n = 1;\quad y = \frac{2(1)-1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

$$n = 2;\quad y = \frac{2(2)-1}{2} = \frac{4-1}{2} = \frac{3}{2}$$

$$n = 3;\quad y = \frac{2(3)-1}{2} = \frac{6-1}{2} = \frac{5}{2}$$

$$n = 4;\quad y = \frac{2(4)-1}{2} = \frac{8-1}{2} = \frac{7}{2}$$

$$\boxed{B = \left\{ \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \right\}}$$

$A - (B \cup C) = (A - B) \cap (A - C)$

LHS :- $A - (B \cup C)$

$B \cup C = \left\{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right\} \cup \left\{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\right\}$

$= \left\{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, -1, 1, 2\right\}$

$A - (B \cup C) = \left\{\cancel{\frac{1}{2}}, \cancel{1}, \cancel{\frac{3}{2}}, \cancel{2}, \cancel{\frac{5}{2}}, 3\right\} - \left\{-\frac{1}{2}, \cancel{\frac{1}{2}}, \cancel{\frac{3}{2}}, \cancel{\frac{5}{2}}, \frac{7}{2}, -1, \cancel{1}, \cancel{2}\right\}$

$\boxed{A - (B \cup C) = \{3\}}$

RHS :- $(A - B) \cap (A - C)$

$(A - B) = \left\{\cancel{\frac{1}{2}}, 1, \cancel{\frac{3}{2}}, 2, \cancel{\frac{5}{2}}, 3\right\} - \left\{-\frac{1}{2}, \cancel{\frac{1}{2}}, \cancel{\frac{3}{2}}, \cancel{\frac{5}{2}}, \frac{7}{2}\right\}$

$= \{1, 2, 3\}$

$(A - C) = \left\{\frac{1}{2}, \cancel{1}, \cancel{\frac{3}{2}}, \cancel{2}, \frac{5}{2}, 3\right\} - \left\{-1, -\frac{1}{2}, \cancel{1}, \cancel{\frac{3}{2}}, \cancel{2}\right\}$

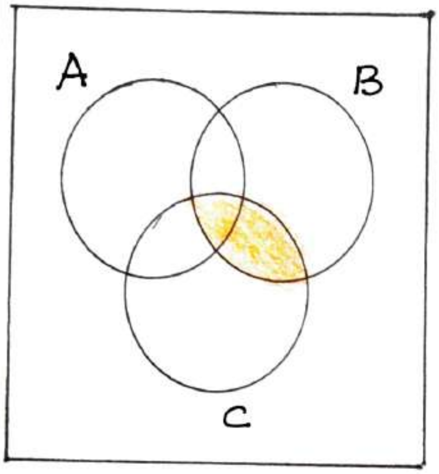
$= \left\{\frac{1}{2}, \frac{5}{2}, 3\right\}$

$(A - B) \cap (A - C) = \{1, 2, 3\} \cap \left\{\frac{1}{2}, \frac{5}{2}, 3\right\}$

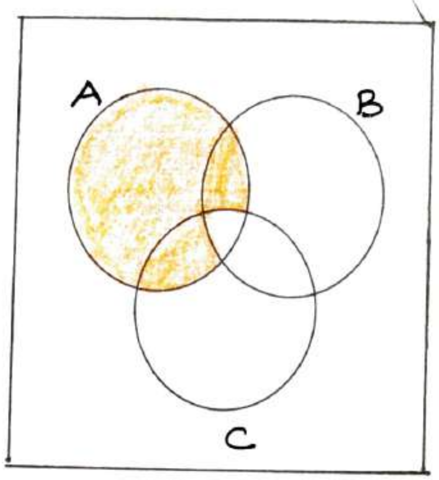
$\boxed{(A - B) \cap (A - C) = \{3\}}$ $\therefore$ $\boxed{LHS = RHS}$

⑨ Verify $A-(B\cap C) = (A-B)\cup(A-C)$ Using Venn diagram:-

LHS:- $A-(B\cap C)$

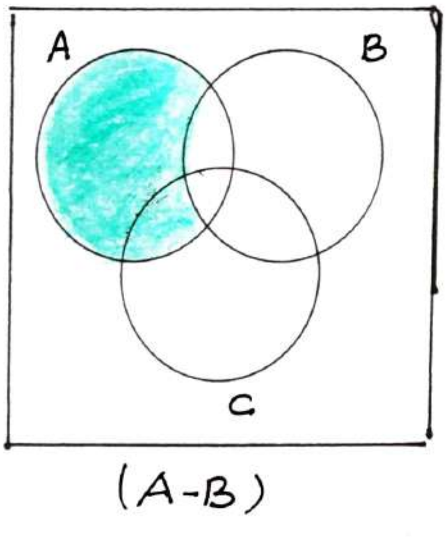


$B\cap C$

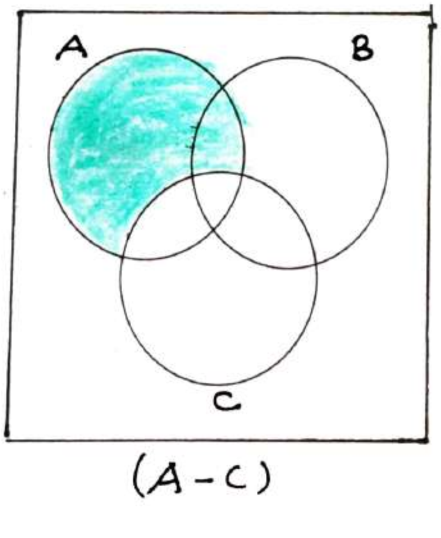


$A-(B\cap C)$

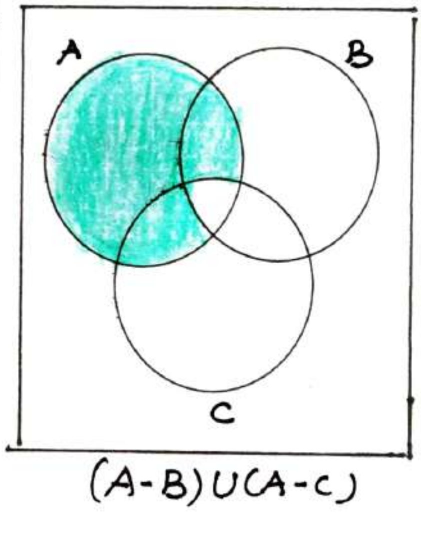
RHS:- $(A-B)\cup(A-C)$



$(A-B)$



$(A-C)$



$(A-B)\cup(A-C)$

$\therefore$ $\boxed{LHS = RHS}$

(10). If $U = \{4, 7, 8, 10, 11, 12, 15, 16\}$, $A = \{7, 8, 11, 12\}$ and $B = \{4, 8, 12, 15\}$ then Verify DeMorgan's Laws for Complementation.

Sol:- Demorgan's laws for Complementation

$$\rightarrow \text{(i)}\ (A \cup B)' = A' \cap B'$$

$$\rightarrow \text{(ii)}\ (A \cap B)' = A' \cup B'$$

(i) $(A \cup B)' = A' \cap B'$

LHS : $(A \cup B)'$

$$A \cup B = \{7, 8, 11, 12\} \cup \{4, 8, 12, 15\}$$

$$= \{7, 8, 11, 12, 4, 15\}$$

$$(A \cup B)' = \{\cancel{4}, \cancel{7}, \cancel{8}, 10, \cancel{11}, \cancel{12}, \cancel{15}, 16\} - \{\cancel{7}, \cancel{8}, \cancel{11}, \cancel{12}, \cancel{4}, \cancel{15}\}$$

$$(A \cup B)' = \{10, 16\}$$

RHS :- $A' \cap B'$

$$A' = \{4, \cancel{7}, \cancel{8}, 10, \cancel{11}, \cancel{12}, 15, 16\} - \{\cancel{7}, \cancel{8}, \cancel{11}, \cancel{12}\}$$

$$= \{4, 10, 15, 16\}$$

$B' = \{\cancel{4},7,\cancel{8},10,11,\cancel{12},\cancel{15},16\} - \{\cancel{4},\cancel{8},\cancel{12},\cancel{15}\}$

$= \{7,10,11,16\}$

$\therefore A' \cap B' = \{4,10,15,16\} \cap \{7,10,11,16\}$

$A' \cap B' = \{10,16\}$

$\therefore \boxed{LHS = RHS}$

(ii) $(A \cap B)' = A' \cup B'$

LHS:- $(A \cap B)'$

$A \cap B = \{7,8,11,12\} \cap \{4,8,12,15\}$

$= \{8,12\}$

$(A \cap B)' = \{4,7,\cancel{8},10,11,\cancel{12},15,16\} - \{\cancel{8},\cancel{12}\}$

$\boxed{(A \cap B)' = \{4,7,10,11,15,16\}}$

RHS:- $A' \cup B'$

$A' = \{4,10,15,16\}$

$B' = \{7,10,11,16\}$

[found in (i) Sum]

$A' \cup B' = \{4,10,15,16\} \cup \{7,10,11,16\}$

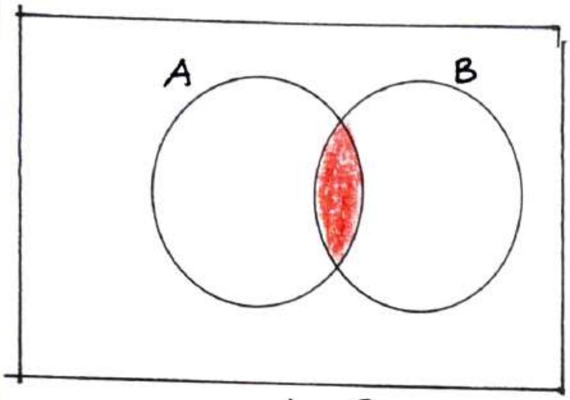
$\boxed{A' \cup B' = \{4,10,15,16,7,11\}}$

$\boxed{LHS = RHS}$

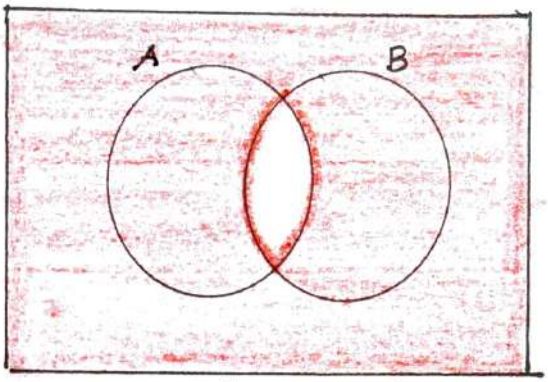
---

(11) Verify $(A \cap B)' = A' \cup B'$ using Venn diagram:-

LHS :- $(A \cap B)'$

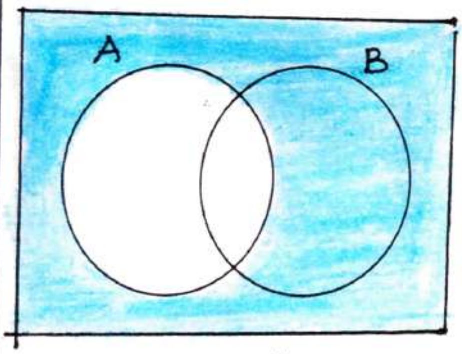


$A \cap B$

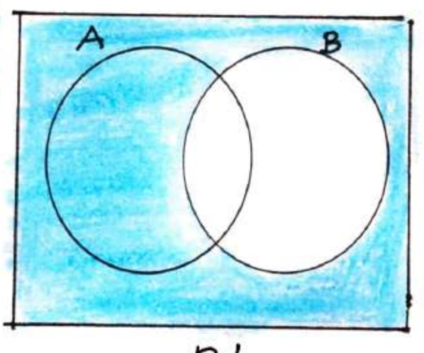


$(A \cap B)'$

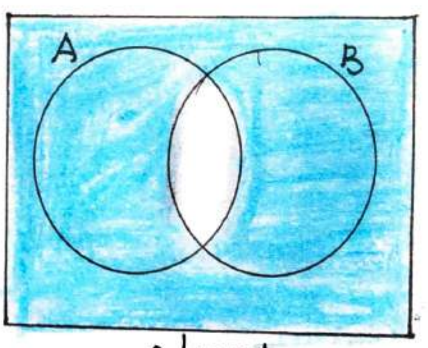
RHS :- $A' \cup B'$



A'



B'



$A' \cup B'$

$$\boxed{L.H.S = R.H.S}$$

Exercise - 1.6

1. (i) If $n(A) = 25$, $n(B) = 40$, $n(A \cup B) = 50$ and $n(B') = 25$, find $n(A \cap B)$ and $n(U)$

<u>Sol</u> :-

* $\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

$$50 = 25 + 40 - n(A \cap B)$$

$$50 = 65 - n(A \cap B)$$

$$n(A \cap B) = 65 - 50$$

$$\boxed{n(A \cap B) = 15}$$

* $n(U) = n(B) + n(B')$

$$= 40 + 25$$

$$\boxed{n(U) = 65}$$

(ii) If $n(A) = 300$, $n(A \cup B) = 500$, $n(A \cap B) = 50$ and $n(B') = 350$, find $n(B)$ and $n(U)$

<u>Sol</u>:- (*) $n(B) = ?$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$500 = 300 + n(B) - 50$$

$$500 = 250 + n(B)$$

$$500 - 250 = n(B)$$

$$\therefore \boxed{n(B) = 250}$$

(*) $n(U) = ?$

$$n(U) = n(B) + n(B')$$

$$= 250 + 350$$

$$\boxed{n(U) = 600}$$

---

2) If $U = \{x : x \in N, x \leq 10\}$, $A = \{2, 3, 4, 8, 10\}$ and $B = \{1, 2, 5, 8, 10\}$, then Verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Sol :- $A = \{2, 3, 4, 8, 10\} \Rightarrow n(A) = 5$

$B = \{1, 2, 5, 8, 10\} \Rightarrow n(B) = 5$

$A \cup B = \{2, 3, 4, 8, 10, 1, 5\} \Rightarrow n(A \cup B) = 7$

$A \cap B = \{2, 8, 10\} \Rightarrow n(A \cap B) = 3$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$7 = 5 + 5 - 3$

$7 = 10 - 3$

$\boxed{7 = 7}$

Hence Verified.

---

(3) Verify $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

for the following sets.

(i) $A = \{a, c, e, f, h\}$

$B = \{c, d, e, f\}$

$C = \{a, b, c, f\}$

Sol:-

$A = \{a, c, e, f, h\} \Rightarrow n(A) = 5$

$B = \{c, d, e, f\} \Rightarrow n(B) = 4$

$C = \{a, b, c, f\} \Rightarrow n(C) = 4$

$A \cap B = \{a, c, e, f, h\} \cap \{c, d, e, f\}$

$A \cap B = \{c, e, f\} \Rightarrow \boxed{n(A \cap B) = 3}$

$B \cap C = \{c, d, e, f\} \cap \{a, b, c, f\}$

$B \cap C = \{c, f\} \Rightarrow \boxed{n(B \cap C) = 2}$

$A \cap C = \{a, c, e, f, h\} \cap \{a, b, c, f\}$

$A \cap C = \{a, c, f\} \Rightarrow \boxed{n(A \cap C) = 3}$

$A \cap B \cap C = \{a, c, e, f, h\} \cap \{c, d, e, f\} \cap \{a, b, c, f\}$

$A \cap B \cap C = \{c, f\} \Rightarrow \boxed{n(A \cap B \cap C) = 2}$

$A \cup B \cup C = \{a, c, e, f, h\} \cup \{c, d, e, f\} \cup \{a, b, c, f\}$

$A \cup B \cup C = \{a, c, e, f, h, d, b\} \Rightarrow \boxed{n(A \cup B \cup C) = 7}$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$7 = 5 + 4 + 4 - 3 - 2 - 3 + 2$

$7 = 15 - 8$

$$\boxed{7 = 7}$$

Hence Verified

(ii) $A = \{1, 3, 5\}$, $B = \{2, 3, 5, 6\}$, $C = \{1, 5, 6, 7\}$

Sol :- $A = \{1, 3, 5\} \Rightarrow n(A) = 3$

$B = \{2, 3, 5, 6\} \Rightarrow n(B) = 4$

$C = \{1, 5, 6, 7\} \Rightarrow n(C) = 4$

$A \cap B = \{1, 3, 5\} \cap \{2, 3, 5, 6\} \rightarrow \{3, 5\} \rightarrow \boxed{n(A \cap B) = 2}$

$B \cap C = \{2, 3, 5, 6\} \cap \{1, 5, 6, 7\} \rightarrow \{5, 6\} \rightarrow \boxed{n(B \cap C) = 2}$

$A \cap C = \{1, 3, 5\} \cap \{1, 5, 6, 7\} \rightarrow \{1, 5\} \rightarrow \boxed{n(A \cap C) = 2}$

$A \cap B \cap C = \{1, 3, 5\} \cap \{2, 3, 5, 6\} \cap \{1, 5, 6, 7\} \rightarrow \{5\}$

$\rightarrow \boxed{n(A \cap B \cap C) = 1}$

$A \cup B \cup C = \{1, 3, 5\} \cup \{2, 3, 5, 6\} \cup \{1, 5, 6, 7\}$

$A \cup B \cup C = \{1, 3, 5, 2, 6, 7\} \rightarrow \boxed{n(A \cup B \cup C) = 6}$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$$
$$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$6 = 3 + 4 + 4 - 2 - 2 - 2 + 1$$

$$6 = 12 - 6$$

$$\boxed{6 = 6}$$

Hence Verified.

---

(4) In a class, all students take part in either music or drama or both. 25 Students take part in music, 30 Students take part in drama and 8 Students take part in both music and drama. Find.

(i) The number of Students who Take part in Only music.

(ii) The number of Students who take part in Only drama.

(iii) The total number of students in the class.

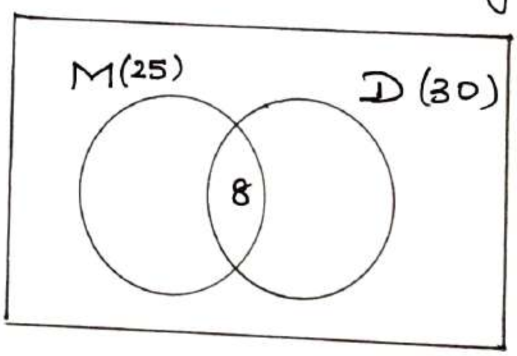
Sol :- Given :-

Number of students take part in Music $= 25$

Number of Students take part in Drama $= 30$

Number of students take part in both $= 8$

(i) No of Students take part in Only Music



$\Rightarrow$ $25 - 8$

$\Rightarrow$ 17

(ii) No of Students take part in Only Drama

$\Rightarrow$ $30 - 8$

$\Rightarrow$ 22

(iii) Total number of Students

$\Rightarrow$ $17 + 8 + 22$

$\Rightarrow$ 47

---

(5) In a party of 45 People, each One likes tea or coffee or both. 35 People like tea and 20 people like coffee. Find the number of people who (i) like both tea and coffee

(ii) do not like Tea.

(iii) do not like Coffee.

Sol :- Given :- $n(U) = n(T \cup C) = 45$

Total number of People $\Rightarrow \boxed{n(T \cup C) = 45}$

Number of People like Tea $\Rightarrow \boxed{n(T) = 35}$

Number of People like Coffee $\Rightarrow \boxed{n(C) = 20}$

(i) No of People who like both tea and coffee $\Rightarrow n(T \cap C) = ?$

Sol:-

$$n(T \cup C) = n(T) + n(C) - n(T \cap C)$$

$$45 = 35 + 20 - n(T \cap C)$$

$$45 = 55 - n(T \cap C)$$

$$n(T \cap C) = 55 - 45$$

$$\boxed{n(T \cap C) = 10}$$

$\therefore$ No of people who like both tea and coffee $= 10$

(ii) No of people who do not like Tea

$\Rightarrow n(T') = ?$

Sol:-

$$n(T') = n(U) - n(T)$$

$$= 45 - 35$$

$$\boxed{n(T') = 10}$$

(iii) No of people who donot like coffee

$\Rightarrow n(C') = ?$

$$n(c') = n(U) - n(c)$$
$$= 45 - 20$$
$$\boxed{n(c') = 25}$$

---

(6). In an examination 50% of the Students passed in Mathematics and 70% of Students passed in Science while 10% Students failed in both Subjects. 300 Students passed in both the Subjects. Find the total number of Students who appeared in the examination, If they took the examination in Only two Subjects.

Sol:- Given :-

$$\left.\begin{array}{r}\text{Percentage of Students passed in}\\ \text{Mathematics}\end{array}\right\} = 50\%$$

Percentage of students passed in Science $\Big\} = 70\%$

Percentage of students failed in both Subjects $\Big\} = 10\%$

Number of students passed in both the Subjects $\Big\} = 300$

$\therefore$ % of students failed in Maths $\Big\} = 100\% - 50\%$

$n(M) = 50\%$

% of students failed in Science $\Big\} = 100\% - 70\%$

$n(S) = 30\%$

$\therefore n(M \cup S) = n(M) + n(S) - n(M \cap S)$

$= 50\% + 30\% - 10\%$

$= 70\%$

$\therefore$ % of students passed in at least one Subject $\Big\} = 100\% - 70\%$

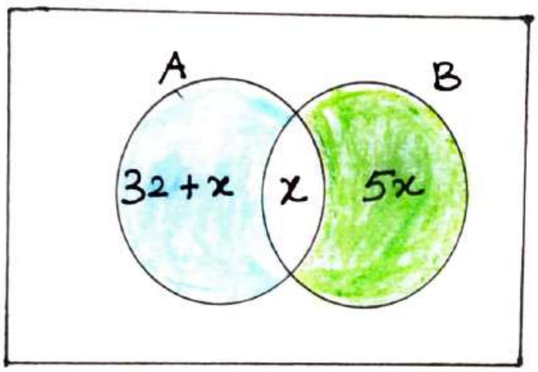
$= 30\% = 300$

$\therefore 100\% = \dfrac{100 \times \cancel{300}^{10}}{\cancel{30}} = 1000$

$\therefore$ No of students appeared the Examination $\Big\} = 1000$

(7) A and B are two sets such that $n(A-B) = 32+x$; $n(B-A) = 5x$ and $n(A \cap B) = x$. Illustrate the information by means of a Venn diagram.
Given that $n(A) = n(B)$, calculate the value of x.

Sol:-



From the Venn diagram

$n(A) = 32 + x + x \Rightarrow 32 + 2x$

$n(B) = x + 5x \Rightarrow 6x$

Given:- $n(A) = n(B)$

$32 + 2x = 6x$

$32 = 6x - 2x$

$$32 = 4x$$

$$4x = 32$$

$$x = \frac{32}{4}$$

$$\boxed{x = 8}$$

$\therefore \boxed{n(A \cap B) = 8}$

---

(8) Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?

Sol :- Given :-

No of Owners of Car A $\Rightarrow n(A) = 400$

No of Owners of Car B $\Rightarrow n(B) = 200$

No of Owners of both cars $\Rightarrow n(A \cap B) = 50$

Total no of Owners investigated

$\Rightarrow n(A \cup B) = 500$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$500 = 400 + 200 - 50$$

$$500 = 600 - 50$$

$$500 \neq 550$$

$\therefore$ The Given data is incorrect.

---

(9) In a colony, 275 families buy Tamil newspaper, 150 families buy English newspaper, 45 families buy Hindi newspaper, 125 families buy Tamil and English newspapers, 17 families buy English and Hindi newspapers, 5 families buy Tamil and Hindi newspapers and 3 families buy all the three newspapers. If each family buy atleast One of these newspapers then find

(i) Number of families buy Only One newspaper

(ii) Number of families buy atleast two newspapers.

(iii) Total number of families in the colony..

Sol: - Given:

No of families buy

Tamil newspaper $\Rightarrow n(A) = 275$

English newspaper $\Rightarrow n(B) = 150$

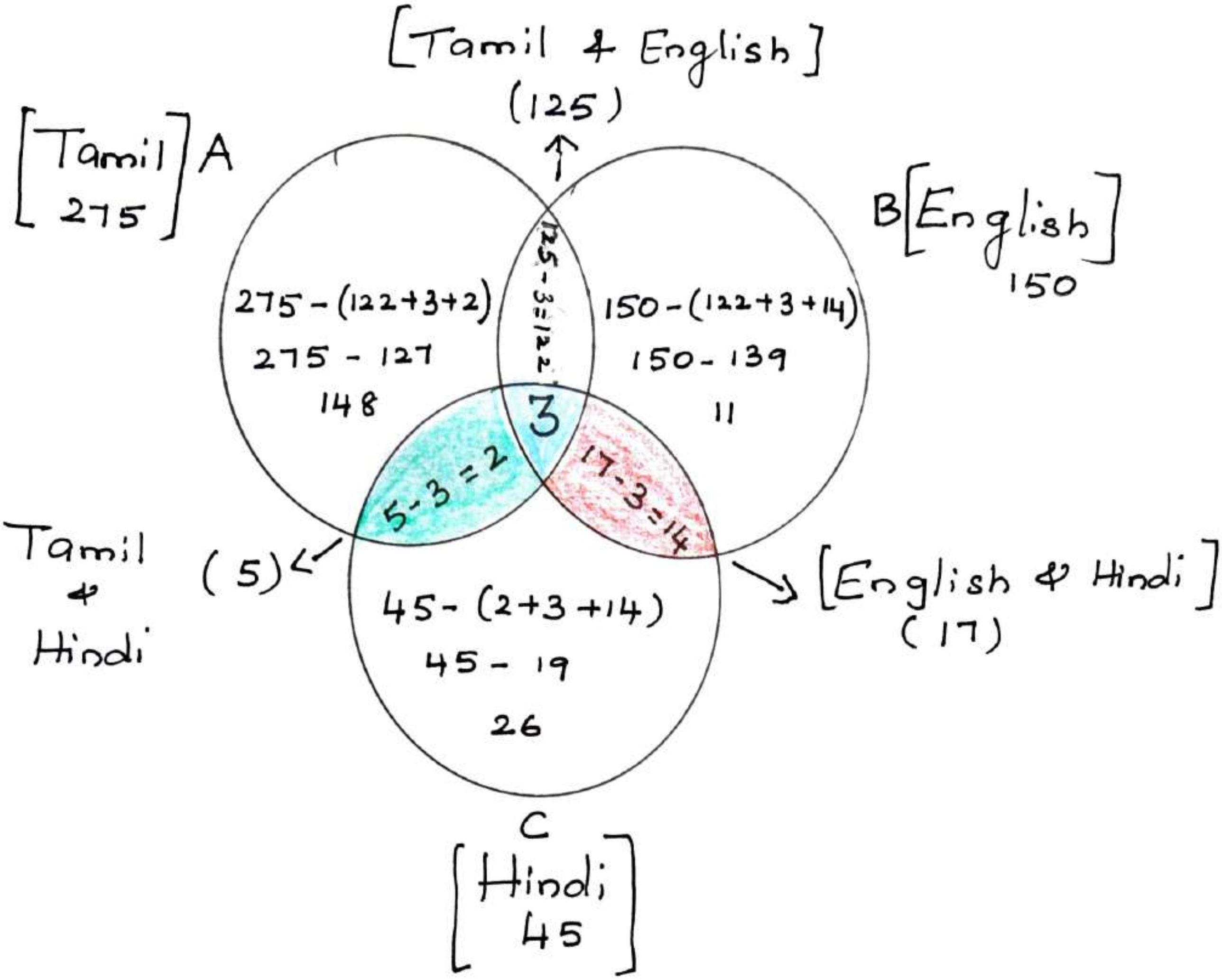
Hindi newspaper $\Rightarrow n(C) = 45$

Tamil and English newspaper $\Rightarrow n(A \cap B) = 125$

English and Hindi newspaper $\Rightarrow n(B \cap C) = 17$

Tamil and Hindi newspaper $\Rightarrow n(A \cap C) = 5$

Tamil, English and Hindi newspaper $\Rightarrow n(A \cap B \cap C) = 3$



(i) Number of families buy Only One news paper.

Sol:- 148 + 11 + 26

➡ 185

(ii) Number of families buy atleast two newspaper.

122 + 14 + 2 + 3

➡ 141

(iii) Total number of families in the colony

148 + 122 + 11 + 14 + 26 + 2 + 3

➡ 326

---

(10) A Survey of 1000 farmers found that 600 grew paddy, 350 grew ragi, 280 grew Corn, 120 grew paddy and ragi, 100 grew ragi and Corn, 80 grew paddy and Corn. If each farmer grew atleast any One of the above three, then find the number of farmers who grew all the three.

Sol:-

No. of farmers Surveyed $\Rightarrow n(A \cup B \cup C) = 1000$

No. of farmers grew paddy $\Rightarrow n(A) = 600$

No. of farmers grew ragi $\Rightarrow n(B) = 350$

No. of farmers grew corn $\Rightarrow n(C) = 280$

$\underset{\because}{\underline{No}}$ of farmers grew

Paddy and ragi $\Rightarrow n(A \cap B) = 120$

ragi and corn $\Rightarrow n(B \cap C) = 100$

Paddy and corn $\Rightarrow n(A \cap C) = 80$

$\therefore$ $\underline{No}$ of farmers grew all three $\Rightarrow n(A \cap B \cap C) = ?$

$$\boxed{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)}$$

$$1000 = 600 + 350 + 280 - 120 - 100 + 80 + n(A \cap B \cap C)$$

$$1000 = 1230 - 300 + n(A \cap B \cap C)$$

$$1000 = 930 + n(A \cap B \cap C)$$

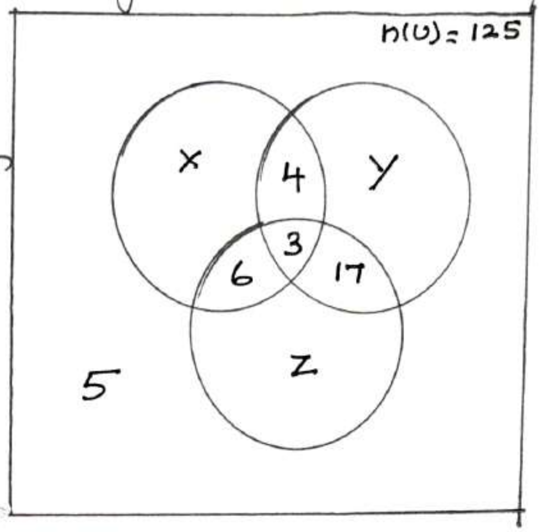
$$1000 - 930 = n(A \cap B \cap C)$$

$$70 = n(A \cap B \cap C)$$

$$\boxed{n(A \cap B \cap C) = 70}$$

$\therefore$ $\underline{No}$ of farmers grew all three $= 70$

(11) In the adjacent diagram, if $n(U)=125$, y is two times of x and z is 10 more than x, then find the value of x, y, and z.



Sol:- Given:-

$$n(U) = 125$$

y is two times of x → $\boxed{y = 2x}$

z is 10 more than x → $\boxed{z = x+10}$

$$x+y+z+\overbrace{4+17+6+3+5} = n(U)$$

$$x+2x+x+10+35 = 125$$

$$4x+45 = 125$$

$$4x = 125-45$$

$$4x = 80$$

$$x = \frac{80}{4}$$

$$\boxed{x = 20}$$

$$y = 2x = 2(20) = 40$$
$$z = x + 10 = 20 + 10 = 30$$

$$\therefore \boxed{\begin{array}{l} x = 20 \\ y = 40 \\ z = 30 \end{array}}$$

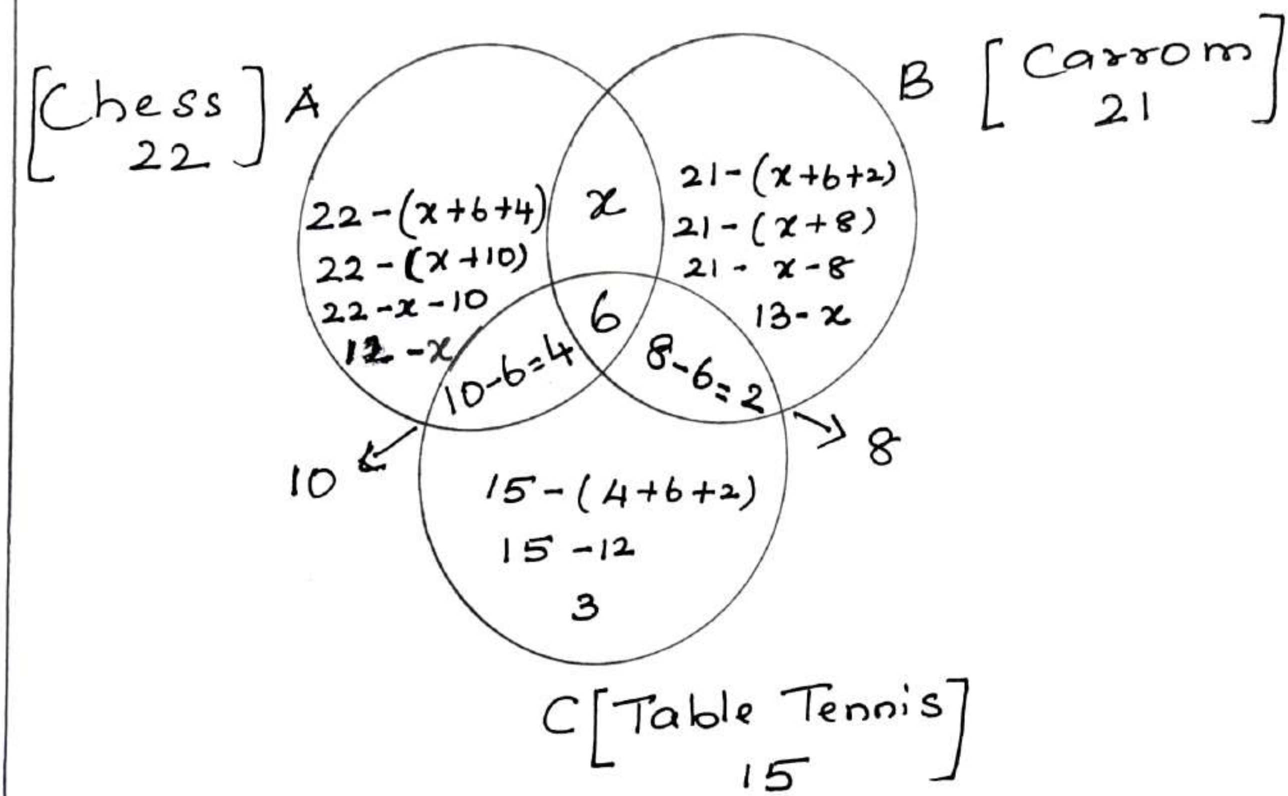
---

(12) Each Student in a class of 35 plays atleast One game among Chess, Carrom and table tennis. 22 play Chess, 21 play Carrom, 15 play table tennis, 10 play Chess and table tennis, 8 play Carrom and table tennis and 6 play all the three games. Find the number of students who play

(i) Chess and Carrom but not table tennis.

(ii) Only Chess

(iii) Only Carrom. [Hint: Use Venn diagram]



(i) No of Students play Chess and Carrom but not table tennis

Sol: $\Rightarrow x$

We know, $n(U) = 35$

$\therefore 12 - x + \cancel{x} + 13 - \cancel{x} + 2 + 3 + 4 + 6 = 35$

$40 - x = 35$

$40 - 35 = x$

$\therefore \boxed{x = 5}$

$\therefore$ Students play chess and Carrom, not Table tennis = 5

(ii) Student play Only Chess.

$12 - x$

$12 - 5$

$\Rightarrow 7$

(iii) Student play Only carrom

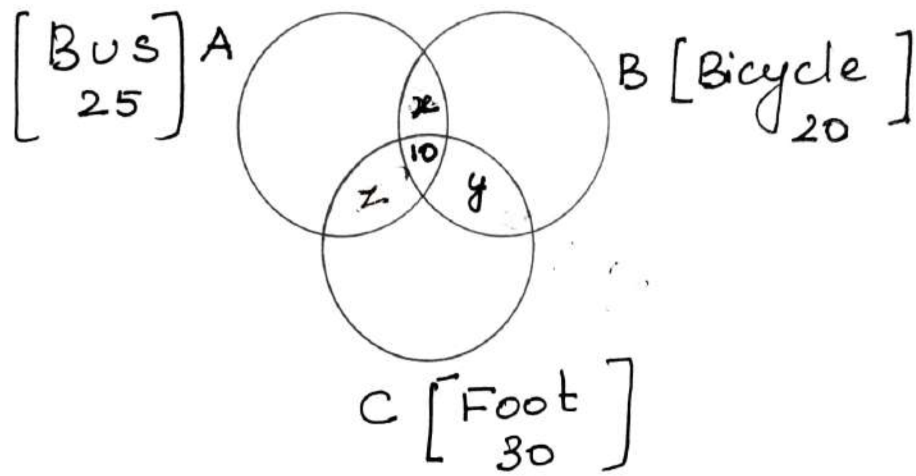
$13 - x$

$13 - 5$

$\Rightarrow 8$

---

13) In a Class of 50 Students, each One come to school by bus or by bicycle Or on foot. 25 by bus, 20 by bicycle, 30 on foot and 10 Students by all the three. Now how many Students Come to School exactly by two modes of transport ?



<u>Sol</u> :-

$$n(A \cup B \cup C) = 50$$

No. of students come by bus $\Rightarrow n(A) = 25$

No. of students come by bicycle $\Rightarrow n(B) = 20$

No. of students come by foot $\Rightarrow n(C) = 30$

No. of students come by all the three $\Big\} \Rightarrow n(A \cap B \cap C) = 10$

$$n(A \cap B) = x + 10$$

$$n(B \cap C) = y + 10$$

$$n(A \cap C) = z + 10$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$50 = 25 + 20 + 30 - (x+10) - (y+10) - (z+10) + 10$$

$$50 = 85 - x - 10 - y - 10 - z - 10$$

$$50 = 85 - 30 - (x+y+z)$$

$$50 = 55 - (x+y+z)$$

$$x+y+z = 55 - 50$$

$$\boxed{x+y+z = 5}$$

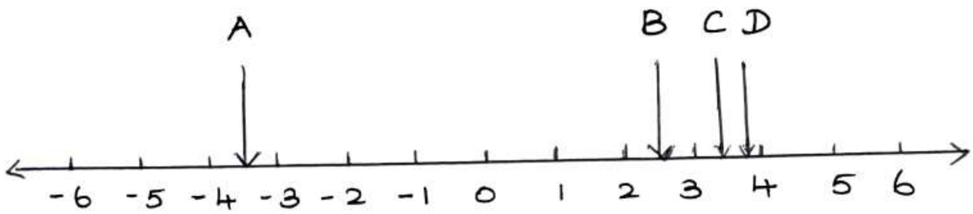
$\therefore$ Total number of students come to school exactly by two modes of transports $= 5$

CHAPTER-2

REAL NUMBERS

EXERCISE 2.1

1) Which arrow best shows the position of $\frac{11}{3}$ on the number line?



$\frac{11}{3} = 3.6666\ldots\ldots$

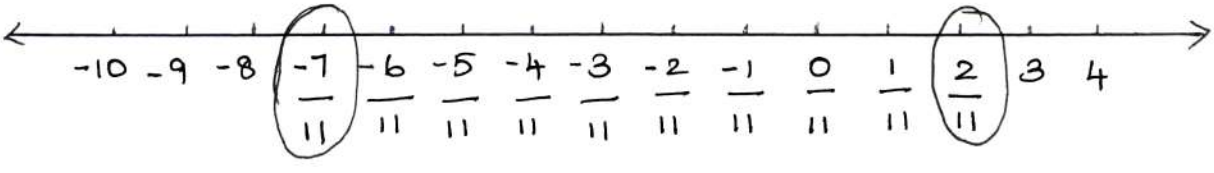
$\frac{11}{3} \simeq 3.7$ (Nearly)

⇒ 'D' arrow best shows the position of $\frac{11}{3}$ on the number line.

$$\begin{array}{r} 3.66\ldots \\ 3\,\overline{)\,11} \\ \underline{9} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ \underline{2} \end{array}$$

---

2) Find any three rational numbers between $\frac{-7}{11}$ and $\frac{2}{11}$



⇒ Three Rational numbers between $-\frac{7}{11}$ and $\frac{2}{11}$ are $\frac{-5}{11}, \frac{-3}{11}$ and $\frac{1}{11}$

---

3) Find any five rational numbers between

(i) $\frac{1}{4}$ and $\frac{1}{5}$

$a = \frac{1}{4}$, $b = \frac{1}{5}$

Five Rational numbers

$\Rightarrow q_1, q_2, q_3, q_4, q_5$

$$\boxed{q_1 = \frac{1}{2}(a+b)}$$

$$= \frac{1}{2}\left(\frac{1}{4} + \frac{1}{5}\right)$$

$$= \frac{1}{2}\left(\frac{1\times5}{4\times5} + \frac{1\times4}{5\times4}\right)$$

$$= \frac{1}{2}\left(\frac{5+4}{20}\right)$$

$$= \frac{1}{2}\left(\frac{9}{20}\right)$$

$$q_1 = \frac{9}{40}$$

$$\boxed{q_2 = \frac{1}{2}(a+q_1)}$$

$$= \frac{1}{2}\left(\frac{1}{4} + \frac{9}{40}\right)$$

$$= \frac{1}{2}\left(\frac{1\times10}{4\times10} + \frac{9\times1}{40\times1}\right)$$

$$= \frac{1}{2}\left(\frac{10+9}{40}\right)$$

$$= \frac{1}{2}\left(\frac{19}{40}\right)$$

$$q_2 = \frac{19}{80}$$

$$\boxed{q_3 = \frac{1}{2}(a+q_2)}$$

$$= \frac{1}{2}\left(\frac{1}{4} + \frac{19}{80}\right)$$

$$= \frac{1}{2}\left(\frac{1\times20}{4\times20} + \frac{19\times1}{80\times1}\right)$$

$$= \frac{1}{2}\left(\frac{20+19}{80}\right)$$

$$= \frac{1}{2}\left(\frac{39}{80}\right)$$

$$q_3 = \frac{39}{160}$$

$$\boxed{q_4 = \frac{1}{2}(a+q_3)}$$

$$= \frac{1}{2}\left(\frac{1}{4} + \frac{39}{160}\right)$$

$$= \frac{1}{2}\left(\frac{1\times 40}{4\times 40} + \frac{39\times 1}{160\times 1}\right)$$

$$= \frac{1}{2}\left(\frac{40+39}{160}\right)$$

$$= \frac{1}{2}\left(\frac{79}{160}\right)$$

$$q_4 = \frac{79}{320}$$

$$\boxed{q_5 = \frac{1}{2}(a+q_4)}$$

$$= \frac{1}{2}\left(\frac{1}{4} + \frac{79}{320}\right)$$

$$= \frac{1}{2}\left(\frac{1\times 80}{4\times 80} + \frac{79\times 1}{320\times 1}\right)$$

$$= \frac{1}{2}\left(\frac{80+79}{320}\right)$$

$$= \frac{1}{2}\left(\frac{159}{320}\right)$$

$$q_5 = \frac{159}{640}$$

$\Rightarrow$ Five Rational numbers are

$$\frac{9}{40}, \frac{19}{80}, \frac{39}{160}, \frac{79}{320} \text{ and } \frac{159}{640}.$$

(ii) 0.1 and 0.11

$\Rightarrow$ 0.100 and 0.110

$\therefore$ Five Rational numbers are

0.102, 0.103, 0.105, 0.107, 0.108

(iii) -1 and -2

$a = -1$, $b = -2$

Five Rational numbers are

q_1, q_2, q_3, q_4, q_5

$$q_1 = \frac{1}{2}(a+b)$$

$$= \frac{1}{2}(-1-2)$$

$$= \frac{1}{2}(-3)$$

$$q_1 = \frac{-3}{2}$$

$$q_2 = \frac{1}{2}(a+q_1)$$

$$= \frac{1}{2}\left(-1-\frac{3}{2}\right)$$

$$= \frac{1}{2}\left(\frac{-2-3}{2}\right)$$

$$= \frac{1}{2}\left(-\frac{5}{2}\right)$$

$$q_2 = \frac{-5}{4}$$

$$q_3 = \frac{1}{2}(a+q_2)$$

$$= \frac{1}{2}\left(-1-\frac{5}{4}\right)$$

$$= \frac{1}{2}\left(\frac{-4-5}{4}\right)$$

$$= \frac{1}{2}\left(-\frac{9}{4}\right)$$

$$q_3 = -\frac{9}{8}$$

$$q_4 = \frac{1}{2}(a+q_3)$$

$$q_4 = \frac{1}{2}\left(-1-\frac{9}{8}\right)$$

$$= \frac{1}{2}\left(\frac{-8-9}{8}\right)$$

$$= \frac{1}{2}\left(\frac{-17}{8}\right)$$

$$q_4 = \frac{-17}{16}$$

$$q_5 = \frac{1}{2}(a+q_4)$$

$$= \frac{1}{2}\left(-1-\frac{17}{16}\right)$$

$$= \frac{1}{2}\left(\frac{-16-17}{16}\right)$$

$$= \frac{1}{2}\left(-\frac{33}{16}\right)$$

$$q_5 = \frac{-33}{32}$$

$\Rightarrow$ Five Rational numbers are

$-\frac{3}{2}, -\frac{5}{4}, -\frac{9}{8}, -\frac{17}{16}$ and $-\frac{33}{32}$

EXERCISE 2.2

1) Express the following rational numbers into decimal and state the kind of decimal expansion.

(i) $\frac{2}{7}$

$$
\begin{array}{r|l}
 & 0.285714 \\
7 & 20 \\
 & \underline{14} \\
 & 60 \\
 & \underline{56} \\
 & 40 \\
 & \underline{35} \\
 & 50 \\
 & \underline{49} \\
 & 10 \\
 & \underline{7} \\
 & 30 \\
 & \underline{28} \\
 & \underline{2}
\end{array}
$$

$\frac{2}{7} = 0.\overline{285714}$

$\Rightarrow$ It is non-terminating and recurring decimal.

(ii) $-5\frac{3}{11}$

$$\boxed{-5\frac{3}{11} = -\frac{58}{11}}$$

$$
\begin{array}{r}
5.2727\ldots \\
11\overline{)58} \\
\underline{55} \\
30 \\
\underline{22} \\
80 \\
\underline{77} \\
30 \\
\underline{22} \\
80 \\
\underline{77} \\
\underline{30}
\end{array}
$$

$-5\frac{3}{11} = -5.\overline{27}$

$\Rightarrow$ It is non-terminating and Recurring decimal.

(iii) $\frac{22}{3}$

$$
\begin{array}{r}
7.33\ldots \\
3\overline{)22} \\
\underline{21} \\
10 \\
\underline{9} \\
10 \\
\underline{9} \\
\underline{10}
\end{array}
$$

$\boxed{\frac{22}{3} = 7.\overline{3}}$

$\Rightarrow$ It is non-terminating and Recurring decimal.

(iv) $\frac{327}{200}$

$$
\begin{array}{r|l}
 & 1.635 \\
200 & 327 \\
 & \underline{200} \\
 & 1270 \\
 & \underline{1200} \\
 & 700 \\
 & \underline{600} \\
 & 1000 \\
 & \underline{1000} \\
 & 0
\end{array}
$$

$$\boxed{\frac{327}{200} = 1.635}$$

⇒ It is a Terminating decimal.

2) Express $\frac{1}{13}$ in decimal form. Find the length of the period of decimal.

$\frac{1}{13}$

$$
\begin{array}{r|l}
 & 0.0769230\cdots \\
13 & 100 \\
 & \underline{91} \\
 & 90 \\
 & \underline{78} \\
 & 120 \\
 & \underline{117} \\
 & 30 \\
 & \underline{26} \\
 & 40 \\
 & \underline{39} \\
 & 100
\end{array}
$$

$$\boxed{\frac{1}{13} = 0.\overline{076923}}$$

⇒ The length of period of decimal is 6.

3) Express the rational number $\frac{1}{33}$ in recurring decimal form by using the recurring decimal expansion of $\frac{1}{11}$. Hence write $\frac{71}{33}$ in recurring decimal form.

$\frac{1}{11}$

$$\begin{array}{r} 0.09090\ldots \\ 11 \overline{)100} \\ 99 \\ \hline 100 \\ 99 \\ \hline 100 \\ \hline \end{array}$$

$$\boxed{\frac{1}{11} = 0.\overline{09}}$$

$\therefore \frac{1}{33} = \frac{1}{3 \times 11} = \frac{0.0909\ldots}{3} = 0.0303\ldots$

$\Rightarrow \frac{1}{33} = 0.\overline{03}$

$$\begin{array}{r} 2 \\ 33 \overline{)71} \\ 66 \\ \hline 5 \\ \hline \end{array}$$

$\frac{71}{33} = 2\frac{5}{33}$

$= 2 + \left(\frac{5}{33}\right)$

$= 2 + \left(5 \times \frac{1}{33}\right)$

$= 2 + (5 \times 0.0303\ldots)$

$= 2 + 0.\overline{15}$

$$\begin{array}{r} 2.00 \\ 0.15 \ (+) \\ \hline 2.15 \\ \hline \end{array}$$

$$\boxed{\frac{71}{33} = 2.\overline{15}}$$

4) Express the following decimal expression into rational numbers.

(i) $0.\overline{24}$

Let $x = 0.242424\ldots \longrightarrow$ ①

Here period of decimal is 2

∴ Multiply by 100 on both sides of ①

$$\begin{aligned} 100x &= 24.242424\ldots \rightarrow ② \\ (-) \quad x &= 0.242424\ldots \rightarrow ① \end{aligned}$$

$\boxed{② - ①}$

$99x = 24$

$x = \frac{24}{99} \Rightarrow \boxed{x = \frac{8}{33}}$ $(\div 3)$

(ii) $2.\overline{327}$

Let $x = 2.327327\ldots \longrightarrow$ ①

Period of decimal is 3

Multiply by 1000 on both sides of ①

$$\begin{aligned} 1000x &= 2327.327327\ldots \rightarrow ② \\ (-) \quad x &= 2.327327\ldots \rightarrow ① \end{aligned}$$

$\boxed{② - ①}$

$999x = 2325$

$x = \frac{2325}{999}$

$x = \frac{775}{333}$ $(\div 3)$

(iii) -5.132

Let $x = -5.132$

$x = -\frac{5132}{1000}$

$\Rightarrow \boxed{x = \frac{-1283}{250}}$ $(\div 4)$

(iv) $3.1\overline{7}$

Let $x = 3.17777\ldots \rightarrow (1)$

Here the repeating digit is '7' which is the second digit after decimal Point.

$\therefore$ Multiply by 10 on b.s of (1)

$10x = 31.7777\ldots \rightarrow (2)$

$\Rightarrow$ Period of decimal is 1

$\therefore$ Multiply by 10 on b.s of (2)

$$\begin{array}{rl} & 100x = 317.7777\ldots \rightarrow (3) \\ (-) & 10x = 31.7777\ldots \rightarrow (2) \\ \hline & 90x = 286 \end{array}$$

$\boxed{(3) - (2)}$

$x = \frac{286}{90}$

$x = \frac{143}{45}$ $(\div 2)$

(v) $17.2\overline{15}$

Let $x = 17.2151515\ldots \rightarrow (1)$

Here the repeating digit is 15 which is the second digit after the decimal point.

Multiply by 10 on b.s of (1)

$$10x = 172.151515\ldots \rightarrow (2)$$

Period of decimal is 2

$\therefore$ Multiply by 100 on b.s of (2)

$$\begin{aligned} 1000x &= 17215.151515\ldots \rightarrow (3) \\ (-)\quad 10x &= 172.151515\ldots \rightarrow (2) \\ \hline 990x &= 17043 \end{aligned}$$

$\boxed{(3)-(2)}$

$$x = \frac{17043}{990}$$

$$\boxed{x = \frac{5681}{330}} \quad (\div 3)$$

(vi) $-21.213\overline{7}$

Let $x = -21.2137777\ldots \rightarrow (1)$

Here the repeating digit is 7 which is the fourth digit after the decimal point.

$\therefore$ Multiply by 1000 on b.s of (1)

$$1000x = -21213.7777\ldots \rightarrow (2)$$

Period of decimal is 1

$\therefore$ Multiply by 10 on b.s of (2)

$$
\begin{aligned}
10000x &= -212137.7777\ldots \rightarrow (3)\\
(-)\quad 1000x &= -\ \ 21213.7777\ldots \rightarrow (2)\\
&\quad (+)
\end{aligned}
$$

(3) - (2)

$$9000x = -190924$$

$$x = \frac{-190924}{9000}$$

$$\boxed{x = \frac{-47731}{2250}} \quad (\div 4)$$

---

5) Without actual division, Find which of the following rational numbers have terminating decimal expansion.

(i) $\frac{7}{128}$

$$
\begin{array}{r|l}
2 & 128 \\
2 & 64 \\
2 & 32 \\
2 & 16 \\
2 & 8 \\
2 & 4 \\
 & 2
\end{array}
$$

$$= \frac{7}{2^7 \times 5^0}$$

This is of the form

$$\frac{p}{2^m \times 5^n}$$

$\Rightarrow$ It is a terminating decimal.

(ii) $\frac{21}{15}$

$$\frac{\cancel{21}^{\,7}}{\cancel{15}_{\,5}} = \frac{7}{5^1 \times 2^0} = \frac{7}{2^0 \times 5^1}$$

This is of the form $\frac{p}{2^m \times 5^n}$

$\Rightarrow$ It is a terminating decimal

(iii) $4\frac{9}{35}$

$$4\frac{9}{35} = \frac{149}{35} = \frac{149}{5 \times 7}$$

This is not in the form $\frac{p}{2^m \times 5^n}$

$\Rightarrow$ It is non-terminating decimal.

(iv) $\frac{219}{2200}$

$$\begin{array}{r|l} 2 & 2200 \\ 2 & 1100 \\ 2 & 550 \\ 5 & 275 \\ 5 & 55 \\ & 11 \end{array}$$

$$\frac{219}{2200} = \frac{219}{2^3 \times 5^2 \times 11}$$

This is not in the form

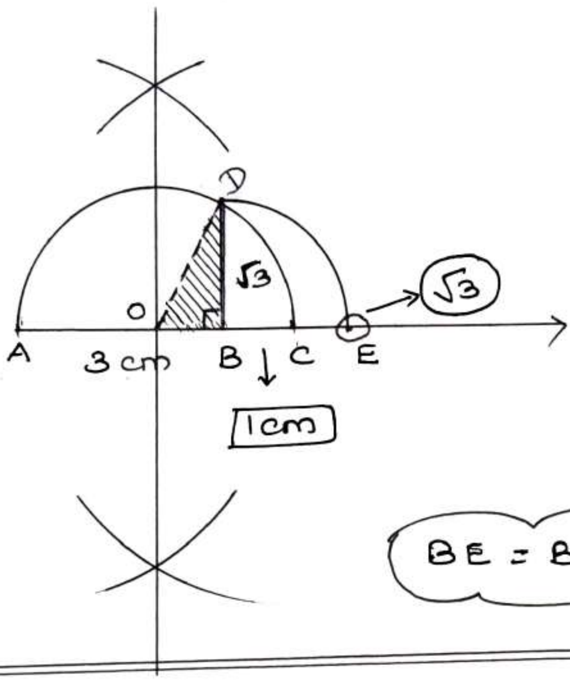
$$\frac{p}{2^m \times 5^n}$$

$\Rightarrow$ It is non-terminating decimal

EXERCISE 2.3

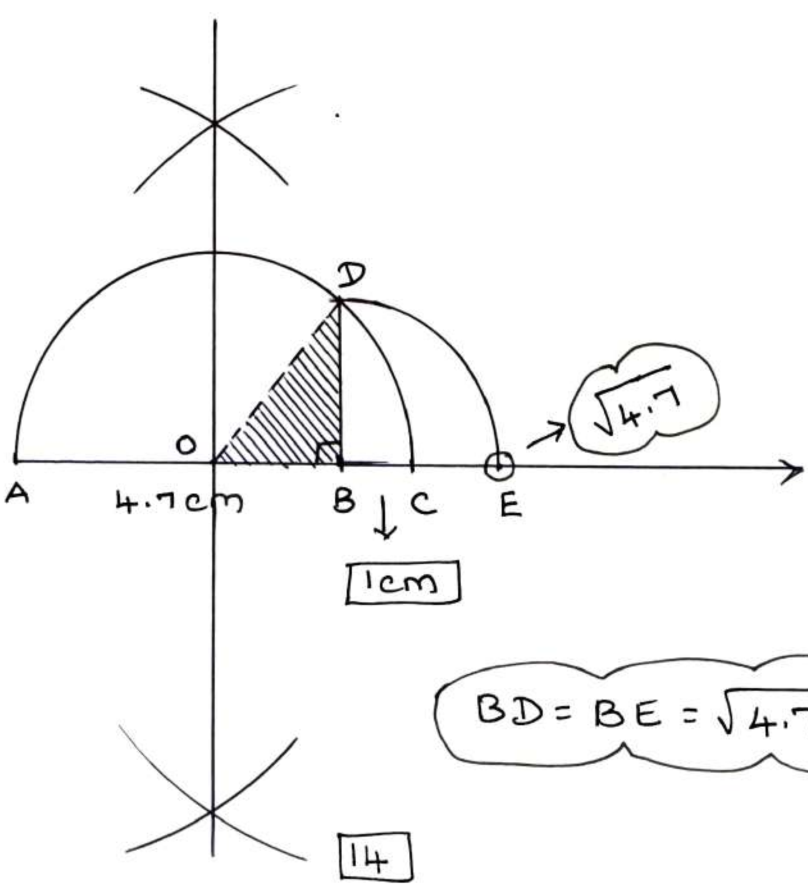
1) Represent the following irrational numbers on the number line.

(i) $\sqrt{3}$



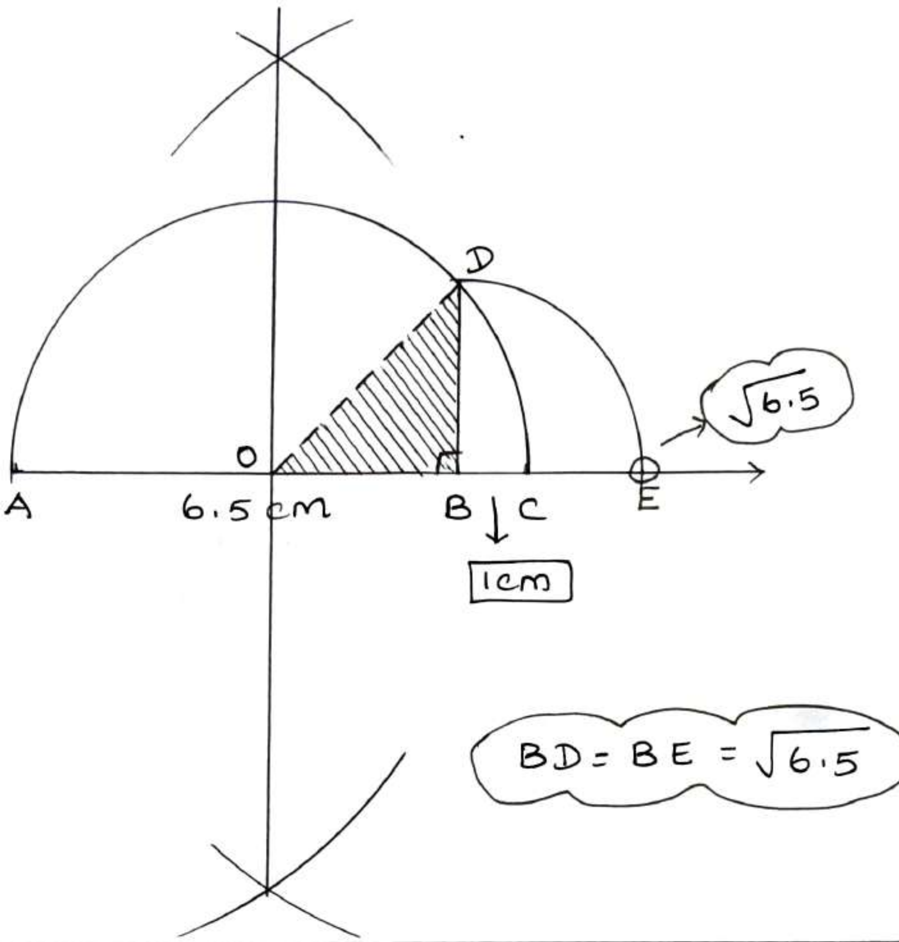
$BE = BD = \sqrt{3}$ cm

(ii) $\sqrt{4.7}$



$BD = BE = \sqrt{4.7}$ cm

(iii) $\sqrt{6.5}$



2) Find any two irrational numbers between

(i) 0.3010011000111 ...... and
0.3020020002 ......

$\therefore$ Two irrational numbers are

0.3011011000011 ..... and
0.3012020002 ....

(ii) $\frac{6}{7}$ and $\frac{12}{13}$

$$
\begin{array}{r|l}
 & 0.85714\ldots \\
7 & 60 \\
 & 56 \\ \hline
 & 40 \\
 & 35 \\ \hline
 & 50 \\
 & 49 \\ \hline
 & 10 \\
 & 7 \\ \hline
 & 30 \\
 & 28 \\ \hline
 & 2
\end{array}
\qquad
\begin{array}{r|l}
 & 0.9230 \\
13 & 120 \\
 & 117 \\ \hline
 & 30 \\
 & 26 \\ \hline
 & 40 \\
 & 39 \\ \hline
 & 100
\end{array}
$$

$\frac{6}{7} = 0.85714\ldots\ldots$

$\frac{12}{13} = 0.9230\ldots\ldots$

$\therefore$ Two irrational numbers are

$0.86714\ldots\ldots, 0.88230\ldots\ldots$

(iii) $\sqrt{2}$ and $\sqrt{3}$

$\sqrt{2} = 1.414\ldots\ldots$

$\sqrt{3} = 1.732\ldots\ldots$

$\therefore$ Two irrational numbers are

$1.514\ldots\ldots, 1.614,\ldots\ldots$

3) Find any two rational numbers between $2.2360679\ldots\ldots$ and $2.236505500\ldots\ldots$

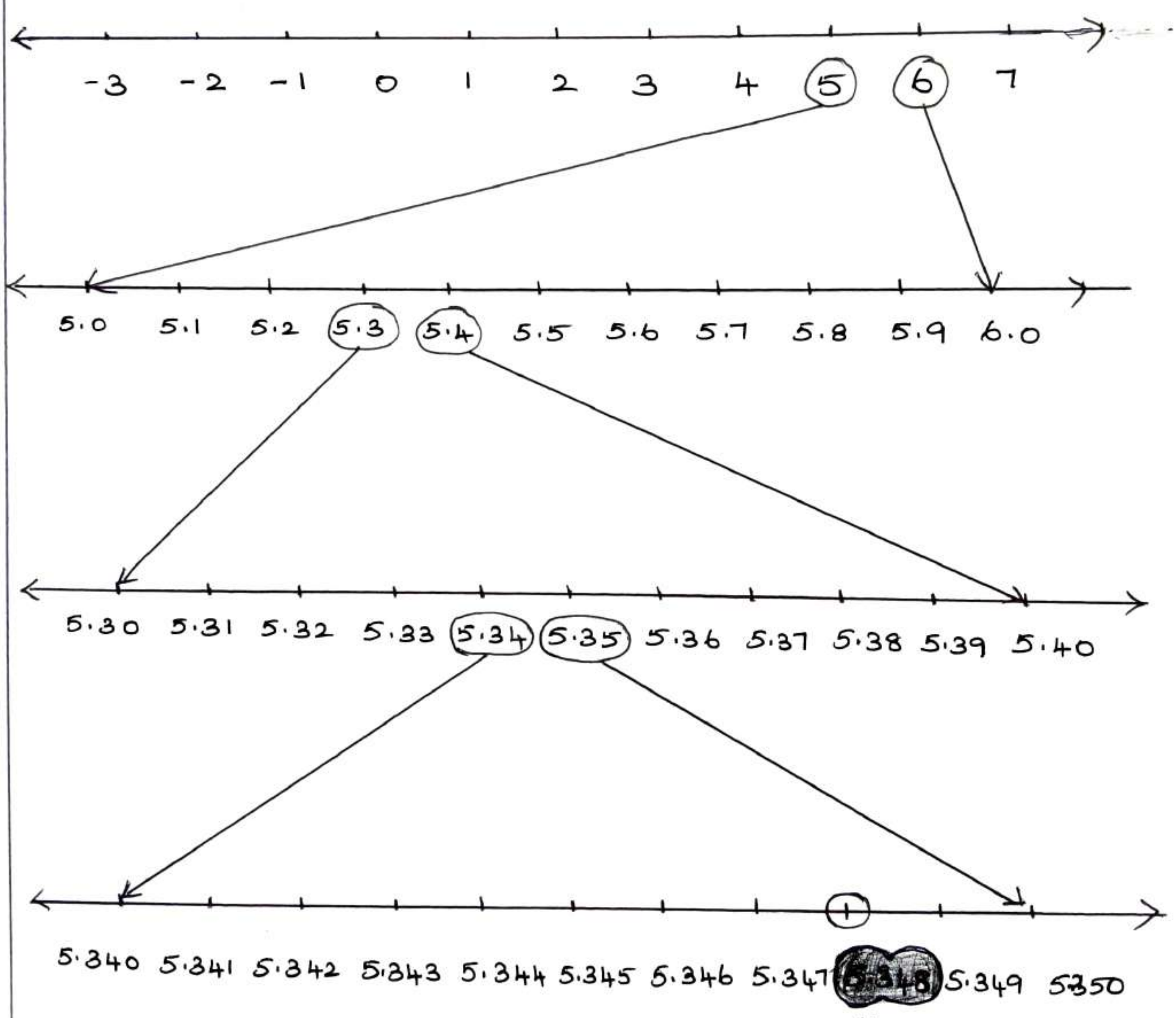
∴ Two Rational numbers are

2.2362 . . . . . . and 2.2364 . . . . .

EXERCISE 2.4

1) Represent the numbers on the number line.

(i) 5.348 ⇒ [Terminating decimal]

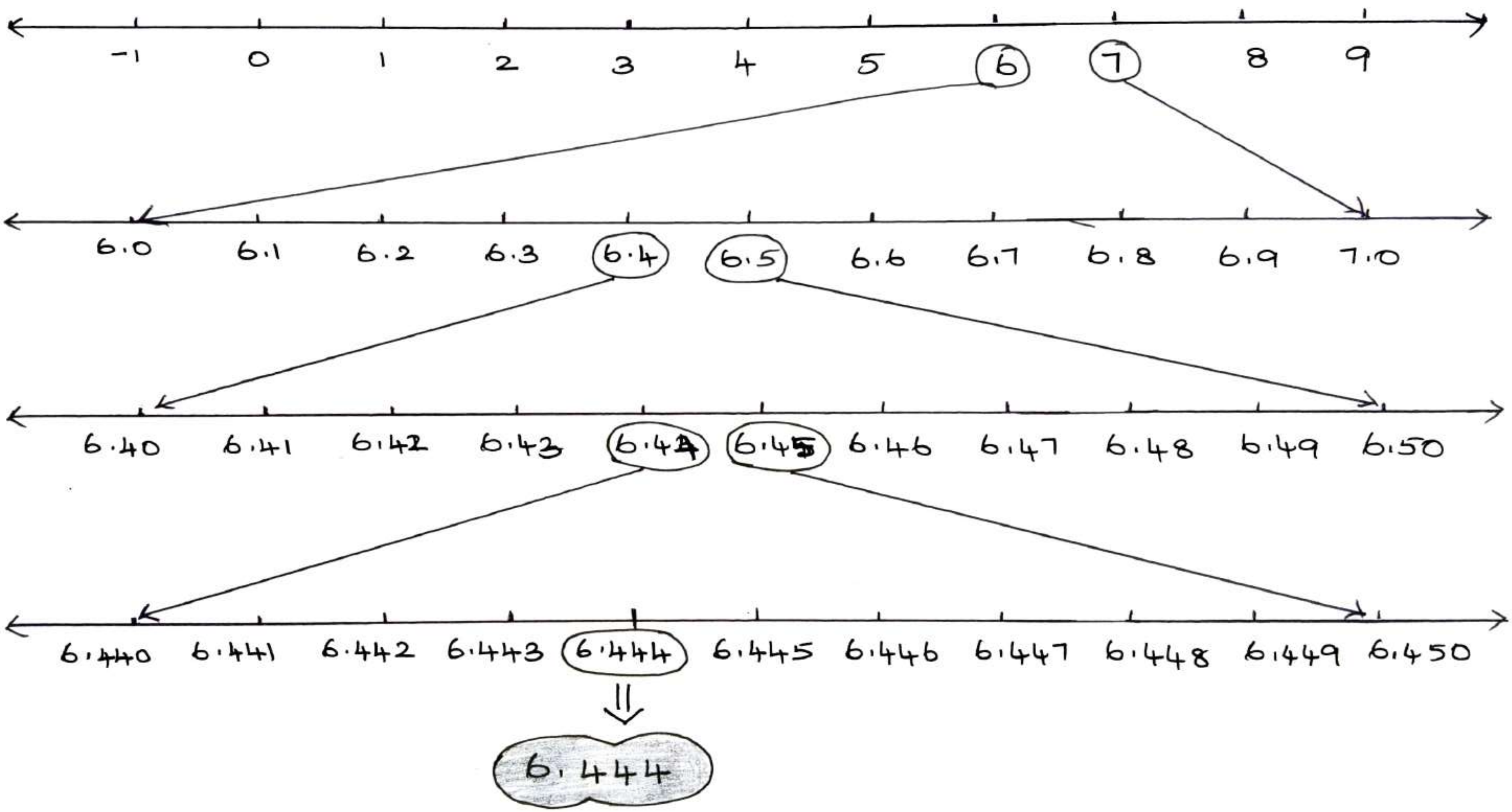


⇓

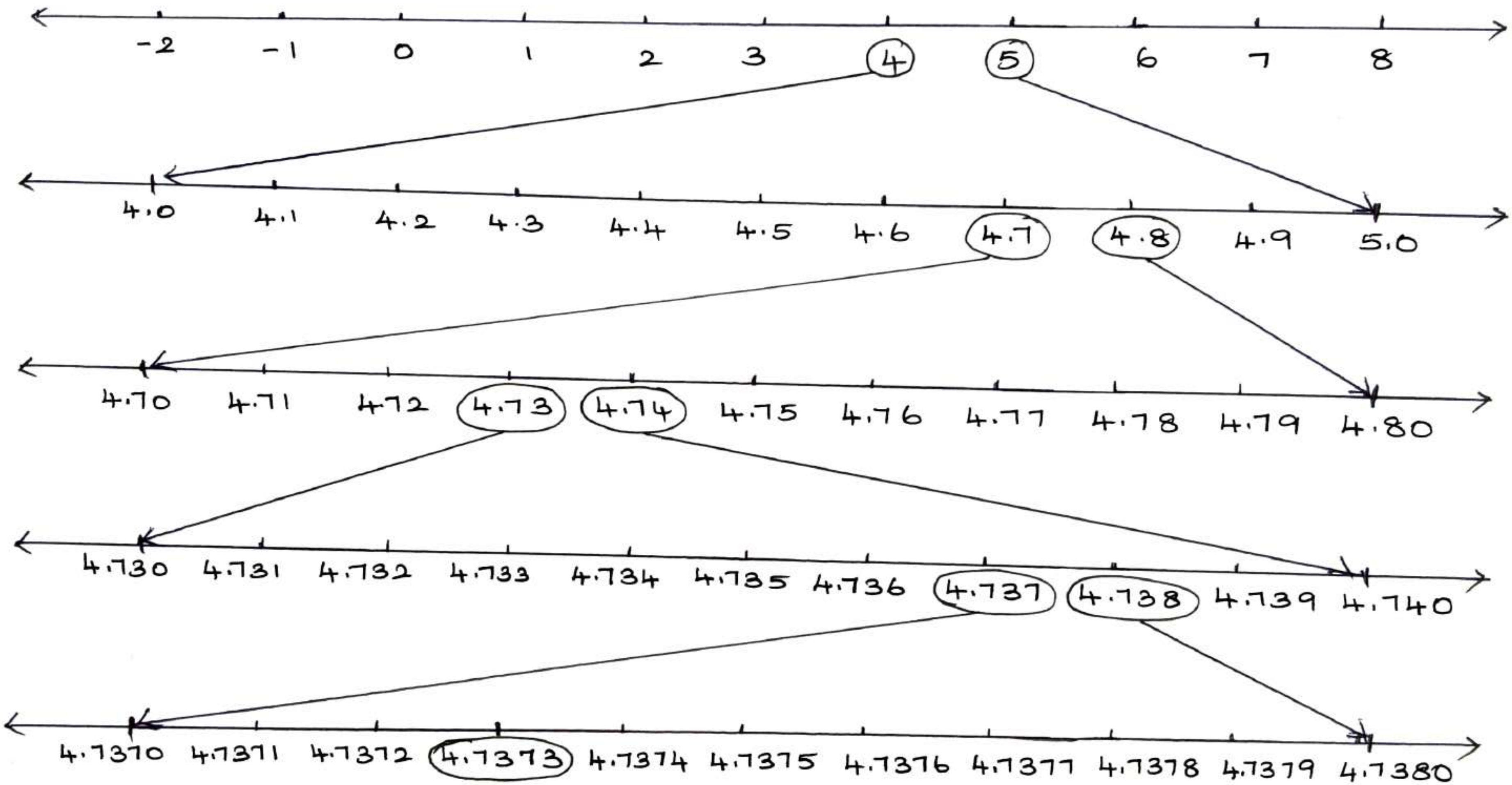
$\boxed{5.348}$

(ii) $6.\overline{4} \Rightarrow 6.444$ (3 decimal places)

[Non-terminating and Recurring decimal]



(iii) $4.\overline{73} \Rightarrow 4.7373$ (4 decimal place) [Non-Terminating Recurring]



$\Downarrow$

$\boxed{4.7373}$

EXERCISE 2.5

1) Write the following in the form of 5^n.

(i) 625

5 | 625
5 | 125
5 | 25
  5

$\Rightarrow 625 = 5^4$

(ii) $\frac{1}{5}$

$\frac{1}{5} = \frac{1}{5^1} = 5^{-1}$

(iii) $\sqrt{5}$

$\boxed{\sqrt{5} = (5)^{1/2}}$

(iv) $\sqrt{125}$

$\sqrt{125} = (125)^{1/2}$

5 | 125
5 | 25
  5

$\therefore (125)^{1/2} = (5^3)^{1/2}$

$= 5^{3 \times \frac{1}{2}}$

$= \boxed{5^{3/2}}$

2) Write the following in the form of 4^n.

(i) 16

$16 = (4)^2$

4 | 16
  4

(ii) 8

$8 = (4)^{3/2}$

1-4 | 8
  2

(iii) 32

$32 = (4)^{5/2}$

1+4 | 32
4 | 8
  2

3) Find the value of:

(i) $(49)^{1/2}$

$(49)^{1/2} = \sqrt{49} = \sqrt{7\times 7} = 7$

(or)

$(49)^{1/2} = (7^2)^{1/2} = 7^{\cancel{2}\times\frac{1}{\cancel{2}}} = 7$

(ii) $(243)^{2/5}$

$$\begin{array}{r|l} 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

$(243)^{2/5} = (3^5)^{2/5}$

$= 3^{\cancel{5}\times\frac{2}{\cancel{5}}}$

$= 3^2$

$\boxed{(243)^{2/5} = 9}$

(iii) $(9)^{-3/2}$

$(9)^{-3/2} = (3^2)^{-\frac{3}{2}} = (3)^{\cancel{2}\times -\frac{3}{\cancel{2}}} = (3)^{-3} = \frac{1}{(3)^3} = \frac{1}{27}$

$\Rightarrow (9)^{-3/2} = \frac{1}{27}$

(iv) $\left(\frac{64}{125}\right)^{-2/3}$

$\left(\frac{64}{125}\right)^{-2/3} = \left(\frac{4^3}{5^3}\right)^{-2/3} = \left(\frac{4}{5}\right)^{\cancel{3}\times -\frac{2}{\cancel{3}}} = \left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{4}\right)^2$

$= \frac{25}{16}$

$\Rightarrow \left(\frac{64}{125}\right)^{-2/3} = \frac{25}{16}$

---

4) Use a fractional index to write:

(i) $\sqrt{5}$

$\sqrt{5} = (5)^{1/2}$

(ii) $\sqrt[2]{7}$

$\sqrt[2]{7} = (7)^{1/2}$

(iii) $\left(\sqrt[3]{49}\right)^5$

$\left(\sqrt[3]{49}\right)^5 = \left[(49)^{1/3}\right]^5 = (49)^{1/3 \times 5} = (49)^{5/3} = (7^2)^{5/3}$

$= (7)^{10/3}$

$\Rightarrow \left(\sqrt[3]{49}\right)^5 = (7)^{10/3}$

(iv) $\left(\frac{1}{\sqrt[3]{100}}\right)^7$

$\left(\frac{1}{\sqrt[3]{100}}\right)^7 = \left[\frac{1}{(100)^{1/3}}\right]^7 = \left[(100)^{-1/3}\right]^7 = (100)^{-\frac{1}{3} \times 7}$

$= (100)^{-\frac{7}{3}}$

$= (10^2)^{-\frac{7}{3}}$

$= (10)^{-\frac{14}{3}}$

$\Rightarrow \left(\frac{1}{\sqrt[3]{100}}\right)^7 = (10)^{-\frac{14}{3}}$

5) Find the 5^{th} root of:

(i) 32

$32 = \sqrt[5]{32} = \sqrt[5]{2\times2\times2\times2\times2} = 2$

$$\begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

(ii) 243

$243 = \sqrt[5]{243} = \sqrt[5]{3\times3\times3\times3\times3} = 3$

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(iii) 100000

$\sqrt[5]{100000} = \sqrt[5]{10\times10\times10\times10\times10} = 10$

(iv) $\frac{1024}{3125}$

$\sqrt[5]{\frac{1024}{3125}} = \sqrt[5]{\frac{4\times4\times4\times4\times4}{5\times5\times5\times5\times5}} = \frac{4}{5}$

$$\begin{array}{r|l} 2 & 1024 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$$\begin{array}{r|l} 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

EXERCISE 2.6

1) Simplify using addition and subtraction properties of surds.

(i) $5\sqrt{3} + 18\sqrt{3} - 2\sqrt{3}$

$= 23\sqrt{3} - 2\sqrt{3}$

$= 21\sqrt{3}$

(ii) $4\sqrt[3]{5} + 2\sqrt[3]{5} - 3\sqrt[3]{5}$

$= 6\sqrt[3]{5} - 3\sqrt[3]{5}$

$= 3\sqrt[3]{5}$

(iii) $3\sqrt{75} + 5\sqrt{48} - \sqrt{243}$

$= 3\sqrt{3 \times 5 \times 5} + 5\sqrt{2 \times 2 \times 2 \times 2 \times 3}$
$- \sqrt{3 \times 3 \times 3 \times 3 \times 3}$

$= (3 \times 5)\sqrt{3} + (5 \times 2 \times 2)\sqrt{3} - (3 \times 3)\sqrt{3}$

$= 15\sqrt{3} + 20\sqrt{3} - 9\sqrt{3}$

$= 35\sqrt{3} - 9\sqrt{3}$

$= 26\sqrt{3}$

$$\begin{array}{r|l} 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$$\begin{array}{r|l} 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

(iv) $5\sqrt[3]{40} + 2\sqrt[3]{625} - 3\sqrt[3]{320}$

$$\begin{array}{r|l} 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

$$\begin{array}{r|l} 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

$$\begin{array}{r|l} 2 & 320 \\ 2 & 160 \\ 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

$= 5\sqrt[3]{2\times2\times2\times5} + 2\sqrt[3]{5\times5\times5\times5}$

$\quad - 3\sqrt[3]{2\times2\times2\times2\times2\times2\times5}$

$= (5\times2)\sqrt[3]{5} + (2\times5)\sqrt[3]{5} - (3\times2\times2)\sqrt[3]{5}$

$= 10\sqrt[3]{5} + 10\sqrt[3]{5} - 12\sqrt[3]{5}$

$= 20\sqrt[3]{5} - 12\sqrt[3]{5}$

$= 8\sqrt[3]{5}.$

2) Simplify the following using multiplication and division property of surds:

(i) $\sqrt{3}\times\sqrt{5}\times\sqrt{2}$

$= \sqrt{3\times5\times2} = \sqrt{30}$

(ii) $\sqrt{35}\div\sqrt{7}$.

$= \dfrac{\sqrt{35}}{\sqrt{7}} = \sqrt{\dfrac{\cancel{35}^{5}}{\cancel{7}}} = \sqrt{5}$

(iii) $\sqrt[3]{27}\times\sqrt[3]{8}\times\sqrt[3]{125}$

$= \sqrt[3]{3\times3\times3}\times\sqrt[3]{2\times2\times2}\times\sqrt[3]{5\times5\times5}$

$= 3\times2\times5$

$= 30$

3 | 27 3 | 9 3	2 | 8 2 | 4 2
5 | 125 5 | 25 5	

(iv) $(7\sqrt{a}-5\sqrt{b})(7\sqrt{a}+5\sqrt{b})$

$a - b \qquad a + b$

We know, $\boxed{(a-b)(a+b) = a^2 - b^2}$

$\Rightarrow (7\sqrt{a})^2 - (5\sqrt{b})^2$

$= (\underbrace{7\times 7}\times \underbrace{\sqrt{a}\times\sqrt{a}}) - (\underbrace{5\times 5}\times \underbrace{\sqrt{b}\times\sqrt{b}})$

$= 49a - 25b$

(v) $\left[\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{144}}\right] \div \left[\sqrt{\frac{16}{81}}\right]$

$= \left[\sqrt{\frac{15\times 15}{27\times 27}} - \sqrt{\frac{5\times 5}{12\times 12}}\right] \div \sqrt{\frac{4\times 4}{9\times 9}}$

$= \left[\frac{\cancel{15}^{5}}{\cancel{27}_{9}} - \frac{5}{12}\right) \div \left(\frac{4}{9}\right)$

$= \left(\frac{5}{9} - \frac{5}{12}\right) \div \left(\frac{4}{9}\right)$

$= \left(\frac{5\times 4}{9\times 4} - \frac{5\times 3}{12\times 3}\right) \div \left(\frac{4}{9}\right)$

$= \left(\frac{20-15}{36}\right) \div \left(\frac{4}{9}\right)$

$= \left(\frac{5}{36}\right) \div \left(\frac{4}{9}\right)$

$= \frac{5}{\cancel{36}_{4}} \times \frac{\cancel{9}}{4}$

$= \frac{5}{16}$

$$\begin{array}{r|l} 3 & 9, 12 \\ 3 & 3, 4 \\ 2 & 1, 4 \\ 2 & 1, 2 \\ & 1, 1 \end{array}$$

L.C.M

$= 3\times 3\times 2\times 2$

$= 36$

3) If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{10} = 3.162$, then find the values of the following. correct it to 3 places of decimals.

(i) $\sqrt{40} - \sqrt{20}$

$= \sqrt{\underbrace{2 \times 2} \times 2 \times 5} - \sqrt{\underbrace{2 \times 2} \times 5}$

$= 2\sqrt{10} - 2\sqrt{5}$

$= 2\left[\sqrt{10} - \sqrt{5}\right]$

$= 2\left[3.162 - 2.236\right]$

$= 2\left[0.926\right]$

$= 1.852$

$$\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array}$$

$$\begin{array}{r} 3.162 \\ 2.236\ (-) \\ \hline 0.926 \end{array}$$

(ii) $\sqrt{300} + \sqrt{90} - \sqrt{8}$

$= \sqrt{\underbrace{2 \times 2} \times 3 \times \underbrace{5 \times 5}} + \sqrt{2 \times \underbrace{3 \times 3} \times 5} - \sqrt{\underbrace{2 \times 2} \times 2}$

$= (2 \times 5)\sqrt{3} + 3\sqrt{10} - 2\sqrt{2}$

$= 10\sqrt{3} + 3\sqrt{10} - 2\sqrt{2}$

$= 10(1.732) + 3(3.162) - (2 \times 1.414)$

$= 17.320 + 9.486 - 2.828$

$= 26.806 - 2.828$

$= 23.978$

$$\begin{array}{r|l} 2 & 300 \\ \hline 2 & 150 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$$\begin{array}{r} 26.806 \\ 2.828\ (-) \\ \hline 23.978 \end{array}$$

4) Arrange in descending order:

(i) $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$

$$\begin{array}{r|l} 2 & 3, 9, 6 \\ \hline 3 & 3, 9, 3 \\ \hline 3 & 1, 3, 1 \\ \hline & 1, 1, 1 \end{array}$$

L.C.M = $2 \times 3 \times 3$ $= 18$

$$\sqrt[3]{5} = \sqrt[18]{(5)^{\cancel{18}^{6}/\cancel{3}}} = \sqrt[18]{(5)^6}$$

$$= \sqrt[18]{15625}$$

$$\sqrt[9]{4} = \sqrt[18]{(4)^{\cancel{18}^{2}/\cancel{9}}} = \sqrt[18]{(4)^2} = \sqrt[18]{16}$$

$$\sqrt[6]{3} = \sqrt[18]{(3)^{\cancel{18}^{3}/\cancel{6}}} = \sqrt[18]{(3)^3} = \sqrt[18]{27}$$

$$\sqrt[18]{15625} > \sqrt[18]{27} > \sqrt[18]{16}$$

$$\Rightarrow \sqrt[3]{5} > \sqrt[6]{3} > \sqrt[9]{4}$$

(ii) $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

$$\begin{array}{r|l} 2 & 6, 12, 4 \\ \hline 2 & 3, 6, 2 \\ \hline 3 & 3, 3, 1 \\ \hline & 1, 1, 1 \end{array}$$

L.C.M $= 2 \times 2 \times 3$ $= 12$

$$\Rightarrow \sqrt[6]{5}, \sqrt[12]{7}, \sqrt[4]{3}$$

$$\sqrt[6]{5} = \sqrt[12]{(5)^{\cancel{12}^{2}/\cancel{6}}} = \sqrt[12]{(5)^2} = \sqrt[12]{25}$$

$$\sqrt[12]{7} = \sqrt[12]{7}$$

$$\sqrt[4]{3} = \sqrt[12]{(3)^{\cancel{12}^{3}/\cancel{4}}} = \sqrt[12]{(3)^3} = \sqrt[12]{27}$$

$$\sqrt[12]{27} > \sqrt[12]{25} > \sqrt[12]{7}$$

$$\Rightarrow \sqrt[4]{3} > \sqrt[6]{5} > \sqrt[12]{7}$$

5) Can you get a pure surd when you find (i) the sum of two surds
(ii) the difference of two surds
(iii) the product of two surds
(iv) the quotient of two surds.
Justify each answer with an example.

(i) $\boxed{\text{yes}}$

example:

$$4\sqrt[3]{21} + (-3\sqrt[3]{21})$$

$$= 4\sqrt[3]{21} - 3\sqrt[3]{21}$$

$$= \sqrt[3]{21}\,[4-3]$$

$$= \sqrt[3]{21}$$

(ii) $\boxed{\text{yes}}$

eg $\Rightarrow$ $7\sqrt[4]{25} - 6\sqrt[4]{25}$

$$= \sqrt[4]{25}\,[7-6]$$

$$= \sqrt[4]{25}$$

(iii) $\boxed{\text{yes}}$

eg $\Rightarrow$ $\sqrt[3]{5} \times \sqrt[3]{4}$

$$= \sqrt[3]{20}$$

(iv) $\boxed{\text{yes}}$

eg $\Rightarrow \dfrac{\sqrt{10}}{\sqrt{2}} = \dfrac{\sqrt{2\times5}}{\sqrt{2}} = \dfrac{\cancel{\sqrt{2}}\times\sqrt{5}}{\cancel{\sqrt{2}}} = \sqrt{5}$

6) Can you get a rational number when you compute

(i) the sum of two surds

(ii) the difference of two surds

(iii) the product of two surds

(iv) the quotient of two surds.

Justify each answer with an example.

(i) $\boxed{\text{Yes}}$

eg $\Rightarrow (5-\sqrt{3})+(5+\sqrt{3})$

$= 5-\cancel{\sqrt{3}}+5+\cancel{\sqrt{3}}$

$=10$, a rational number

(ii) $\boxed{\text{Yes}}$

eg $\Rightarrow (5+\sqrt[3]{7})-(-6+\sqrt[3]{7})$

$=5+\cancel{\sqrt[3]{7}}+6-\cancel{\sqrt[3]{7}}$

$=11$, a rational number

(iii) $\boxed{\text{yes}}$

eg $\Rightarrow (5+\sqrt{3})(5-\sqrt{3})$

$=(5)^2-(\sqrt{3})^2$

$=25-3=22$, a rational number

(iv) $\boxed{\text{Yes}}$

eg $\Rightarrow \dfrac{5\sqrt{3}}{\sqrt{3}} = 5$, a rational number.

EXERCISE 2.7

1) Rationalise the Denominator:

(i) $\dfrac{1}{\sqrt{50}} = \dfrac{1}{\sqrt{2\times \underbrace{5\times 5}}}$

$= \dfrac{1}{5\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{2}}{5\times 2}$

$= \dfrac{\sqrt{2}}{10}$

$$\begin{array}{r|l} 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

(ii) $\dfrac{5}{3\sqrt{5}}$.

$\dfrac{5}{3\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\cancel{5}\sqrt{5}}{3\times \cancel{5}} = \dfrac{\sqrt{5}}{3}$

(iii) $\dfrac{\sqrt{75}}{\sqrt{18}} = \dfrac{\sqrt{3\times 5\times 5}}{\sqrt{2\times 3\times 3}}$

$= \dfrac{5\sqrt{3}}{3\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{5\sqrt{6}}{3\times 2} = \dfrac{5\sqrt{6}}{6}$

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(iv) $\dfrac{3\sqrt{5}}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{\cancel{3}\sqrt{30}}{\cancel{6}\,2} = \dfrac{\sqrt{30}}{2}$

2) Rationalise the Denominator and Simplify:

$2\,\vert\,48$ $2\,\vert\,24$ $2\,\vert\,12$ $2\,\vert\,6$ 3	$2\,\vert\,32$ $2\,\vert\,16$ $2\,\vert\,8$ $2\,\vert\,4$ 2
$3\,\vert\,27$ $3\,\vert\,9$ 3	$2\,\vert\,18$ $3\,\vert\,9$ 3

(i) $\dfrac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}}$

$$= \frac{\sqrt{48}+\sqrt{32}}{\underset{(a\ -\ b)}{\sqrt{27}-\sqrt{18}}} \times \frac{\sqrt{27}+\sqrt{18}}{\underset{(a\ +\ b)}{\sqrt{27}+\sqrt{18}}}$$

$(a+b)(a-b) = a^2-b^2$

$$= \frac{(\sqrt{48}+\sqrt{32})(\sqrt{27}+\sqrt{18})}{(\sqrt{27})^2-(\sqrt{18})^2}$$

$$\frac{\sqrt{48\times27}+\sqrt{48\times18}+\sqrt{32\times27}+\sqrt{32\times18}}{27-18}$$

$$= \frac{\sqrt{2\times2\times2\times2\times3\times3\times3\times3}+\sqrt{2\times2\times2\times2\times3\times2\times3\times3}+\sqrt{2\times2\times2\times2\times2\times3\times3\times3}+\sqrt{2\times2\times2\times2\times2\times2\times3\times3}}{9}$$

$$= \frac{(2\times2\times3\times3)+(2\times2\times3)\sqrt{6}+(2\times2\times3)\sqrt{6}+(2\times2\times3\times3)}{9}$$

$$= \frac{36+12\sqrt{6}+12\sqrt{6}+24}{9}$$

$$= \frac{60+24\sqrt{6}}{9}$$

$$= \frac{\cancel{60}^{20}}{\cancel{9}_{3}} + \frac{\cancel{24}^{8}\sqrt{6}}{\cancel{9}_{3}}$$

$$= \frac{20+8\sqrt{6}}{3}$$

$$= \frac{4(5+2\sqrt{6})}{3}$$

(ii) $\frac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{5\sqrt{3}+\sqrt{2}}{\underset{(a+b)}{\sqrt{3}+\sqrt{2}}} \times \frac{\sqrt{3}-\sqrt{2}}{\underset{(a-b)}{\sqrt{3}-\sqrt{2}}}$$

$$= \frac{(5\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{(5\sqrt{3}\times\sqrt{3}) - (5\sqrt{3}\times\sqrt{2}) + (\sqrt{2}\times\sqrt{3}) - (\sqrt{2}\times\sqrt{2})}{3-2}$$

$$= \frac{(5\times3) - 5\sqrt{6} + \sqrt{6} - 2}{1}$$

$$= 15 - 4\sqrt{6} - 2$$

$$= 13 - 4\sqrt{6}$$

(iii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

$$= \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \times \frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}}$$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5})^2-(2\sqrt{6})^2}$$

$$= \frac{(2\sqrt{6}\times 3\sqrt{5})+(2\sqrt{6}\times 2\sqrt{6})-(\sqrt{5}\times 3\sqrt{5})-(\sqrt{5}\times 2\sqrt{6})}{(9\times 5)-(4\times 6)}$$

$$= \frac{6\sqrt{30}+24-15-2\sqrt{30}}{45-24}$$

$$= \frac{4\sqrt{30}+9}{21}$$

(iv) $\dfrac{\sqrt{5}}{\sqrt{6}+2} - \dfrac{\sqrt{5}}{\sqrt{6}-2}$

$$= \frac{\sqrt{5}(\sqrt{6}-2)-\sqrt{5}(\sqrt{6}+2)}{\underset{a+b}{(\sqrt{6}+2)}\;\underset{a-b}{(\sqrt{6}-2)}}$$

$$= \frac{\cancel{\sqrt{30}}-2\sqrt{5}-\cancel{\sqrt{30}}-2\sqrt{5}}{(\sqrt{6})^2-(2)^2}$$

$$= \frac{-4\sqrt{5}}{6-4} = \frac{-\overset{2}{\cancel{4}}\sqrt{5}}{\cancel{2}} = -2\sqrt{5}$$

---

3) Find the value of 'a' and 'b' if

$$\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$$

L.H.S $\dfrac{\sqrt{7}-2}{\sqrt{7}+2}$

$$= \frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2}$$

$$\boxed{\begin{gathered}(a-b)^2 = a^2+b^2-2ab \\ a^2-b^2 = (a+b)(a-b)\end{gathered}}$$

$$= \frac{(\sqrt{7}-2)^2}{(\sqrt{7})^2-(2)^2}$$

$$= \frac{(\sqrt{7})^2+(2)^2-2\sqrt{7}(2)}{7-4}$$

$$= \frac{7+4-4\sqrt{7}}{3}$$

$$= \frac{11-4\sqrt{7}}{3}$$

$$= \frac{11}{3} - \frac{4\sqrt{7}}{3}$$

$$= -\frac{4\sqrt{7}}{3} + \frac{11}{3} \qquad [R.H.S \Rightarrow a\sqrt{7}+b]$$

$$\Rightarrow a = -\frac{4}{3} \qquad b = \frac{11}{3}$$

---

4) If $x = \sqrt{5}+2$, then find the value of

$x^2 + \frac{1}{x^2}$

$$\boxed{x = \sqrt{5}+2}$$

$$(a+b)^2 = a^2+b^2+2ab$$

$$x^2 = (\sqrt{5}+2)^2$$

$$= (\sqrt{5})^2+(2)^2+2(\sqrt{5})(2)$$

$$= 5+4+4\sqrt{5}$$

$$x^2 = 9+4\sqrt{5}$$

$$\Rightarrow x^2+\frac{1}{x^2}$$

$$=(9+4\sqrt{5})+\frac{1}{(9+4\sqrt{5})}$$

$$=\frac{(9+4\sqrt{5})^2+1}{9+4\sqrt{5}}$$

$$=\frac{(9)^2+(4\sqrt{5})^2+2(9)(4\sqrt{5})+1}{9+4\sqrt{5}}$$

$$=\frac{81+(16\times5)+72\sqrt{5}+1}{9+4\sqrt{5}}$$

$$=\frac{81+80+72\sqrt{5}+1}{9+4\sqrt{5}}$$

$$=\frac{81+81+72\sqrt{5}}{9+4\sqrt{5}}$$

$$=\frac{9\left[9+9+8\sqrt{5}\right]}{9+4\sqrt{5}}$$

$$=\frac{9\,(18+8\sqrt{5})}{9+4\sqrt{5}}$$

$$=\frac{9\times2\,\cancel{(9+4\sqrt{5})}}{\cancel{9+4\sqrt{5}}}$$

$$=9\times2$$

$$=18$$

5) Given $\sqrt{2} = 1.414$. Find the value of $\frac{8-5\sqrt{2}}{3-2\sqrt{2}}$ in 3 decimal places.

$$\frac{8-5\sqrt{2}}{3-2\sqrt{2}} = \frac{8-5\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{(8-5\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$$

$$= \frac{(8\times 3)+(8\times 2\sqrt{2})-(5\sqrt{2}\times 3)-(5\sqrt{2}\times 2\sqrt{2})}{(3)^2-(2\sqrt{2})^2}$$

$$= \frac{24+16\sqrt{2}-15\sqrt{2}-(10\times 2)}{9-(4\times 2)}$$

$$= \frac{24+16\sqrt{2}-15\sqrt{2}-20}{9-8}$$

$$= \frac{24-20+16\sqrt{2}-15\sqrt{2}}{1}$$

$$= \frac{4+\sqrt{2}}{1}$$

$$= \frac{4+1.414}{1} \quad (\text{Given} \Rightarrow \sqrt{2}=1.414)$$

$$= \frac{5.414}{1}$$

$$= 5.414$$

EXERCISE 2.8

1) Represent the following numbers in the Scientific Notation:

(i) 569430000000

$= 5.6943 \times 10^{11}$

(ii) 2000.57

$= 2.00057 \times 10^{3}$

(iii) 0.0000006000

$= 6.0 \times 10^{-7}$

(iv) 0.0009000002

$= 9.000002 \times 10^{-4}$

2) Write the following numbers in decimal form.

(i) 3.459×10^{6}

$= 3.459000$

$= 3459000$

(ii) 5.678×10^{4}

$= 5.6780$

$= 56780$

(iii) 1.00005×10^{-5}

$= 00001.00005$

$= 0.0000100005$

(iv) 2.530009×10^{-7}

$= 0000002.530009$

$= 0.0000002530009$

3) Represent the following numbers in scientific Notation.

(i) $(300000)^2 \times (20000)^4$

$= (3.0 \times 10^5)^2 \times (2.0 \times 10^4)^4$

$= (3)^2 \times 10^{10} \times (2)^4 \times 10^{16}$

$= 9 \times 10^{10} \times 16 \times 10^{16}$

$= 144 \times 10^{26}$

$= 1.44 \times 10^2 \times 10^{26}$

$= 1.44 \times 10^{28}$

(ii) $(0.000001)^{11} \div (0.005)^3$

$= \dfrac{(1.0 \times 10^{-6})^{11}}{(5.0 \times 10^{-3})^3}$

$= \dfrac{(1)^{11} \times 10^{-66}}{(5)^3 \times 10^{-9}}$

$= \dfrac{1}{125} \times 10^{-66+9}$

$= 0.008 \times 10^{-57}$

$$\begin{array}{r} 0.008 \\ 125 \overline{)1000} \\ 1000 \\ \hline 0 \\ \hline \end{array}$$

$$= 8.0 \times 10^{-3} \times 10^{-57}$$

$$= 8.0 \times 10^{-60}$$

(iii) $\{(0.00003)^6 \times (0.00005)^4\} \div \{(0.009)^3 \times (0.05)^2\}$

$$= \frac{(3.0 \times 10^{-5})^6 \times (5.0 \times 10^{-5})^4}{(9.0 \times 10^{-3})^3 \times (5.0 \times 10^{-2})^2}$$

$$= \frac{(3)^6 \times 10^{-30} \times (5)^{\cancel{4}\,2} \times 10^{-20}}{(9)^3 \times 10^{-9} \times \cancel{(5)^2} \times 10^{-4}}$$

$$= \frac{(3)^6 \times 10^{-50} \times (5)^2}{(3^2)^3 \times 10^{-13}}$$

$$= \frac{\cancel{(3)^6} \times 10^{-50} \times 25}{\cancel{(3)^6} \times 10^{-13}}$$

$$= 10^{-50+13} \times 25$$

$$= 25 \times 10^{-37}$$

$$= 2.5 \times 10^{1} \times 10^{-37}$$

$$= 2.5 \times 10^{1-37}$$

$$= 2.5 \times 10^{-36}$$

---

4) Represent the following information in Scientific Notation.

(i) The world population is nearly
7000,000,000

$7000,000,000 = 7.0 \times 10^9$

(ii) One light year means the distance
9460528400000000 km

$= 9.4605284 \times 10^{15}$ km

(iii) mass of an electron is
0.000 000 000 000 000 000 000 000
000 000 9109822 kg

$= 9.109822 \times 10^{-31}$ kg

5) Simplify:

(i) $(2.75 \times 10^7) + (1.23 \times 10^8)$

$2.75 \times 10^7 = 27500000$
$1.23 \times 10^8 = 123000000$ (+)

150500000

$\Rightarrow 1.505 \times 10^8$

(ii) $(1.598 \times 10^{17}) - (4.58 \times 10^{15})$

$\Rightarrow$ 159800000000000000
4580000000000000 (−)

155220000000000000

$\Rightarrow 1.5522 \times 10^{17}$

(iii) $(1.02 \times 10^{10}) \times (1.20 \times 10^{-3})$

$= 1.02 \times 1.20 \times 10^{10-3}$

$= 1.2240 \times 10^{7}$

$$\begin{array}{r} 102 \times 120 \\ \hline 000 \\ 204 \\ 102 \\ \hline 12240 \\ \hline \end{array}$$

(iv) $(8.41 \times 10^{4}) \div (4.3 \times 10^{5})$

$= \dfrac{8.41 \times 10^{4}}{4.3 \times 10^{5}}$

$= \dfrac{8.41 \times 100 \times 10^{4}}{4.3 \times 100 \times 10^{5}}$

$= \dfrac{841}{430} \times 10^{4-5}$

$= \dfrac{841}{43 \times 10^{1}} \times 10^{-1}$

$= \dfrac{841}{43} \times 10^{-1-1}$

$= 19.558 \times 10^{-2}$

$= 1.9558 \times 10^{-2+1}$

$= 1.9558 \times 10^{-1}$

$$\begin{array}{r} 19.558 \\ 43\overline{)841} \\ 43 \\ \hline 411 \\ 387 \\ \hline 240 \\ 215 \\ \hline 250 \\ 215 \\ \hline 350 \\ 344 \\ \hline 6 \\ \hline \end{array}$$

Chapter - 3

Algebra

(1) Which of the following expressions are polynomials. If not give reason:

(i) $\frac{1}{x^2} + 3x - 4$

Sol:- $\frac{1}{x^2} + 3x - 4 \Rightarrow x^{-2} + 3x - 4$

* It is not a polynomial
* One of the power of the Variable is negative

(ii) $x^2(x-1)$

Sol: $x^2(x-1)$

* It is a polynomial.
* Non-negative integral power.

(iii) $\frac{1}{x}(x+5)$

Sol:- $\frac{1}{x}(x+5) \Rightarrow x^{-1}(x+5)$

* It is not a polynomial.
* One of the power of the Variable is negative.

(iv) $\frac{1}{x^{-2}} + \frac{1}{x^{-1}} + 7$

Sol:- $\frac{1}{x^{-2}} + \frac{1}{x^{-1}} + 7 \Rightarrow x^2 + x + 7$

* It is a polynomial.
* Non negative integral power.

(v) $\sqrt{5}x^2 + \sqrt{3}x + \sqrt{2}$

Sol:- $\sqrt{5}\,x^2 + \sqrt{3}x + \sqrt{2}$

* It is a polynomial.
* Non-negative integral power.

(vi) $m^2 - \sqrt[3]{m} + 7m - 10$

Sol:- $m^2 - \sqrt[3]{m} + 7m - 10$

* It is not a polynomial.
* One of the power of Variable is fraction.

---

(2) Write the Coefficient of x^2 and x in each of the following Polynomials.

Sol :-

Polynomial	Coefficient of x^2	coefficient of x
$4+\frac{2}{5}x^2-3x$	$\frac{2}{5}$	-3
$6-2x^2+3x^3-\sqrt{7}x$	-2	$-\sqrt{7}$
πx^2-x+2	π	-1
$\sqrt{3}x^2+\sqrt{2}x+0\cdot5$	$\sqrt{3}$	$\sqrt{2}$
$x^2-\frac{7}{2}x+8$	1	$-\frac{7}{2}$

(3) Find the degree of the following polynomial

Sol :-

Polynomial	Degree
$1-\sqrt{2}y^2+y^7$	7
$\frac{x^3-x^4+6x^6}{x^2} \Rightarrow$	4
$x^3(x^2+x) \Rightarrow$	5
$3x^4+9x^2+27x^6$	6
$2\sqrt{5}p^4-\frac{8p^3}{\sqrt{3}}+\frac{2p^2}{7}$	4

$\Rightarrow (x^3-x^4+6x^6)x^{-2}$

$\Rightarrow x-x^2+6x^4$

$\Rightarrow x^5+x^4$

(4) Rewrite the following polynomial in standard form:-

Sol:-

Polynomial	Standard Form
$x-9+\sqrt{7}x^3+6x^2$	$\sqrt{7}x^3+6x^2+x-9$
$\sqrt{2}x^2-\frac{7}{2}x^4+x-5x^3$	$-\frac{7}{2}x^4-5x^3+\sqrt{2}x^2+x$
$7x^3-\frac{6}{5}x^2+4x-1$	$7x^3-\frac{6}{5}x^2+4x-1$
$y^2+\sqrt{5}y^3-11-\frac{7}{3}y+9y^4$	$9y^4+\sqrt{5}y^3+y^2-\frac{7}{3}y-11$

(5) Add the following polynomials and find the degree of the resultant polynomial.

(i) $p(x)=6x^2-7x+2$, $q(x)=6x^3-7x+15$

Sol:- $p(x)+q(x) =$

$$\begin{array}{r} 6x^2-7x+2 \\ +\ 6x^3+0-7x+15 \\ \hline 6x^3+6x^2-14x+17 \end{array}$$

$\therefore$ Degree = 3

(ii) $h(x) = 7x^3 - 6x + 1$; $f(x) = 7x^2 + 17x - 9$

Sol:-

$$
\begin{array}{rrrrr}
h(x) + f(x) = & 7x^3 & + 0x^2 & - 6x & + 1 \\
 & & 7x^2 & + 17x & - 9 \\
\hline
 & 7x^3 & + 7x^2 & + 11x & - 8 \\
\hline
\end{array}
$$

$\therefore$ Degree = 3

(iii) $f(x) = 16x^4 - 5x^2 + 9$; $g(x) = -6x^3 + 7x - 15$

Sol:-

$$
\begin{array}{rrrrrr}
f(x) + g(x) = & 16x^4 & + 0x^3 & - 5x^2 & + 0x & + 9 \\
 & & - 6x^3 & + 0 & + 7x & - 15 \\
\hline
 & 16x^4 & - 6x^3 & - 5x^2 & + 7x & - 6 \\
\hline
\end{array}
$$

$\therefore$ Degree = 4

---

(6) Subtract the Second polynomial from the first polynomial and find the degree of the resultant polynomial.

(i) $p(x) = 7x^2 + 6x - 1$; $q(x) = 6x - 9$

Sol :-

$$P(x) - q(x) = \begin{array}{r} 7x^2 + \cancel{6x} - 1 \\ \cancel{6x} - 9 \\ - \quad (+) \\ \hline 7x^2 \quad + 8 \\ \hline \end{array}$$

$\therefore$ Degree $= 2$

(ii) $f(y) = 6y^2 - 7y + 2$; $g(y) = 7y + y^3$

$\therefore g(y) = y^3 + 7y$

Sol :-

$$f(y) - g(y) = \begin{array}{r} 0 + 6y^2 - 7y + 2 \\ y^3 + 0 \pm 7y \\ - \qquad\quad - \\ \hline \end{array}$$

$$= \underline{-y^3 + 6y^2 - 14y + 2}$$

$\therefore$ Degree $= 3$

(iii) $h(z) = z^5 - 6z^4 + z$; $f(z) = 6z^2 + 10z - 7$

Sol

$$h(z) - f(z) = z^5 - 6z^4 + 0 + 0 + z + 0$$

$$\underline{\ -6z^2 \underset{-}{+} 10z \underset{+}{-} 7}$$

$$z^5 - 6z^4 + 0 - 6z^2 - 9z + 7$$

$$\Rightarrow z^5 - 6z^4 - 6z^2 - 9z + 7$$

$\therefore$ Degree = 5

---

(7) What should be added to $2x^3 + 6x^2 - 5x + 8$ to get $3x^3 - 2x^2 + 6x + 15$?

Sol :-

Let the added polynomial = A

$$2x^3 + 6x^2 - 5x + 8 + A = 3x^3 - 2x^2 + 6x + 15$$

$$A = (3x^3 - 2x^2 + 6x + 15) - (2x^3 + 6x^2 - 5x + 8)$$

$$= 3x^3 - 2x^2 + 6x + 15 - 2x^3 - 6x^2 + 5x - 8$$

$$\boxed{A = x^3 - 8x^2 + 11x + 7}$$

---

(8) What must be Subtracted from $2x^4+4x^2-3x+7$ to get $3x^3-x^2+2x+1$?

Sol

Let the polynomial to be Subtracted $\Big\}$ = B

$$\therefore 2x^4+4x^2-3x+7-B = 3x^3-x^2+2x+1$$

$$(2x^4+4x^2-3x+7)-(3x^3-x^2+2x+1) = B$$

$$2x^4+4x^2-3x+7-3x^3+x^2-2x-1 = B$$

$$2x^4-3x^3+5x^2-5x+6 = B$$

$$\therefore \boxed{B = 2x^4-3x^3+5x^2-5x+6}$$

---

(9) Multiply the following polynomials and find the degree of the resultant Polynomial.

(i) $P(x) = x^2-9$ $\quad q(x) = 6x^2+7x-2$

Sol:-

$$P(x)\times q(x) = (x^2-9)(6x^2+7x-2)$$

$$= 6x^4+7x^3-2x^2-54x^2-63x+18$$

$$P(x)\times q(x) = 6x^4+7x^3-56x^2-63x+18$$

$\therefore$ Degree = 4

(ii) $f(x) = 7x+2$; $g(x) = 15x-9$

Sol:-

$$f(x)\times g(x) = (7x+2)\times(15x-9)$$

$$= 105x^2-63x+30x-18$$

$$f(x)\times g(x) = 105x^2-33x-18$$

$\therefore$ Degree = 2

(iii) $h(x) = 6x^2-7x+1$; $f(x) = 5x-7$

Sol:-

$$h(x)\times f(x) = (6x^2-7x+1)(5x-7)$$

$$= 30x^3-42x^2-35x^2+49x+5x-7$$

$$h(x)\times f(x) = 30x^3-77x^2+54x-7$$

Degree = 3

(10) The cost of a Chocolate is Rs $(x+y)$ and Amir bought $(x+y)$ Chocolates. Find the total amount paid by him in terms of x and y. If $x=10$, $y=5$ find the amount paid by him.

Sol:-

cost of a chocolate = ₹ $(x+y)$

No of chocolate Amir bought = $(x+y)$

$\therefore$ Total Amount paid by Amir = $(x+y)(x+y)$

$= (x+y)^2$

Given:-
$x = 10$
$y = 5$

$= (10+5)^2$

$= (15)^2$

$\therefore$ Total Amount paid by Amir = ₹ 225

(11) The length of a rectangle is $(3x+2)$ units and its breadth is $(3x-2)$ units. Find its area in terms of x. what will be the area if $x = 20$ units.

Sol

Rectangle

length $= (3x+2)$ units

breadth $= (3x-2)$ units

$a^2-b^2 = (a+b)(a-b)$

$$\begin{aligned}
\text{Area} &= l \times b \\
&= (3x+2)(3x-2) \\
&= (3x)^2 - 2^2 \\
&= 9x^2 - 4 \\
&= 9(20)^2 - 4 \\
&= 9(400) - 4 \\
&= 3600 - 4
\end{aligned}$$

Given

$x = 20$ units

$$\begin{array}{r} 3600 \\ -\ \ \ 4 \\ \hline 3596 \end{array}$$

Area $= 3596$ sq. units

(12) $p(x)$ is a polynomial of degree 1 and $q(x)$ is a polynomial of degree 2. What kind of the polynomial $p(x) \times q(x)$ is ?

Sol :-

$$\text{Degree of } p(x) = 1$$

$$\text{Degree of } q(x) = 2$$

$$\therefore \text{Degree of } p(x) \times q(x) = 3$$

Exercise - 3.2

(1) Find the Value of the polynomial $f(y) = 6y - 3y^2 + 3$ at

(i) $y = 1$

Sol :- $f(y) = 6y - 3y^2 + 3$

$$f(1) = 6(1) - 3(1)^2 + 3$$

$$= 6 - \cancel{3} + \cancel{3}$$

$$\boxed{f(1) = 6}$$

(ii) $y = -1$

Sol :- $f(y) = 6y - 3y^2 + 3$

$f(-1) = 6(-1) - 3(-1)^2 + 3$

$= -6 - 3(1) + 3$

$= -6 - \cancel{3} + \cancel{3}$

$\boxed{f(-1) = -6}$

(iii) $y = 0$

Sol :-

$f(y) = 6y - 3y^2 + 3$

$f(0) = 6(0) - 3(0) + 3$

$= 0 - 0 + 3$

$\boxed{f(0) = 3}$

---

(2) If $P(x) = x^2 - 2\sqrt{2}x + 1$, find $P(2\sqrt{2})$

Sol :- $P(x) = x^2 - 2\sqrt{2}x + 1$ $\quad x = 2\sqrt{2}$

$P(2\sqrt{2}) = (2\sqrt{2})^2 - (2\sqrt{2})(2\sqrt{2}) + 1$

$= 4(2) - (4 \times 2) + 1$

$= \cancel{8} - \cancel{8} + 1$

$\boxed{P(2\sqrt{2}) = 1}$

(3) Find the Zeros of the polynomial in each of the following :-

(i) $p(x) = x - 3$

Sol :- $p(x) = x - 3$

$p(3) = 3 - 3$

$p(3) = 0$

$\therefore x = 3$ is the Zero of $p(x)$

(ii) $p(x) = 2x + 5$

Sol :- $p(x) = 2x + 5$

$p(x) = 2\left(x + \frac{5}{2}\right)$

$p\left(-\frac{5}{2}\right) = 2\left(-\frac{5}{2} + \frac{5}{2}\right)$

$\therefore p\left(-\frac{5}{2}\right) = 0$

$\therefore x = -\frac{5}{2}$ is the Zero of $p(x)$

(iii) $q(y) = 2y - 3$

Sol :- $q(y) = 2y - 3$

$q(y) = 2\left(y - \frac{3}{2}\right)$

$$q\left(\frac{3}{2}\right) = 2\left(\frac{3}{2} - \frac{3}{2}\right)$$

$$q\left(\frac{3}{2}\right) = 2(0)$$

$$q\left(\frac{3}{2}\right) = 0$$

$\therefore$ $y = \frac{3}{2}$ is the Zero of $q(x)$

(iv) $f(z) = 8z$

Sol :-

$f(0) = 8(0)$

$f(0) = 0$

$\therefore$ $z = 0$ is the Zero of $f(z)$.

(v) $p(x) = ax$ when $a \neq 0$

Sol :- $p(x) = ax$

$p(0) = a(0)$

$p(0) = 0$

$x = 0$ is the Zero of $p(x)$

(vi) $h(x) = ax + b$; $a \neq 0$, $a, b \in R$

Sol :- $h(x) = ax + b$

$$h(x) = a\left(x + \frac{b}{a}\right)$$

$$h\left(\frac{-b}{a}\right) = a\left(\frac{-b}{a} + \frac{b}{a}\right)$$

$$h\left(\frac{-b}{a}\right) = a(0)$$

$$h\left(\frac{-b}{a}\right) = 0$$

$\therefore$ $x = -\frac{b}{a}$ is Zero of $h(x)$

(4) Find the roots of the polynomial equations.

(i) $5x-6=0$

Sol:

$$5x-6=0$$

$$5x = 6$$

$$\boxed{x = \frac{6}{5}}$$

(ii) $x+3=0$

Sol:-

$$x+3=0$$

$$\boxed{x = -3}$$

(iii) $10x+9=0$

Sol:-

$$10x+9=0$$

$$10x = -9$$

$$\boxed{x = \frac{-9}{10}}$$

(iv) $9x-4=0$

Sol:-

$$9x-4=0$$

$$9x = 4$$

$$\boxed{x = \frac{4}{9}}$$

(5) Verify whether the following are Zeros of the Polynomial indicated against them, or not.

(i) $P(x) = 2x-1, \quad x = \frac{1}{2}$

Sol:- $P(x) = 2x-1$

$P(\frac{1}{2}) = \cancel{2}(\frac{1}{\cancel{2}}) - 1$

$= 1 - 1$

$P(\frac{1}{2}) = 0$

$\therefore x = \frac{1}{2}$ is Zero of the polynomial.

(ii) $P(x) = x^3 - 1, \quad x = 1$

Sol:- $P(x) = x^3 - 1$

$P(1) = 1^3 - 1$

$P(1) = 1 - 1$

$P(1) = 0$

$\therefore x = 1$ is Zero of the polynomial

(iii) $P(x) = ax + b, \quad x = -\frac{b}{a}$

Sol :-

$$p(x) = ax+b$$

$$p\left(\frac{-b}{a}\right) = \cancel{a}\left(\frac{-b}{\cancel{a}}\right)+b$$

$$= -b+b$$

$$p\left(\frac{-b}{a}\right) = 0$$

$\therefore$ $x = \frac{-b}{a}$ is Zero of the Polynomial

(iv) $p(x) = (x+3)(x-4)$, $x=4$, $x=-3$

Sol :-

$$p(x) = (x+3)(x-4) \qquad x=4$$

$$p(4) = (4+3)(4-4)$$

$$= 7(0)$$

$$p(4) = 0$$

$\therefore$ $x=4$ is Zero of $p(x)$

$$p(-3) = (-3+3)(-3-4) \qquad x=-3$$

$$= 0(-7)$$

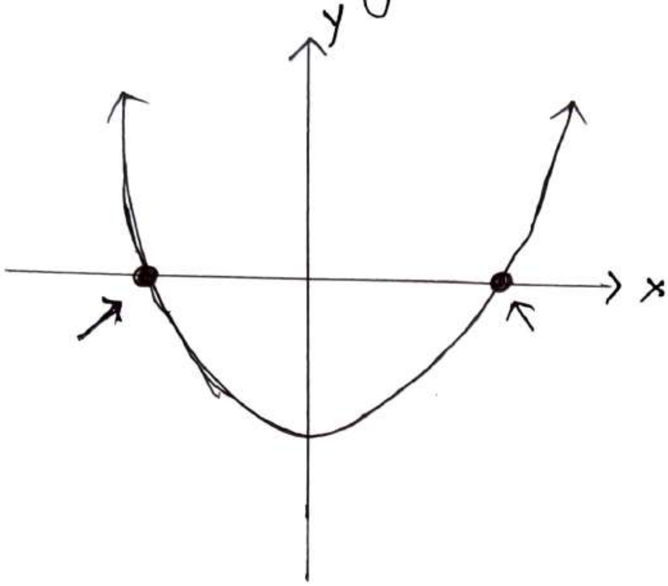
$$p(-3) = 0$$

$\therefore$ $x=-3$ is Zero of $p(x)$

---

⑥ Find the number of Zeros of the following polynomials represented by their graph.

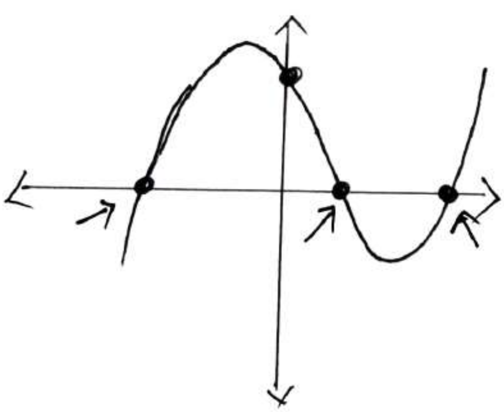
(i)



Sol :- The Curve touches the x-axis at 2 points

∴ Number of Zeros = 2

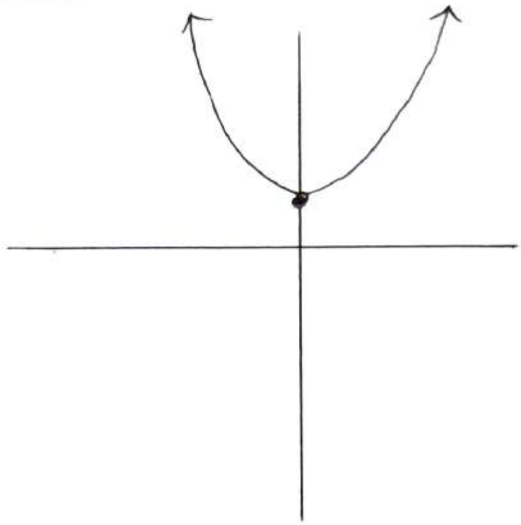
(ii)



Sol :- The Curve touches the x-axis at 3 points.

$\therefore$ Number of Zeros = 3

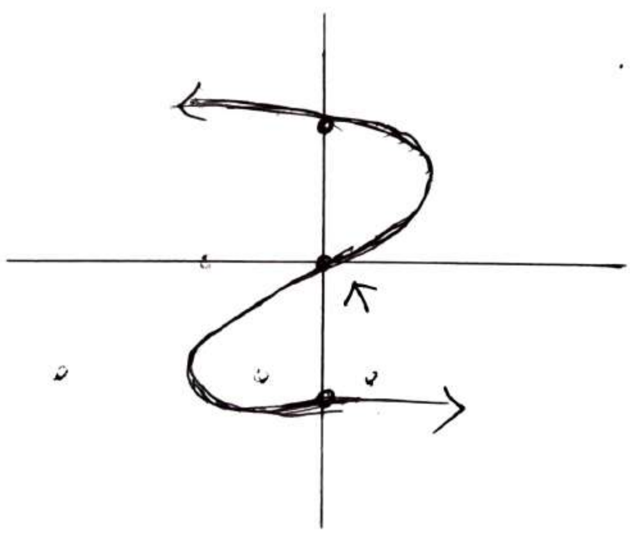
(iii)



Sol:- The Curve doesnot touch the x-axis at any point

$\therefore$ Number of Zeros = 0

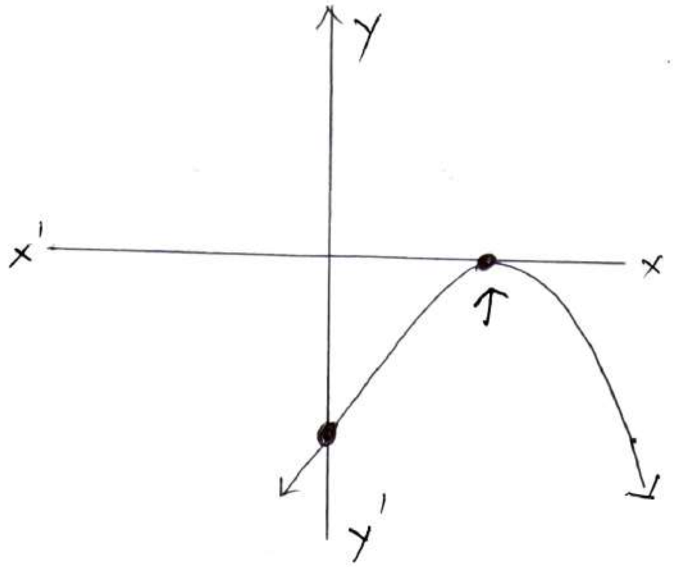
(iv)



Sol:- The Curve touches the x-axis at any point

$\therefore$ Number of Zeros = 1

(v)



Sol: The curve touches the x-axis at one point

$\therefore$ Number of zeros = 1

Exercise - 3.3

1. Check whether $p(x)$ is a multiple of $g(x)$ or not.

$p(x) = x^3 - 5x^2 + 4x - 3$; $g(x) = x - 2$

Sol: - $p(x) = x^3 - 5x^2 + 4x - 3$

$g(x) = x - 2$

$g(x) = 0$

$$x-2=0$$

$$x=2$$

$$P(x) = x^3-5x^2+4x-3$$

$$P(2) = 2^3-5(2)^2+4(2)-3$$

$$= 8-5(4)+8-3$$

$$= 8-20+8-3$$

$$= 16-23$$

$$P(2) = -7$$

$$\therefore P(2) \neq 0$$

$\therefore$ $P(x)$ is not a multiple of $g(x)$

---

(2) By remainder theorem, find the remainder when, $P(x)$ is divided by $g(x)$ where

(i) $P(x) = x^3-2x^2-4x-1$; $g(x) = x+1$

Sol: -

$$g(x) = 0$$

$$x+1 = 0$$

$$\boxed{x = -1}$$

$$P(x) = x^3 - 2x^2 - 4x - 1$$

$$p(-1) = (-1)^3 - 2(-1)^2 - 4(-1) - 1$$

$$= -1 - 2(1) + 4 - 1$$

$$= -1 - 2 + 4 - 1$$

$$= -4 + 4$$

$$\boxed{P(-1) = 0}$$

$\therefore$ Remainder $= 0$

(ii) $P(x) = 4x^3 - 12x^2 + 14x - 3$; $g(x) = 2x - 1$

Sol :-

$$g(x) = 0$$

$$2x - 1 = 0$$

$$2x = 1$$

$$\boxed{x = \frac{1}{2}}$$

$$P(x) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + \overset{7}{\cancel{14}}\left(\frac{1}{\cancel{2}}\right) - 3$$

$$= \overset{1}{\cancel{4}}\left(\frac{1}{\underset{2}{\cancel{8}}}\right) - \overset{3}{\cancel{12}}\left(\frac{1}{\underset{1}{\cancel{4}}}\right) + 7 - 3$$

$$= \frac{1}{2} - 3 + 7 - 3$$

$$= \frac{1}{2} + 7 - 6$$

$= \frac{1}{2} + 1$

$= \frac{1+2}{2}$

$$\boxed{P\left(\frac{1}{2}\right) = \frac{3}{2}}$$

$\therefore$ Remainder $= \frac{3}{2}$

(iii) $P(x) = x^3 - 3x^2 + 4x + 50$; $g(x) = x - 3$

Sol :-

$g(x) = 0$

$x - 3 = 0$

$$\boxed{x = 3}$$

$P(x) = x^3 - 3x^2 + 4x + 50$

$P(3) = (3)^3 - 3(3)^2 + 4(3) + 50$

$= 27 - 3(9) + 12 + 50$

$= \cancel{27} - \cancel{27} + 62$

$$\boxed{P(3) = 62}$$

$\therefore$ Remainder $= 62$

---

(3) Find the remainder when $3x^3-4x^2+7x-5$ is divided by $(x+3)$

Sol :-

Let $P(x) = 3x^3-4x^2+7x-5$

$\therefore \; x+3 = 0$

$$\boxed{x = -3}$$

$\therefore \; P(-3) = 3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

$= 3(-27) - 4(9) - 21 - 5$

$= -81 - 36 - 26$

$$\boxed{P(-3) = -143}$$

$\therefore$ Remainder $= -143$

---

(4) What is the remainder when $x^{2018} + 2018$ is divided by $x-1$

Sol :-

Let $P(x) = x^{2018} + 2018$

$\therefore \; x - 1 = 0$

$$\boxed{x = 1}$$

$$\therefore \; P(1) = 1^{2018} + 2018$$

$$= 1 + 2018$$

$$\boxed{P(1) = 2019}$$

$\therefore$ Remainder $= 2019$

---

(5) For what Value of k is the Polynomial :-

$P(x) = 2x^3 - kx^2 + 3x + 10$ exactly divisible by $(x-2)$

Sol:- $P(x) = 2x^3 - kx^2 + 3x + 10$

$P(x)$ is exactly divisible by $x-2$

$\Rightarrow \; P(x) = 0$

$$x - 2 = 0$$

$$\boxed{x = 2}$$

$$P(x) = 0$$

$$2(2)^3 - k(2)^2 + 3(2) + 10 = 0$$

$$2(8) - k(4) + 6 + 10 = 0$$

$$16 - 4k + 16 = 0$$

$$32 - 4k = 0$$

$$\cancel{-}4k = \cancel{-}32$$

$$k = \frac{32}{4}$$

$$\boxed{k = 8}$$

---

6) If two polynomial $2x^3 + ax^2 + 4x - 12$ and $x^3 + x^2 - 2x + a$ leaves the Same remainder when divided by $(x-3)$, find the value of a and also find the remainder.

Sol:- Let $p(x) = 2x^3 + ax^2 + 4x - 12$

$$q(x) = x^3 + x^2 - 2x + a$$

$p(x)$ and $q(x)$ has Same remainder When divided by $x-3$

$\therefore \quad x - 3 = 0$

$x = 3$

$\therefore \quad p(3) = q(3)$ [Remainder Same]

$2(3)^3 + a(3)^2 + 4(3) - 12 = (3)^3 + (3)^2 - 2(3) + a$

$2(27) + a(9) + \cancel{12} - \cancel{12} = 27 + 9 - 6 + a$

$54 + 9a = 36 - 6 + a$

$54 + 9a = 30 + a$

$9a - a = 30 - 54$

$8a = -24$

$a = \dfrac{-\cancel{24}^{\,3}}{\cancel{8}_{\,1}}$

$\boxed{a = -3}$

To Find the Remainder

$q(x) = x^3 + x^2 - 2x - 3$

$x - 3 = 0$

$x = 3$

$q(3) = (3)^3 + 3^2 - 2(3) - 3$

$= 27 + 9 - 6 - 3$

$= 27 + \cancel{9} - \cancel{9}$

$\boxed{q(3) = 27}$

$\therefore$ Remainder $= 27$

7) Determine whether $(x-1)$ is a factor of the following polynomials:

(i) $x^3+5x^2-10x+4$

Sol:- $p(x) = x^3+5x^2-10x+4$

To Check:- $x-1$ is a factor
i.e., $\boxed{P(1) = 0}$

$x-1=0$
$x=1$

$$P(1) = 1^3+5(1)^2-10(1)+4$$
$$= 1+5(1)-10+4$$
$$= 1+5-10+4$$
$$= 10 - 10$$

$\therefore \boxed{P(1) = 0}$

$\therefore (x-1)$ is a factor

(ii) x^4+5x^2-5x+1

Sol:- $p(x) = x^4+5x^2-5x+1$

To Check: $x-1$ is a factor
i.e., $\boxed{P(1) = 0}$

$x-1=0$
$x=1$

$$P(1) = (1)^4 + 5(1)^2 - 5(1) + 1$$

$$= 1 + 5(1) - 5 + 1$$

$$= 2 + \cancel{5} - \cancel{5}$$

$$\boxed{P(1) = 2}$$

$\therefore (x-1)$ is not a factor.

(8) Using factor theorem, Show that $(x-5)$ is a factor of the polynomial $2x^3 - 5x^2 - 28x + 15$.

Sol :- Let $P(x) = 2x^3 - 5x^2 - 28x + 15$

To Check :- $x-5$ is a factor

i.e, $\boxed{P(5) = 0}$

$x - 5 = 0$

$x = 5$

$$P(5) = 2(5)^3 - 5(5)^2 - 28(5) + 15$$

$$= 2(125) - 5(25) - 140 + 15$$

$$= 250 - 125 - 140 + 15$$

$$= 265 - 265$$

$$\begin{array}{r} 250 \\ 15 \\ \hline 265 \end{array} \qquad \begin{array}{r} -125 \\ -140 \\ \hline 265 \end{array}$$

$\boxed{P(5) = 0}$ $\therefore (x-5)$ is a factor.

(9) Determine the Value of m, if $(x+3)$ is a factor of $x^3 - 3x^2 - mx + 24$

Sol :- To find :- $\boxed{m = ?}$

Let; $P(x) = x^3 - 3x^2 - mx + 24$

Given :- $(x+3)$ is a factor

ie, $P(x) = 0$

ie; $P(-3) = 0$

$x + 3 = 0$

$x = -3$

$$(-3)^3 - 3(-3)^2 - m(-3) + 24 = 0$$

$$-27 - 3(9) + 3m + 24 = 0$$

$$-27 - 27 + 3m + 24 = 0$$

$$-54 + 3m + 24 = 0$$

$$-30 + 3m = 0$$

$$3m = 30$$

$$m = \frac{30}{3}$$

$$\boxed{m = 10}$$

---

(10) If both $(x-2)$ and $(x - \frac{1}{2})$ are the factor of $ax^2 + 5x + b$, then show that $a = b$

Given :- $P(x) = ax^2 + 5x + b$

$x - 2$ is a factor

ie, $x - 2 = 0$

$\boxed{x = 2}$

$\therefore P(2) = 0$

$a(2)^2 + 5(2) + b = 0$

$a(4) + 10 + b = 0$

$4a + b + 10 = 0 \longrightarrow (1)$

$(x - \frac{1}{2})$ is a factor

ie, $x - \frac{1}{2} = 0$

$\boxed{x = \frac{1}{2}}$

$\therefore P(\frac{1}{2}) = 0$

$a(\frac{1}{2})^2 + 5(\frac{1}{2}) + b = 0$

$a(\frac{1}{4}) + \frac{5}{2}^{\times 2} + b^{\times 4} = 0$

$\frac{a + 10 + 4b}{4} = 0$

$a + 4b + 10 = 0 \longrightarrow (2)$

To Show :- $\boxed{a = b}$

$4a + b + \cancel{10} = a + 4b + \cancel{10}$

$4a + b = a + 4b$

$4a - a = 4b - b$

$\cancel{3}a = \cancel{3}b$

$\boxed{a = b}$

Hence Proved.

(11) If $(x-1)$ divides the polynomial $kx^3 - 2x^2 + 25x - 26$ without Remainder, then find the Value of k.

<u>Sol</u>:-

$$p(x) = kx^3 - 2x^2 + 25x - 26$$

$(x-1)$ divides $p(x)$, Without Remainder

iè,

$$x - 1 = 0$$

$$\boxed{x = 1}$$

$\Rightarrow \quad p(1) = 0$ [Without Remainder]

$$k(1)^3 - 2(1)^2 + 25(1) - 26 = 0$$

$$k(1) - 2(1) + 25 - 26 = 0$$

$$k - 2 + 25 - 26 = 0$$

$$k - 28 + 25 = 0$$

$$k - 3 = 0$$

$$\boxed{k = 3}$$

(12) Check if $(x+2)$ and $(x-4)$ are the sides of a rectangle whose area is $x^2 - 2x - 8$ by using factor theorem.

Sol :- To Check :- $(x+2)$ and $(x-4)$ are sides of Rectangle

Given :- Area $= x^2 - 2x - 8$

ie, $P(x) = x^2 - 2x - 8$

$x + 2 = 0$	$x - 4 = 0$
$\boxed{x = -2}$	$\boxed{x = 4}$
$P(-2) = (-2)^2 - 2(-2) - 8$	$P(4) = (4)^2 - 2(4) - 8$
$= 4 + 4 - 8$	$= 16 - 8 - 8$
$= 8 - 8$	$= 16 - 16$
$\boxed{P(-2) = 0}$	$\boxed{P(4) = 0}$

$\therefore$ $(x+2)$ and $(x-4)$ are the sides of the rectangle.

Algebraic Identities.

* $(a+b)^2 = a^2+b^2+2ab$

* $(a-b)^2 = a^2+b^2-2ab$

* $a^2-b^2 = (a+b)(a-b)$

* $(x+a)(x+b) = x^2+(a+b)x+ab$

* $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

* $(x+a)(x+b)(x+c) = x^3+(a+b+c)x^2 + (ab+bc+ca)x+abc$

* $(x+y)^3 = x^3+3x^2y+3xy^2+y^3$

  (or)

  $x^3+y^3+3xy(x+y)$

* $(x-y)^3 = x^3-3x^2y+3xy^2-y^3$

  (or)

  $x^3-y^3-3xy(x-y)$

* $x^3+y^3+z^3-3xyz \equiv (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

If $x+y+z=0$

then; $x^3+y^3+z^3=3xyz$

* $x^3+y^3=(x+y)^3-3xy(x+y)$

* $x^3-y^3=(x-y)^3+3xy(x-y)$

Exercise - 3.4

(1.) Expand the following :-

(i) $(x+2y+3z)^2$

Sol: - $\boxed{a=x}$ $\boxed{b=2y}$ $\boxed{c=3z}$

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$$

$$(x+2y+3z)^2=(x)^2+(2y)^2+(3z)^2+2(x)(2y)+2(2y)(3z)+2(3z)(x)$$

$$(x+2y+3z)^2=x^2+4y^2+9z^2+4xy+12yz+6xz$$

(ii) $(-p+2q+3r)^2$

<u>Sol</u>:- $\boxed{a=-p}$ $\boxed{b=2q}$ $\boxed{c=3r}$

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(-p+2q+3r)^2 = (-p)^2+(2q)^2+(3r)^2+2(-p)(2q)$$
$$+2(2q)(3r)+2(3r)(-p)$$

$$(-p+2q+3r) = p^2+4q^2+9r^2-4pq+12qr-6rp$$

(iii) $(2p+3)(2p-4)(2p-5)$

<u>Sol</u> $x=2p$ $a=3$ $b=-4$ $c=-5$

$$(x+a)(x+b)(x+c) = x^3+(a+b+c)x^2$$
$$+(ab+bc+ca)x+abc$$

$$(2p+3)(2p-4)(2p-5) = (2p)^3+(3-4-5)(2p)^2$$
$$+\left[(3)(-4)+(-4)(-5)+(-5)(3)\right](2p)$$
$$+(3)(-4)(-5)$$

$$= 8p^3+(3-9)(4p^2)+\left[-12+20-15\right]2p$$
$$+60$$

$= 8p^3 + (-6)4p^2 + [20-27]\,2p + 60$

$(2p+3)(2p-4)(2p-5) = 8p^3 - 24p^2 + (-7)(2p) + 60$

$(2p+3)(2p-4)(2p-5) = 8p^3 - 24p^2 - 14p + 60$

(iv) $(3a+1)(3a-2)(3a+4)$

Sol:- $x = 3a$ $a = 1$ $b = -2$ $c = 4$

$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

$(3a+1)(3a-2)(3a+4) = (3a)^3 + (1-2+4)(3a)^2 + [(1)(-2) + (-2)(4) + (4)(1)](3a) + (1)(-2)(4)$

$= 27a^3 + (5-2)(9a^2) + [-2-8+4](3a) + (-8)$

$= 27a^3 + 3(9a^2) + [-10+4](3a) - 8$

$= 27a^3 + 27a^2 + (-6)(3a) - 8$

$(3a+1)(3a-2)(3a+4) = 27a^3 + 27a^2 - 18a - 8$

(2) Using algebraic identity, find the coefficients of x^2, x and constant term without actual expansion.

Sol:-

(i) $(x+5)(x+6)(x+7)$

$x = x$ $a = 5$ $b = 6$ $c = 7$

$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

$\Rightarrow$ Coefficient of $x^2 = a+b+c$

$= 5+6+7$

$= 18$

$\Rightarrow$ Coefficient of $x = ab+bc+ca$

$= (5\times6)+(6\times7)+(7\times5)$

$= 30+42+35$

$= 107$

$\Rightarrow$ Constant $= abc$

$= (5)(6)7$

$= 210$

(ii) $(2x+3)(2x-5)(2x-6)$

<u>Sol</u> :- $x = 2x$ $a = 3$ $b = -5$ $c = -6$

$$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$$

$$\text{Coefficient of } x^2 = (3-5-6)2^2$$
$$= (3-11)4$$
$$= (-8)4$$

Coefficient of $x^2 = -32$

$$\text{Coefficient of } x = [(3)(-5)+(-5)(-6)+(-6)(3)](2)$$
$$= (-15+30-18)(2)$$
$$= (-33+30)(2)$$
$$= (-3)(2)$$

Coefficient of $x = -6$

$$\text{Constant} = (3)(-5)(-6)$$

Constant $= 90$

(3) If $(x+a)(x+b)(x+c) = x^3+14x^2+59x+70$ find the Value of

(i) $a+b+c$

<u>Sol</u> :- $(x+a)(x+b)(x+c) = x^3+14x^2+59x+70$

$$(x+a)(x+b)(x+c) = x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$$

$$a+b+c = 14$$
$$ab+bc+bc = 59$$
$$abc = 70$$

(i) $a+b+c = 14$

(ii) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \frac{bc+ac+ab}{abc}$

$$= \frac{59}{70}$$

(iii) $a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca)$

$$= (14)^2 - 2(59)$$
$$= 196 - 118$$
$$= 78$$

(iv) $\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab}$ $\boxed{a^2+b^2+c^2=78}$

Sol :- $\frac{a^{\times a}}{bc} + \frac{b^{\times b}}{ac} + \frac{c^{\times c}}{ab} = \frac{a^2+b^2+c^2}{abc}$

$= \frac{78}{70}$

④ Expand :-

(i) $(3a-4b)^3$

Sol :- $a = 3a$ $b = 4b$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$(3a-4b)^3 = (3a)^3 - (4b)^3 - 3(3a)(4b)(3a-4b)$

$= 27a^3 - 64b^3 - 36ab(3a-4b)$

$(3a-4b)^3 = 27a^3 - 64b^3 - 108a^2b + 144ab^2$

(ii) $\left(x+\frac{1}{y}\right)^3$

Sol :- $a = x$ $b = \frac{1}{y}$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\left(x+\frac{1}{y}\right)^3 = x^3 + \left(\frac{1}{y}\right)^3 + 3(x)\left(\frac{1}{y}\right)\left(x+\frac{1}{y}\right)$

$$= x^3 + \frac{1}{y^3} + \frac{3x}{y}\left(x + \frac{1}{y}\right)$$

$$= x^3 + \frac{1}{y^3} + \frac{3x^2}{y} + \frac{3x}{y^2}$$

(5) Evaluate the following by using identities :-

(i) 98^3

Sol :- $98^3 = (100-2)^3$

$a = 100$
$b = 2$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$98^3 = 100^3 - 2^3 - 3(100)(2)(100-2)$$

$$= 1000000 - 8 - 600(98)$$

$$= 999992 - 58800$$

$$\boxed{98^3 = 941192}$$

$$\begin{array}{r} 1000000 \\ -\ \ \ \ \ 8 \\ \hline 999992 \end{array}$$

$$\begin{array}{r} 98 \\ \times\ 6 \\ \hline 588 \end{array}$$

$$\begin{array}{r} 999992 \\ -58800 \\ \hline 941192 \end{array}$$

(ii) 1001^3

Sol :- $1001^3 = (1000+1)^3$

$$(a+b)^3 = a^3+b^3+3ab(a+b)$$ $a=1000$, $b=1$

$$(1000+1)^3 = (1000)^3+1^3+3(1000)(1)[1000+1]$$

$$= 1000000000+1+3000(1001)$$

$$= 1000000001+3003000$$

$$(1000+1)^3 = 1003003001$$

(6) If $(x+y+z)=9$ and $(xy+yz+zx)=26$, find the Value of $x^2+y^2+z^2$.

Sol: Given:- $x+y+z=9$

$xy+yz+zx=26$

$x^2+y^2+z^2=?$

$$x^2+y^2+z^2 = (x+y+z)^2-2(xy+yz+zx)$$

$$= (9)^2-2(26)$$

$$= 81-52$$

$$\boxed{x^2+y^2+z^2 = 29}$$

(7) Find $27a^3+64b^3$; If $3a+4b=10$ and $ab=2$

Sol:- $3a+4b=10$ cubing on b.s

$(3a+4b)^3=10^3$

$(a+b)^3=a^3+b^3+3ab(a+b)$

$a=3a$
$b=4b$

$(3a)^3+(4b)^3+3(3a)(4b)[3a+4b]=1000$

$27a^3+64b^3+36ab[3a+4b]=1000$ $ab=2$

$27a^3+64b^3+36(2)[10]=1000$

$27a^3+64b^3+72(10)=1000$

$27a^3+64b^3+720=1000$

$27a^3+64b^3=1000-720$

$\boxed{27a^3+64b^3=280}$

---

(8) Find x^3-y^3, if $x-y=5$ and $xy=14$

Sol:- $x-y=5$ cubing on b.s

$(x-y)^3=5^3$

$(a-b)^3=a^3-b^3-3ab(a-b)$

$$x^3 - y^3 - 3xy(x-y) = 125$$

$xy = 14$

$x - y = 5$

$$x^3 - y^3 - 3(14)(5) = 125$$

$$x^3 - y^3 - 210 = 125$$

$$\begin{array}{r} 14 \\ \times 3 \\ \hline 42 \\ \times 5 \\ \hline 210 \end{array}$$

$$x^3 - y^3 = 125 + 210$$

$$\boxed{x^3 - y^3 = 335}$$

---

(9) If $a + \frac{1}{a} = 6$, then find the Value of $a^3 + \frac{1}{a^3}$

<u>Sol</u>:-

$$a + \frac{1}{a} = 6$$

Cubing on. b.s

$$\left(a + \frac{1}{a}\right)^3 = 6^3$$

$$\begin{array}{r} 6 \times 6 \times 6 \\ 36 \times 6 \\ \hline 216 \end{array}$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$a^3 + \left(\frac{1}{a}\right)^3 + 3(\cancel{a})\left(\frac{1}{\cancel{a}}\right)\left[a + \frac{1}{a}\right] = 216$$

$a + \frac{1}{a} = 6$

$$a^3 + \frac{1}{a^3} + 3(6) = 216$$

$$a^3 + \frac{1}{a^3} + 18 = 216$$

$$a^3 + \frac{1}{a^3} = 216 - 18$$

$$216 - 18 = 198$$

$$\boxed{a^3 + \frac{1}{a^3} = 198}$$

---

(10) If $x^2 + \frac{1}{x^2} = 23$, then find the Value of $x + \frac{1}{x}$ and $x^3 + \frac{1}{x^3}$

<u>Sol</u>:-

$$x^2 + \frac{1}{x^2} = 23 \longrightarrow (1)$$

(*) $x + \frac{1}{x}$ Squaring on b.s

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)$$

$$= x^2 + \frac{1}{x^2} + 2 \quad \text{from (1)}$$

$$= 23 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 25$$

$$x + \frac{1}{x} = \sqrt{25}$$

$$\boxed{x + \frac{1}{x} = \pm 5} \longrightarrow (2)$$

(*) $x + \frac{1}{x}$ Cubing on b.s

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3(\cancel{x})\left(\frac{1}{\cancel{x}}\right)\left(x+\frac{1}{x}\right)$$

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$$ Put (2)

$$(5)^3 = x^3 + \frac{1}{x^3} + 3(5)$$

$$125 = x^3 + \frac{1}{x^3} + 15$$

$$125 - 15 = x^3 + \frac{1}{x^3}$$

$$110 = x^3 + \frac{1}{x^3}$$

$$\therefore \boxed{x^3 + \frac{1}{x^3} = 110}$$

---

(11) If $\left(y-\frac{1}{y}\right)^3 = 27$, then find the Value of $y^3 - \frac{1}{y^3}$.

Sol
$\therefore$

$$\left(y-\frac{1}{y}\right)^3 = 27 \longrightarrow (1)$$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$y^3 - \frac{1}{y^3} - 3(\cancel{y})\left(\frac{1}{\cancel{y}}\right)\left(y-\frac{1}{y}\right) = 27$$

$$y^3 - \frac{1}{y^3} - 3\left(y - \frac{1}{y}\right) = 27$$

From (1)

$$\left(y-\frac{1}{y}\right)^3 = 27$$

$$y - \frac{1}{y} = \sqrt[3]{27}$$

$$\boxed{y - \frac{1}{y} = 3}$$

$$\sqrt[3]{27} = \sqrt[3]{3\times3\times3} = 3$$

$$y^3 - \frac{1}{y^3} - 3(3) = 27$$

$$y^3 - \frac{1}{y^3} - 9 = 27$$

$$y^3 - \frac{1}{y^3} = 27 + 9 \Rightarrow \boxed{y^3 - \frac{1}{y^3} = 36}$$

(12) Simplify :-

(i) $(2a+3b+4c)(4a^2+9b^2+16c^2-6ab-12bc-8ca)$

Sol :-

$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx) =$$

$$x^3+y^3+z^3-3xyz$$

$$\Rightarrow (2a+3b+4c)(4a^2+9b^2+16c^2-6ab-12bc-8ca)$$

$\therefore$ $x = 2a$
$y = 3b$
$z = 4c$

$$\Rightarrow x^3+y^3+z^3-3xyz$$

$$\Rightarrow (2a)^3+(3b)^3+(4c)^3-3(2a)(3b)(4c)$$

$$\Rightarrow 8a^3+27b^3+16c^3-72abc$$

(ii) $(x-2y+3z)(x^2+4y^2+9z^2+2xy+6yz-3xz)$

Sol :-

$x = x$
$y = -2y$
$z = 3z$

$\Rightarrow x^3+y^3+z^3-3xyz$

$\Rightarrow x^3+(-2y)^3+(3z)^3-3(x)(-2y)(3z)$

$\Rightarrow x^3+(-8y^3)+27z^3+18xyz$

$\Rightarrow x^3-8y^3+27z^3+18xyz$

---

(13) By Using identity evaluate the following:-

(i) $7^3-10^3+3^3$

Sol:- $7^3-10^3+3^3$

If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

$a=7$
$b=-10$
$c=3$

$\Rightarrow 7-10+3=0$

$\therefore 7^3+(-10)^3+3^3=3(7)(-10)(3)$

$7^3-10^3+3^3=-630$

(ii) $1+\frac{1}{8}-\frac{27}{8}$

Sol :- $1+\frac{1}{8}-\frac{27}{8}$

If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

$\Rightarrow 1^3+\left(\frac{1}{2}\right)^3+\left(\frac{-3}{2}\right)^3$

$a=1$

$b=\frac{1}{2}$

$c=\frac{-3}{2}$

$\Rightarrow 1+\frac{1}{2}-\frac{3}{2} \Rightarrow \frac{2+1-3}{2}=\frac{3-3}{2}=0$

$\therefore 1^3+\left(\frac{1}{2}\right)^3+\left(\frac{-3}{2}\right)^3=3(1)\left(\frac{1}{2}\right)\left(\frac{-3}{2}\right)$

$$\boxed{1^3+\left(\frac{1}{2}\right)^3+\left(\frac{-3}{2}\right)^3=\frac{-9}{4}}$$

(14) If $2x-3y-4z=0$, then find $8x^3-27y^3-64z^3$.

Sol :- If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

$$\Rightarrow 8x^3 - 27y^3 - 64z^3 = (2x)^3 + (-3y)^3 + (-4z)^3$$

$\therefore$

$$a = 2x$$
$$b = -3y$$
$$c = -4z$$

$$\therefore (2x)^3 + (-3y)^3 + (-4z)^3 = 3(2x)(-3y)(-4z)$$

$$(2x)^3 + (-3y)^3 + (-4z)^3 = 72xyz$$

Identity

* $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
* $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Exercise - 3.5

(1) Factorise the following expressions:-

(i) $2a^2 + 4a^2b + 8a^2c$

<u>Sol</u>:- $2a^2 + 4a^2b + 8a^2c$

$\Rightarrow 2a^2(1 + 2b + 4c)$

(ii) $ab - ac - mb + mc$

Sol $ab - ac - mb + mc$

$\Rightarrow a(b-c) - m(b-c)$

$\Rightarrow (b-c)(a-m)$

(2) Factorize the following :-

(i) $x^2 + 4x + 4$

Sol :- $x^2 + 4x + 4 = x^2 + 2(2x) + 2^2$

$= \boxed{a^2 + 2(a)(b) + b^2}$

$= (x+2)^2$

(ii) $3a^2 - 24ab + 48b^2$

Sol :- $3a^2 - 24ab + 48b^2$

$\Rightarrow 3(a^2 - 8ab + 16b^2)$

$\Rightarrow 3(a^2 - 2(4b)(a) + (4b)^2)$ $\boxed{a^2 - 2ab + b^2}$

$a = a$
$b = 4b$

$\Rightarrow 3(a - 4b)^2$

(iii) $x^5 - 16x$

Sol: $x^5 - 16x = x(x^4 - 16)$

$= x((x^2)^2 - 4^2)$

$a = x^2$
$b = 4$

$= x(x^2 - 4)(x^2 + 4)$

$a^2 - b^2 = (a+b)(a-b)$

$= x(x^2 - 2^2)(x^2 + 4)$

$\therefore x^5 - 16x = x(x-2)(x+2)(x^2+4)$

(iv) $m^2 + \frac{1}{m^2} - 23$

Sol:- $m^2 + \frac{1}{m^2} - 23$

$a^2 + b^2 = (a+b)^2 - 2ab$

$a = m$
$b = \frac{1}{m}$

$\Rightarrow \left(m + \frac{1}{m}\right)^2 - 2(\cancel{m})\left(\frac{1}{\cancel{m}}\right) - 23$

$\Rightarrow \left(m + \frac{1}{m}\right)^2 - 2 - 23$

$\Rightarrow \left(m + \frac{1}{m}\right)^2 - 25$

$a^2 - b^2 = (a+b)(a-b)$

$\Rightarrow \left(m + \frac{1}{m}\right)^2 - 5^2$

$\Rightarrow \left[\left(m + \frac{1}{m}\right) + 5\right]\left[\left(m + \frac{1}{m}\right) - 5\right]$

(V) $6 - 216x^2$

Sol :- $6 - 216x^2 = 6(1 - 36x^2)$

$a = 1$
$b = 6x$

$= 6\left[1 - (6x)^2\right]$

$a^2 - b^2 = (a+b)(a-b)$

$= 6\left[(1+6x)(1-6x)\right]$

(Vi) $a^2 + \frac{1}{a^2} - 18$

Sol :- $a^2 + \frac{1}{a^2} - 18$

$a^2 + b^2 = (a-b)^2 + 2ab$

$\Rightarrow \left(a - \frac{1}{a}\right)^2 + 2(\cancel{a})\left(\frac{1}{\cancel{a}}\right) - 18$

$a = a$
$b = \frac{1}{a}$

$\Rightarrow \left(a - \frac{1}{a}\right)^2 + 2 - 18$

$\Rightarrow \left(a - \frac{1}{a}\right)^2 - 16$

$a^2 - b^2 = (a+b)(a-b)$

$\Rightarrow \left(a - \frac{1}{a}\right)^2 - 4^2$

$a = a - \frac{1}{a}$
$b = 4$

$\Rightarrow \left(a - \frac{1}{a} + 4\right)\left(a - \frac{1}{a} - 4\right)$

(3) Factorise the following :-

(i) $4x^2+9y^2+25z^2+12xy+30yz+20xz$

Sol:- $4x^2+9y^2+25z^2+12xy+30yz+20xz$

$\Rightarrow (2x)^2+(3y)^2+(5z)^2+2(2x)(3y)+$
$2(3y)(5z)+2(5z)(2x)$

$\Rightarrow$ $a^2+b^2+c^2+2ab+2bc+2ca = (a+b+c)^2$

$\Rightarrow (2x+3y+5z)^2$

(ii) $25x^2+4y^2+9z^2-20xy+12yz-30xz$

Sol :-
$25x^2+4y^2+9z^2-20xy+12yz-30xz$

x value is $-ve$

$\Rightarrow$
$(-5x)^2+(2y)^2+(3z)^2+2(-5x)(2y)+$
$2(2y)(3z)+2(3z)(-5x)$

$\Rightarrow (-5x+2y+3z)^2$

(4) Factorise the following

(i) $8x^3+125y^3$

Sol $8x^3+125y^3$

$a^3+b^3=(a+b)(a^2+b^2-ab)$

$\Rightarrow (2x)^3+(5y)^3$

$\Rightarrow (2x+5y)\left((2x)^2+(5y)^2-(2x)(5y)\right)$

$\Rightarrow (2x+5y)\left[4x^2+25y^2-10xy\right]$

(ii) $27x^3-8y^3$

$a=3x$
$b=2y$

Sol: $27x^3-8y^3=(3x)^3-(2y)^3$

$\boxed{a^3-b^3=(a-b)(a^2+b^2+ab)}$

$=(3x-2y)\left[(3x)^2+(2y)^2+(3x)(2y)\right]$

$=(3x-2y)\left[9x^2+4y^2+6xy\right]$

(iii) a^6-64

$a=a^2$
$b=4$

Sol:-

$a^6-64=(a^2)^3-4^3$

$=(a^2-4)\left[(a^2)^2+4^2+(a^2)(4)\right]$

$=(a^2-2^2)\left[a^4+16+4a^2\right]$

$=(a-2)(a+2)(a^4+16+4a^2)$

(5) Factorise the following :-

(i) $x^3+8y^3+6xy-1$

Sol :- $x^3+8y^3-1+6xy$

$\Rightarrow x^3+(2y)^3+(-1)^3-3(x)(2y)(-1)$

$a^3+b^3+c^3-3abc \equiv (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\boxed{\begin{array}{l} a=x \\ b=2y \\ c=-1 \end{array}}$

$\Rightarrow (x+2y-1)\left[x^2+(2y)^2+(-1)^2-(x)(2y)-(2y)(-1)-(-1)(x)\right]$

$\Rightarrow (x+2y-1)(x^2+4y^2+1-2xy+2y+x)$

(ii) $l^3-8m^3-27n^3-18lmn$

Sol: $l^3+(-2m)^3+(-3n)^3-3(l)(-2m)(-3n)$

$\boxed{\begin{array}{l} a=l \\ b=-2m \\ c=-3n \end{array}}$

$\Rightarrow (l-2m-3n)\left[l^2+(-2m)^2+(-3n)^2-(l)(-2m)-(-2m)(-3n)-(-3n)(l)\right]$

$\Rightarrow (l-2m-3n)(l^2+4m^2+9n^2+2lm-6mn+3nl)$

Exercise - 3.6

① Factorise the following

(i) $x^2+10x+24$

24

6 4

Sol:- $(x+6)(x+4)$

(ii) $z^2+4z-12$

12

6 -2

Sol $(z+6)(z-2)$

(iii) $p^2-6p-16$

16

-8 2

Sol:- $(p-8)(p+2)$

(iv) $t^2+72-17t$

Sol:- $t^2+72-17t$

72

-9 -8

$\Rightarrow t^2-17t+72$

$(t-9)(t-8)$

(v) $y^2-16y-80$

80

-20 4

Sol $(y-20)(y+4)$

(vi) $a^2+10a-600$

Sol:- $(a+30)(a-20)$

600

$30 \quad -20$

---

② Factorise the following:-

(i) $2a^2+9a+10$

Sol:- $2a^2+9a+10$

20

$\frac{{}^{2}\cancel{4}}{\cancel{2}_{1}a} \quad \frac{5}{2a}$

$\Rightarrow (a+2)(2a+5)$

(ii) $5x^2-29xy-42y^2$

Sol:- $5x^2-29xy-42y^2$

$\Rightarrow (x-7y)(5x+6y)$

$210y^2$

$\frac{-\cancel{35}^{7}y}{\cancel{5}_{1}x} \quad \frac{6y}{5x}$

(iii) $9-18x+8x^2$

Sol:- $8x^2-18x+9$

$\Rightarrow (2x-3)(4x-3)$

72

$\frac{-\cancel{12}^{3}}{\cancel{8}_{2}x} \quad \frac{-\cancel{6}^{3}}{\cancel{8}_{4}x}$

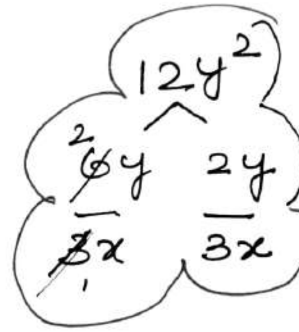
(iv) $6x^2 + 16xy + 8y^2$

Sol

$6x^2 + 16xy + 8y^2$

$2(3x^2 + 8xy + 4y^2)$

$2(x+2y)(3x+2y)$



(v) $12x^2 + 36x^2y + 27y^2x^2$

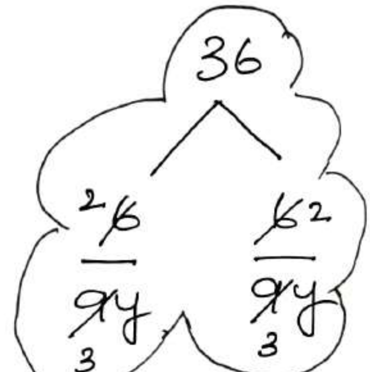
Sol:-

$12x^2 + 36x^2y + 27y^2x^2$

$3x^2(4 + 12y + 9y^2)$

$\Rightarrow 3x^2(9y^2 + 12y + 4)$

$\Rightarrow 3x^2(3y+2)(3y+2)$



(vi) $(a+b)^2 + 9(a+b) + 18$

Sol:- Let $a+b = x$

$\Rightarrow x^2 + 9x + 18$

$(x+6)(x+3)$

$(a+b+6)(a+b+3)$

---

(3) Factorise the following :-

(i) $(p-q)^2 - 6(p-q) - 16$

Sol :- $(p-q)^2 - 6(p-q) - 16$

16
-8 6

Let $p-q = x$

$\Rightarrow \quad x^2 - 6x - 16$

$(x-8)(x+2)$

$\Rightarrow (p-q-8)(p-q+2)$

(ii) $m^2 + 2mn - 24n^2$

$-24n^2$
$6n \quad -4n$

Sol :- $(m+6n)(m-4n)$

(iii) $\sqrt{5}a^2 + 2a - 3\sqrt{5}$

3×5
15
$\frac{-3}{\sqrt{5}a} \quad \frac{5}{\sqrt{5}a}$

Sol :- $\sqrt{5}a^2 + 2a - 3\sqrt{5}$

$(\sqrt{5}a - 3)(\sqrt{5}a + 5)$

(iv) $a^4 - 3a^2 + 2$

Let $a^2 = x$

2
$-2 \quad -1$

Sol :- $a^4 - 3a^2 + 2$

$(a^2)^2 - 3a^2 + 2 \Rightarrow x^2 - 3x + 2$

$\Rightarrow (x-2)(x-1)$

$\Rightarrow (a^2-2)(a^2-1)$

(v) $8m^3 - 2m^2n - 15mn^2$

Sol :- $8m^3 - 2m^2n - 15mn^2$

$m(8m^2 - 2mn - 15n^2)$

$m(2m-3n)(4m+5n)$

$120n^2$

$\frac{-\cancel{12}n}{\cancel{8}m} = \frac{-3n}{2m}$ $\quad$ $\frac{\cancel{10}n}{\cancel{8}m} = \frac{5n}{4m}$

(vi) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{2}{xy}$

Sol :- $\frac{1}{x^2} + \frac{1}{y^2} + \frac{2}{xy}$

$\Rightarrow \left(\frac{1}{x}\right)^2 + \left(\frac{1}{y}\right)^2 + 2\left(\frac{1}{x}\right)\left(\frac{1}{y}\right)$

$\Rightarrow \left(\frac{1}{x} + \frac{1}{y}\right)^2$

Exercise 3.7

(1) Find the quotient and remainder of the following.

(i) $(4x^3+6x^2-23x+18) \div (x+3)$

Sol:-

$$
\begin{array}{r|l}
 & 4x^2-6x-5 \\
\hline
x+3 & 4x^3+6x^2-23x+18 \\
 & \underset{(-)}{4x^3} \underset{(-)}{+} 12x^2 \\
\hline
 & -6x^2-23x \\
 & \underset{(+)}{-6x^2} \underset{(+)}{-} 18x \\
\hline
 & -5x+18 \\
 & \underset{(+)}{-5x} \underset{(+)}{-15} \\
\hline
 & 33
\end{array}
$$

$$\frac{4x^3}{x} = 4x^2$$

$$\frac{-6x^2}{x} = -6x$$

$$\frac{-5x}{x} = -5$$

$\therefore$ Quotient $= 4x^2-6x-5$

Remainder $= 33$

(ii) $(8y^3 - 16y^2 + 16y - 15) \div (2y - 1)$

Sol:-

$$
\begin{array}{r|l}
 & 4y^2 - 6y + 5 \\
\hline
2y-1 & \cancel{8y^3} - 16y^2 + 16y - 15 \\
 & \cancel{8y^3} - 4y^2 \\
 & (-) \quad (+) \\
\hline
 & -\cancel{12y^2} + 16y \\
 & -\cancel{12y^2} + 6y \\
 & (+) \quad (-) \\
\hline
 & \cancel{10y} - 15 \\
 & \cancel{10y} - 5 \\
 & (-) \quad (+) \\
\hline
 & -10 \\
\hline
\end{array}
$$

$\frac{\overset{4}{\cancel{8}}y^{\cancel{3}}}{\cancel{2}\cancel{y}} = 4y^2$

$\frac{-\overset{6}{\cancel{12}}y^{\cancel{2}}}{\cancel{2}\cancel{y}} = -6y$

$\frac{\overset{5}{\cancel{10}}\cancel{y}}{\cancel{2}\cancel{y}} = 5$

Quotient $= 4y^2 - 6y + 5$

Remainder $= -10$

(iii) $(8x^3 - 1) \div (2x - 1)$

<u>Sol</u>:- $8x^3 + 0x^2 + 0x - 1$

$$
\begin{array}{r|l}
 & 4x^2 + 2x + 1 \\
\hline
2x - 1 & \cancel{8x^3} + 0x^2 + 0x - 1 \\
 & \cancel{8x^3} - 4x^2 \\
 & (-) \quad (+) \\
\cline{2-2}
 & \qquad \cancel{4x^2} + 0x \\
 & \qquad \cancel{4x^2} - 2x \\
 & \qquad (-) \quad (+) \\
\cline{2-2}
 & \qquad\qquad \cancel{2x} \cancel{- 1} \\
 & \qquad\qquad \cancel{2x} \cancel{- 1} \\
 & \qquad\qquad (-) \quad (+) \\
\cline{2-2}
 & \qquad\qquad\qquad 0 \\
\end{array}
$$

$$\frac{\cancel{8}^{4} x^{\cancel{3}}}{\cancel{2}\cancel{x}} = 4x^2$$

$$\frac{\cancel{4}^{2} x^{\cancel{2}}}{\cancel{2}\cancel{x}} = 2x$$

$$\frac{\cancel{2x}}{\cancel{2x}} = 1$$

Quotient $= 4x^2 + 2x + 1$

Remainder $= 0$

(iv) $(-18z + 14z^2 + 24z^3 + 18) \div (3z + 4)$

Sol:- $(-18z + 14z^2 + 24z^3 + 18) \div (3z + 4)$

$\Rightarrow (24z^3 + 14z^2 - 18z + 18) \div (3z + 4)$

$$
\begin{array}{r|l}
 & 8z^2 - 6z + 2 \\
\hline
3z+4 & 24z^3 + 14z^2 - 18z + 18 \\
 & 24z^3 + 32z^2 \\
 & (-) \quad (-) \\
\hline
 & -18z^2 - 18z \\
 & -18z^2 - 24z \\
 & (+) \quad (+) \\
\hline
 & 6z + 18 \\
 & 6z + 8 \\
 & (-) \quad (-) \\
\hline
 & 10
\end{array}
$$

$\frac{24z^3}{3z} = 8z^2$

$\frac{-18z^2}{3z} = -6z$

$\frac{6z}{3z} = 2$

$\therefore$ Quotient $= 8z^2 - 6z + 2$

Remainder $= 10$

(2) The area of a rectangle is $x^2+7x+12$. If its breadth is $(x+3)$ then find its length.

Sol:-

Area of rectangle $= x^2+7x+12$

breadth $\Rightarrow b = x+3$

length $\Rightarrow l = ?$

$$l \times b = \text{Area}$$

$$l(x+3) = x^2+7x+12$$

$$l = \frac{x^2+7x+12}{x+3}$$

$$\begin{array}{r|l} & x+4 \\ \hline x+3 & x^2+7x+12 \\ & x^2+3x \\ & (-)\ (-) \\ \hline & 4x+12 \\ & 4x+12 \\ & (-)\ (-) \\ \hline & 0 \end{array}$$

$\therefore$ length $= \boxed{l = x+4}$

(3) The base of a parallelogram is $(5x+4)$. Find its height, if the area is $25x^2-16$.

Sol:- Parallelogram

$$\text{base} = 5x+4$$

$$\text{Area} = b \times h = 25x^2-16$$

$$\text{height} = ?$$

$$b \times h = 25x^2-16$$

$$h = \frac{25x^2-16}{5x+4}$$

$a^2-b^2=(a-b)(a+b)$

$$= \frac{(5x)^2-4^2}{5x+4}$$

$$= \frac{(5x-4)\cancel{(5x+4)}}{\cancel{(5x+4)}}$$

$$\boxed{h = 5x-4}$$

(4) The Sum of $(x+5)$ Observations is (x^3+125). Find the mean of the Observation.

Sol:-

Number of Observation $= x+5$

Sum of Observation $= x^3+125$

Mean of Observation $= ?$

$$\text{Mean} = \frac{\text{Sum of observation}}{\text{Number of Observation}}$$

$$= \frac{x^3+125}{x+5}$$

$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$$= \frac{x^3+5^3}{x+5}$$

$$= \frac{\cancel{(x+5)}(x^2+5^2-(5)(x))}{\cancel{(x+5)}}$$

$$\text{Mean} = x^2+25-5x$$

(5) Find the quotient and remainder for the following using synthetic division.

(i) $(x^3+x^2-7x-3) \div (x-3)$

<u>Sol</u>:- $(x^3+x^2-7x-3) \div (x-3)$

$$x-3=0$$

$$\boxed{x=3}$$

$$\begin{array}{c|cccc} 3 & 1 & 1 & -7 & -3 \\ & 0 & 3 & 12 & 15 \\ \hline & 1 & 4 & 5 & \boxed{12} \\ & x^2 & x & c & \end{array}$$

Quotient $= x^2+4x+5$

Remainder $= 12$

(ii) $(x^3+2x^2-x-4) \div x+2$

<u>Sol</u>:-

$$\begin{array}{c|cccc} & 1 & 2 & -1 & -4 \\ -2 & 0 & -2 & 0 & 2 \\ \hline & 1 & 0 & -1 & \boxed{-2} \\ & x^2 & x & c & \end{array}$$

$x+2=0$

$\boxed{x=-2}$

Quotient $= x^2-1$

Remainder $= -2$

(iii) $(3x^3 - 2x^2 + 7x - 5) \div (x+3)$

Sol:-

$$x + 3 = 0$$
$$\boxed{x = -3}$$

$$\begin{array}{r|rrrr} -3 & 3 & -2 & 7 & -5 \\ & 0 & -9 & 33 & -120 \\ \hline & 3 & -11 & 40 & \boxed{-125} \\ & x^2 & x & c & \end{array}$$

Quotient $= 3x^2 - 11x + 40$

Remainder $= -125$

(iv) $(8x^4 - 2x^2 + 6x + 5) \div (4x+1)$

Sol:-

$$4x + 1 = 0$$
$$4x = -1$$
$$\boxed{x = \frac{-1}{4}}$$

$$\begin{array}{r|rrrrr} -\frac{1}{4} & 8 & 0 & -2 & 6 & 5 \\ & 0 & -2 & \frac{1}{2} & \frac{3}{8} & -51/32 \\ \hline & 8 & -2 & \frac{-3}{2} & \frac{51}{8} & \boxed{\frac{109}{32}} \\ & x^3 & x^2 & x & c & \end{array}$$

Quotient $= \frac{1}{4}\left[8x^3 - 2x^2 - \frac{3}{2}x + \frac{51}{8}\right]$

Remainder $= 109/32$

$\left(\frac{-1}{\cancel{4}}\right)\overset{2}{\cancel{8}} = -2$

$\left(\frac{-1}{\underset{2}{\cancel{4}}}\right)(\overset{1}{\cancel{-2}}) = \frac{1}{2}$

$-2 + \frac{1}{2} = \frac{-4+1}{2} = \frac{-3}{2}$

$\left(\frac{-1}{4}\right)\left(\frac{-3}{2}\right) = \frac{3}{8}$

$6 + \frac{3}{8} = \frac{48+3}{8}$

$= \frac{51}{8}$

$\left(\frac{-1}{4}\right)\left(\frac{51}{8}\right) = -\frac{51}{32}$

$5 - \frac{51}{32} = \frac{160-51}{32} = \frac{109}{32}$

(6) If the quotient obtained on dividing $(8x^4 - 2x^2 + 6x - 7)$ by $(2x+1)$ is $(4x^3 + px^2 - qx + 3)$, then find p, q and also the remainder.

Sol:-

Given quotient $= 4x^3 + px^2 - qx + 3$

$$(8x^4 - 2x^2 + 6x - 7) \div 2x + 1$$

$$2x + 1 = 0$$

$$2x = -1$$

$$\boxed{x = -\frac{1}{2}}$$

$\frac{-1}{\cancel{2}_1} \times \cancel{8}^4 = -4$

$\frac{-1}{\cancel{2}_1} \times -\cancel{4}^2 = 2$

$\frac{-1}{\cancel{2}_1} \times \cancel{6}^3 = -3$

$-\frac{1}{2}$	8	0	-2	6	-7
	0	-4	2	0	-3
	8	-4	0	6	$\boxed{-10}$
	x^3	x^2	x	c	

Quotient obtained =

$$= \frac{1}{2}[8x^3 - 4x^2 + 6]$$

$$= \frac{1}{\cancel{2}} \times \cancel{2}[4x^3 - 2x^2 + 3]$$

Obtained Quotient $= 4x^3 - 2x^2 + 0x + 3$

Given Quotient $= 4x^3 + px^2 - qx + 3$

$\therefore$ $p =$ coeff of x^2 | $q =$ coeff of x

$\therefore \boxed{p = -2}$ | $\therefore \boxed{q = 0}$

$\therefore$ Remainder $= -10$

(7) If the quotient obtained on dividing $3x^3 + 11x^2 + 34x + 106$ by $x-3$ is $3x^2 + ax + b$, then find a, b and also the remainder.

Sol :-

Given quotient $= 3x^2 + ax + b$

$(3x^3 + 11x^2 + 34x + 106) \div (x-3)$

$$x - 3 = 0$$

$$\boxed{x = 3}$$

$$\begin{array}{c|cccc} 3 & 3 & 11 & 34 & 106 \\ & 0 & 9 & 60 & 282 \\ \hline & 3 & 20 & 94 & \boxed{388} \\ & x^2 & x & c & \end{array}$$

Obtained Quotient = $3x^2 + 20x + 24$

Given Quotient = $3x^2 + ax + b$

a = Coeff of x

$\therefore \boxed{a = 20}$

b = Constant

$\boxed{b = 24}$

$\therefore$ Remainder = 388

Exercise - 3.8

(1.) Factorise each of the following Polynomials using Synthetic division :-

(i) $x^3 - 3x^2 - 10x + 24$

Sol :-

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -10 & 24 \\ & 0 & 1 & -2 & -12 \\ \hline & 1 & -2 & -12 & \boxed{12} \end{array} \quad \boxed{r \neq 0}$$

$r \neq 0$

$\therefore (x-1)$ is not a factor

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -10 & 24 \\ & 0 & -1 & 4 & 6 \\ \hline & 1 & -4 & -6 & \boxed{18} \end{array}$$

$r \neq 0$
$\therefore (x+1)$ is not a factor

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & 0 & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$\therefore (x-2)$ is a factor

12
3 -4

$$x^2 - x - 12 = (x+3)(x-4)$$

$$\Rightarrow (x-2)(x+3)(x-4)$$

(ii) $2x^3 - 3x^2 - 3x + 2$

Sol

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -3 & 2 \\ & 0 & 2 & -1 & -4 \\ \hline & 2 & -1 & -4 & \boxed{-2} \end{array}$$

$r \neq 0$
$x-1$ is not a factor

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -3 & 2 \\ & 0 & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$r = 0$

$\therefore (x+1)$ is a factor

$$\Rightarrow 2x^2 - 5x + 2 = (x-2)(2x-1)$$

$$\Rightarrow (x+1)(x-2)(2x-1)$$

4

$\frac{\cancel{-4}^{\,2}}{\cancel{2}_{1}x} \quad \frac{-1}{2x}$

(iii) $-7x + 3 + 4x^3$

Sol:- $4x^3 + 0x^2 - 7x + 3$

$$\begin{array}{c|cccc} 1 & 4 & 0 & -7 & 3 \\ & 0 & 4 & 4 & -3 \\ \hline & 4 & 4 & -3 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$r = 0$

$(x-1)$ is a factor

$$4x^2 + 4x - 3 = (2x+3)(2x-1)$$

$$\Rightarrow (x-1)(2x+3)(2x-1)$$

12

$\frac{{}^{3}\cancel{6}}{\cancel{4}_{2}x} \quad \frac{-\cancel{2}^{\,1}}{\cancel{4}_{2}x}$

(iv) $x^3 + x^2 - 14x - 24$

Sol:- Let $P(x) = x^3 + x^2 - 14x - 24$

when $x = 1$, $P(1) = 1^3 + 1^2 - 14(1) - 24$

$$= 1 + 1 - 14 - 24$$

$$= 2 - 38$$

$$P(1) = -36$$

$\therefore r \neq 0$

$(x-1)$ is not a factor

When $x = -1$; $P(-1) = (-1)^3 + (-1)^2 - 14(-1) - 24$

$$= \cancel{-1} + \cancel{1} + 14 - 24$$

$$P(-1) = -10$$

$\therefore r \neq 0$

$(x+1)$ is not a factor

When $x = 2$; $P(2) = (2)^3 + (2)^2 - 14(2) - 24$

$$= 8 + 4 - 28 - 24$$

$$= 12 - 52$$

$$P(2) = 40$$

$\therefore r \neq 0$

$(x-2)$ is not a factor

When $x = -2$; $P(-2) = (-2)^3 + (-2)^2 - 14(-2) - 24$

$$= -8 + 4 + 28 - 24$$

$$= -32 + 32$$

$$P(-2) = 0$$

$r = 0$

$\therefore (x+2)$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & 0 & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$\therefore (x+2)$ is a factor

$$\begin{array}{c} -12 \\ \wedge \\ -4 \quad 3 \end{array}$$

$\Rightarrow x^2 - x - 12 = (x-4)(x+3)$

$\Rightarrow (x+2)(x-4)(x+3)$

(v) $x^3 - 7x + 6$

Sol:- $x^3 + 0x^2 - 7x + 6$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & 0 & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$\therefore (x-1)$ is a factor

$$\begin{array}{c} -6 \\ \wedge \\ 3 \quad -2 \end{array}$$

$\Rightarrow x^2 + x - 6 = (x+3)(x-2)$

$\Rightarrow (x-1)(x+3)(x-2)$

(vi) $x^3 - 10x^2 - x + 10$

Sol $x^3 - 10x^2 - x + 10$

$$\begin{array}{c|cccc} 1 & 1 & -10 & -1 & 10 \\ & 0 & 1 & -9 & -10 \\ \hline & 1 & -9 & -10 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

-10

$-10 \quad 1$

$\therefore (x-1)$ is a factor

$\Rightarrow x^2 - 9x - 10 = (x-10)(x+1)$

$\Rightarrow (x-1)(x-10)(x+1)$

Exercise - 3.9

(1) Find the G.C.D of the following:-

(i) p^5, p^{11}, p^9

Sol p^5, p^{11}, p^9

$GCD = p^5$

(ii) $4x^3, y^3, z^3$

Sol:- $4x^3, y^3, z^3 \Rightarrow \boxed{G.C.D = 1}$

(iii) $9a^2b^2c^3,\ 15a^3b^2c^4$

Sol :- $9a^2b^2c^3 = 3\times 3\, a^2b^2c^3$

$15a^3b^2c^4 = 3\times 5\, a^3b^2c^4$

$\therefore \boxed{GCD = 3a^2b^2c^3}$

(iv) $64x^8,\ 240x^6$

Sol :- $64x^8 = 2\times 2\times 2\times 2\times 2\times 2\times x^8$

$240x^6 = 2\times 2\times 2\times 2\times 3\times 5\times x^6$

$\therefore GCD = 2\times 2\times 2\times 2\times x^6$

$\boxed{GCD = 16x^6}$

$$\begin{array}{r|l} 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

(v) $ab^2c^3,\ a^2b^3c,\ a^3bc^2$

Sol :- $ab^2c^3,\ a^2b^3c,\ a^3bc^2$

$\boxed{GCD = abc}$

(vi) $35x^5y^3z^4,\ 49x^2yz^3,\ 14xy^2z^2$

Sol :- $35x^5y^3z^4 = 7\times 5\, x^5y^3z^4$

$49x^2yz^3 = 7\times 7\, x^2yz^3$

$14xy^2z^2 = 7\times 2\, xy^2z^2$

$\therefore \boxed{GCD = 7xyz^2}$

(vii) $25ab^3c$, $100a^2bc$, $125ab$

Sol:- $25ab^3c = 5\times5ab^3c$

$100a^2bc = 2\times2\times5\times5a^2bc$

$125ab = 5\times5\,ab$

$GCD = 5\times5ab$

$\boxed{GCD = 25ab}$

(viii) $3abc$, $5xyz$, $7pqr$

Sol:- $3abc$, $5xyz$, $7pqr$

$\therefore \boxed{GCD = 1}$

(2) Find the G.C.D of the following:

i) $(2x+5)$; $(5x+2)$

Sol:- $\boxed{GCD = 1}$

(ii) a^{m+1}, a^{m+2}, a^{m+3}

Sol:-

$$a^{m+1} = a^m \times a^1$$

$$a^{m+2} = a^m \times a^2$$

$$a^{m+3} = a^m \times a^3$$

$$\therefore GCD = a^m \times a^1$$

$$\boxed{GCD = a^{m+1}}$$

(iii) $2a^2+a,\ 4a^2-1$

Sol:- $2a^2+a = a(2a+1)$

$$4a^2-1 = (2a)^2 - 1^2$$

$$= (2a-1)(2a+1)$$

$$\therefore \boxed{GCD = 2a+1}$$

(iv) $3a^2,\ 5b^3,\ 7c^4$

Sol:- $3a^2,\ 5b^3,\ 7c^4$

$$GCD = 1$$

(v) x^4-1 ; x^2-1

Sol :- $x^4-1 = (x^2)^2-1$

$= (x^2-1)(x^2+1)$

$= (x-1)(x+1)(x^2+1)$

$x^2-1 = (x-1)(x+1)$

$\therefore \boxed{GCD = (x-1)(x+1)}$

(vi) a^3-9ax^2 ; $(a-3x)^2$

Sol :- $a^3-9ax^2 = a(a^2-9x^2)$

$= a(a^2-(3x)^2)$

$= a(a-3x)(a+3x)$

$(a-3x)^2 = (a-3x)(a-3x)$

$\therefore \boxed{GCD = (a-3x)}$

Exercise - 3.10

(1) Draw the graph for the following

(i) $y = 2x$

<u>Sol</u>:- $y = 2x$

$x = -2$; $y = 2(-2) = -4$

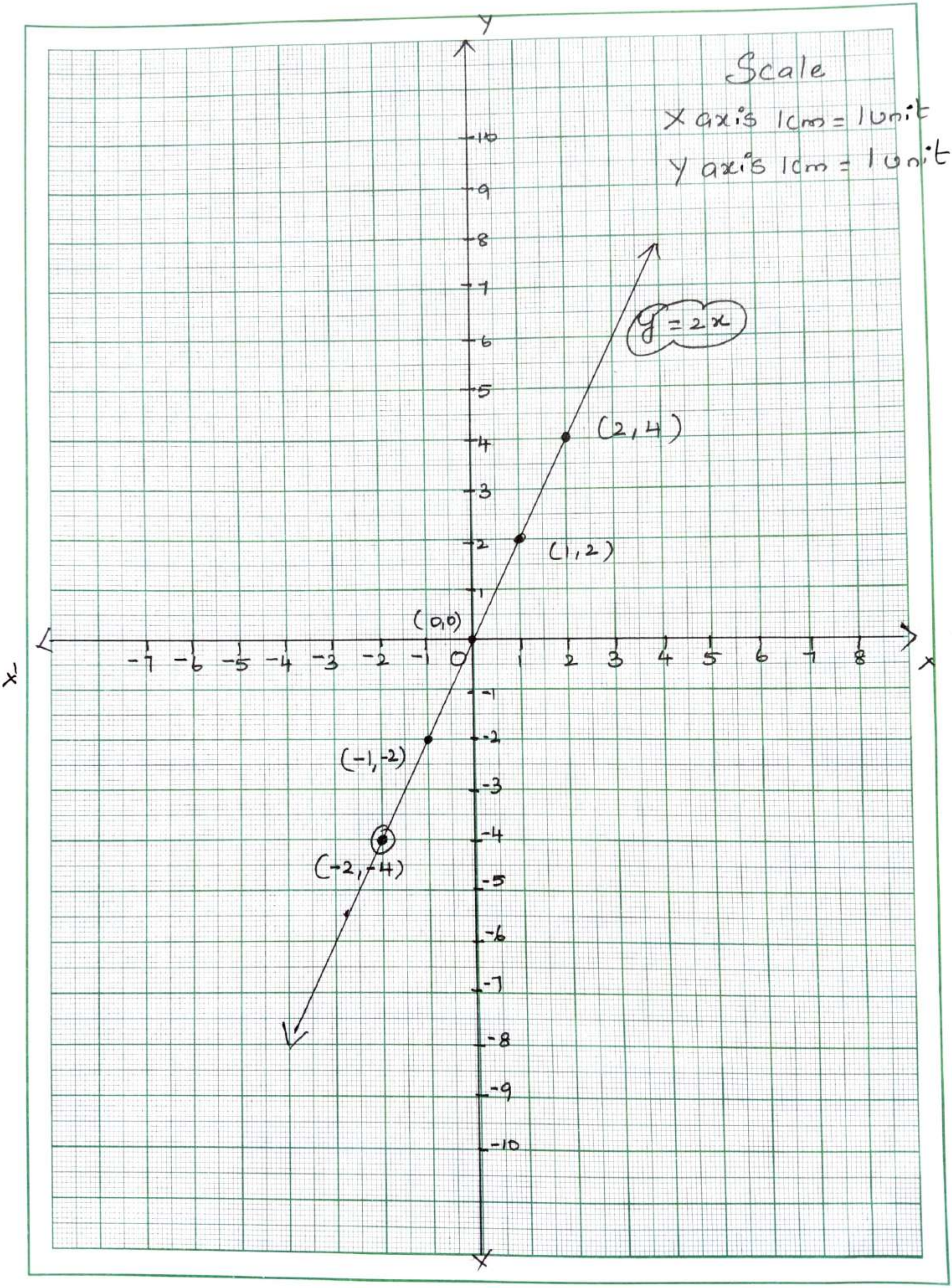
$x = -1$; $y = 2(-1) = -2$

$x = 0$; $y = 2(0) = 0$

$x = 1$; $y = 2(1) = 2$

$x = 2$; $y = 2(2) = 4$

x	-2	-1	0	1	2
y	-4	-2	0	2	4


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit
y = 2x
(2,4)
(1,2)
(0,0)
(-1,-2)
(-2,-4)
X
X'
Y
Y'

(iii) $y = \left(\frac{3}{2}\right)x + 3$

Sol:- $y = \left(\frac{3}{2}\right)x + 3$

$x = -4$; $y = \left(\frac{3}{\cancel{2}_1}\right)(-\cancel{4}^{2}) + 3$

$= 3(-2) + 3$

$= -6 + 3$

$y = -3$

$x = -2$; $y = \left(\frac{3}{\cancel{2}_1}\right)(-\cancel{2}^{1}) + 3$

$= 3(-1) + 3$

$= -3 + 3$

$y = 0$

$x = 0$; $y = \frac{3}{2}(0) + 3$

$= 0 + 3$

$y = 3$

$x = 2$; $y = \frac{3}{\cancel{2}}(\cancel{2}^{1}) + 3$

$$= 3+3$$

$$y = 6$$

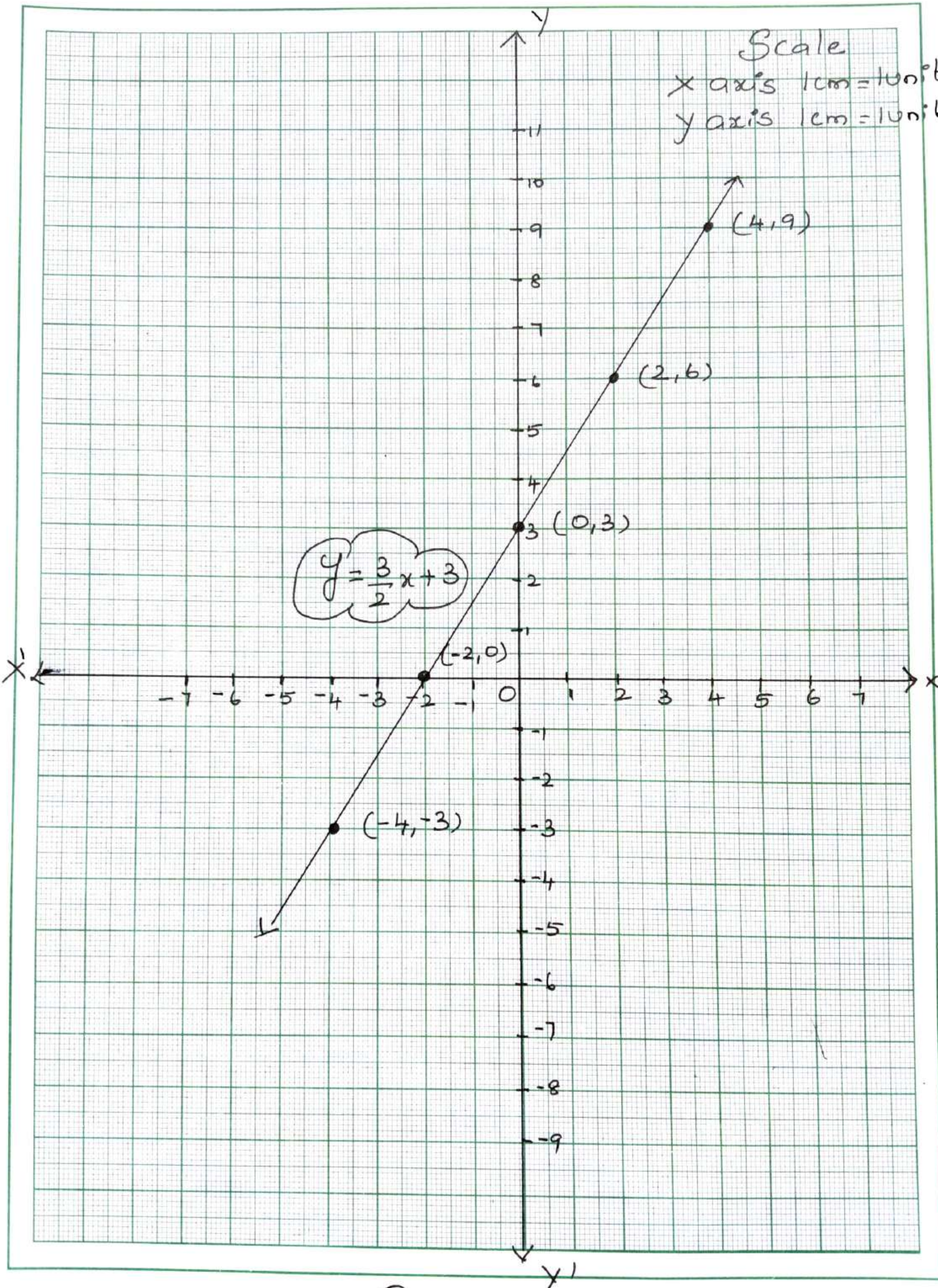
$x = +4$; $y = \frac{3}{\cancel{2}}(\cancel{4}^{2}) + 3$

$$= 3(2) + 3$$

$$= 6 + 3$$

$$y = 9$$

x	-4	-2	0	2	4
y	-3	0	3	6	9


Scale
x axis 1cm = 1unit
y axis 1cm = 1unit
$y = \frac{3}{2}x + 3$
(4,9)
(2,6)
(0,3)
(-2,0)
(-4,-3)
X'
X
Y
Y'
O

(ii) $y = 4x - 1$

<u>Sol</u> $y = 4x - 1$

$x = -1$; $y = 4(-1) - 1$

$= -4 - 1$

$\boxed{y = -5}$

$x = 0$; $y = 4(0) - 1$

$= 0 - 1$

$\boxed{y = -1}$

$x = 1$; $y = 4(1) - 1$

$= 4 - 1$

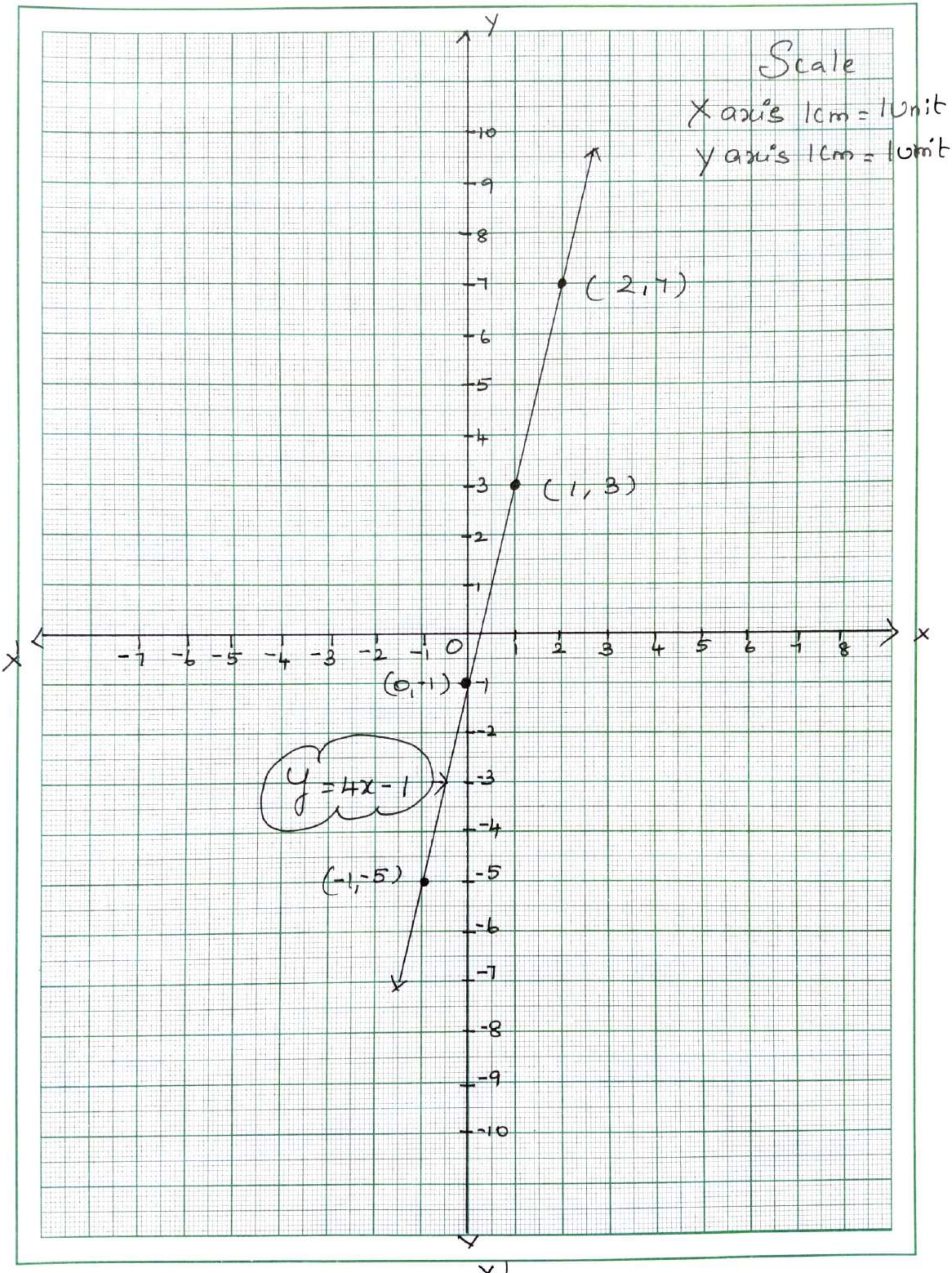
$\boxed{y = 3}$

$x = 2$; $y = 4(2) - 1$

$= 8 - 1$

$\boxed{y = 7}$

X	-1	0	1	2
Y	-5	-1	3	7


Scale
X axis 1cm = 1unit
y axis 1cm = 1unit
(2, 7)
(1, 3)
(0, -1)
(-1, -5)
y = 4x - 1
X
X'
Y
Y'
0

(iv) $3x + 2y = 14$

Sol:- $3x + 2y = 14$

$$2y = 14 - 3x$$

$$y = \frac{14 - 3x}{2}$$

$x = -2$; $y = \dfrac{14 - 3(-2)}{2}$

$$= \frac{14 + 6}{2}$$

$$= \frac{20}{2}$$

$$\boxed{y = 10}$$

$x = 0$; $y = \dfrac{14 - 3(0)}{2}$

$$= \frac{14}{2}$$

$$\boxed{y = 7}$$

$x = 2$; $y = \dfrac{14 - 3(2)}{2}$

$$= \frac{14 - 6}{2} = \frac{8}{2}$$

$$\boxed{y = 4}$$

$x = 4$; $y = \dfrac{14 - 3(4)}{2}$

$= \dfrac{14 - 12}{2}$

$= \dfrac{2}{2}$

$\boxed{y = 1}$

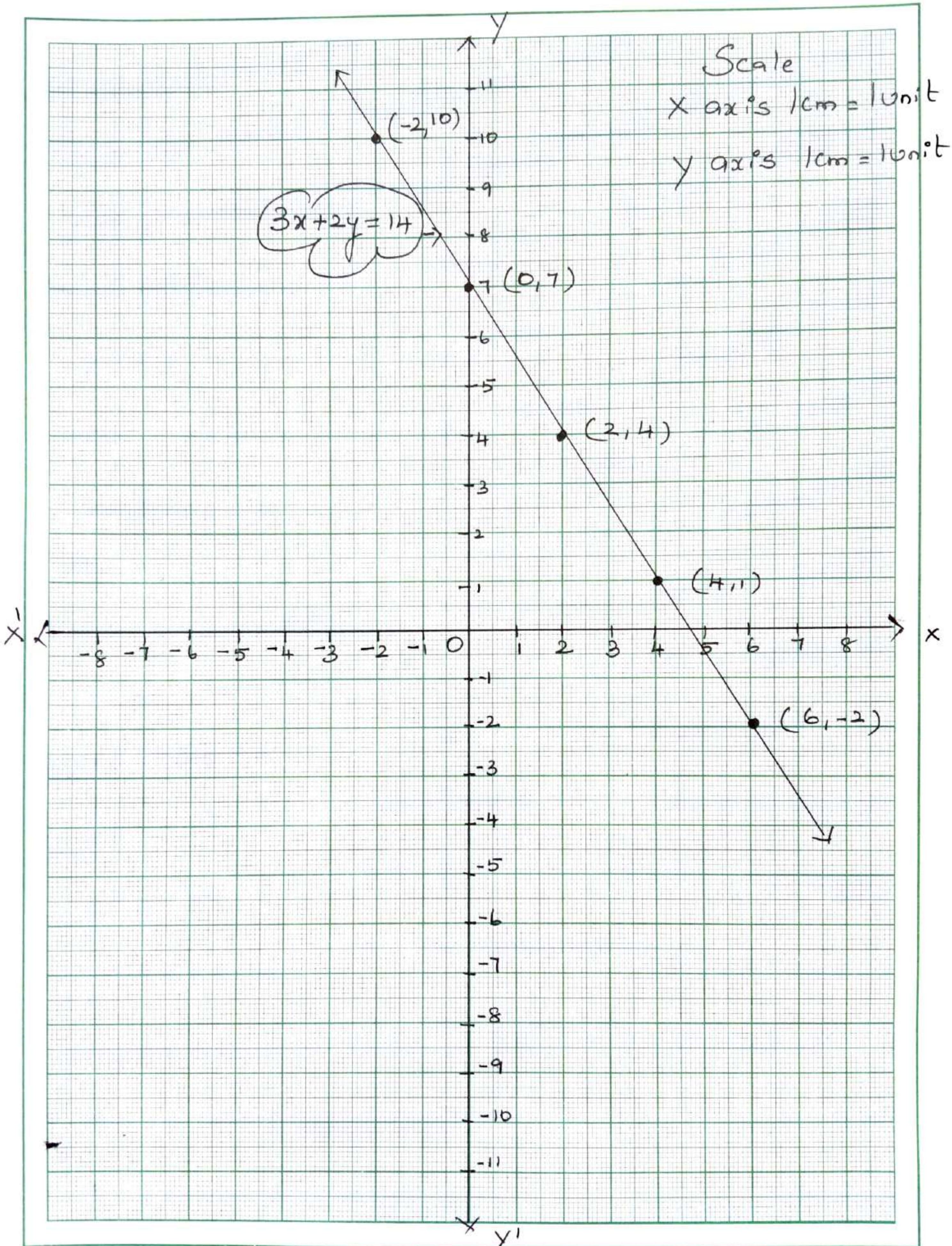
$x = 6$; $y = \dfrac{14 - 3(6)}{2}$

$= \dfrac{14 - 18}{2}$

$= \dfrac{-4}{2}$

$\boxed{y = -2}$

x	-2	0	2	4	6
y	10	7	4	1	-2


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit.
$3x+2y=14$
(-2,10)
(0,7)
(2,4)
(4,1)
(6,-2)
X
X'
Y
Y'

(2) Solve graphically

(i) $x+y=7$; $x-y=3$

Sol:- $x+y=7 \rightarrow (1) \Rightarrow \boxed{y=7-x}$

$x-y=3 \rightarrow (2) \Rightarrow x-3=y$

$\Rightarrow \boxed{y=x-3}$

From (1)

x	-2	-1	0	1	2
7	7	7	7	7	7
$-x$	2	1	0	-1	-2
y	9	8	7	6	5

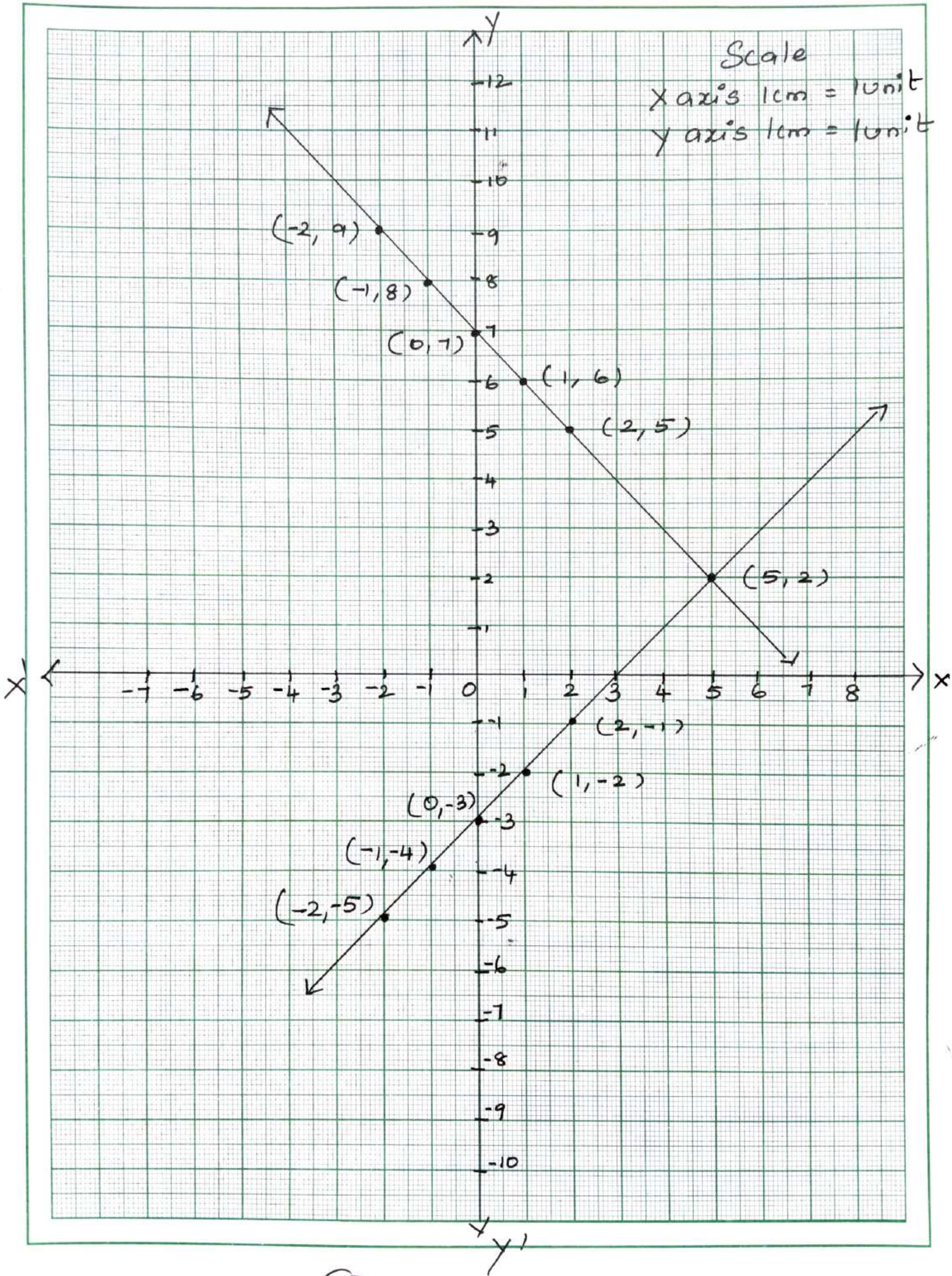
Plot: $(-2,9)$ $(-1,8)$ $(0,7)$ $(1,6)$ $(2,5)$

From (2)

x	-2	-1	0	1	2
-3	-3	-3	-3	-3	-3
y	-5	-4	-3	-2	-1

Plot: $(-2,-5)$ $(-1,-4)$ $(0,-3)$ $(1,-2)$ $(2,-1)$

$\therefore$ Point of intersection $= \{5,2\}$


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit
(-2, 9)
(-1, 8)
(0, 7)
(1, 6)
(2, 5)
(5, 2)
(2, -1)
(1, -2)
(0, -3)
(-1, -4)
(-2, -5)
X
X'
Y
Y'

(ii) $3x+2y=4$; $9x+6y-12=0$

Sol :- $3x+2y=4 \longrightarrow (1)$

$9x+6y=12$ ÷ by 3

$3x+2y=4 \longrightarrow (2)$

From (1)

$3x+2y=4$

$2y=4-3x$

$y=\frac{4-3x}{2}$

x	-4	-2	0	2	4
4	4	4	4	4	4
$-3x$	12	6	0	-6	-12
$4-3x$	16	10	4	-2	-8
$y=\frac{4-3x}{2}$	$\frac{16}{2}=8$	$\frac{10}{2}=5$	$\frac{4}{2}=2$	$\frac{-2}{2}=-1$	$\frac{-8}{2}=-4$

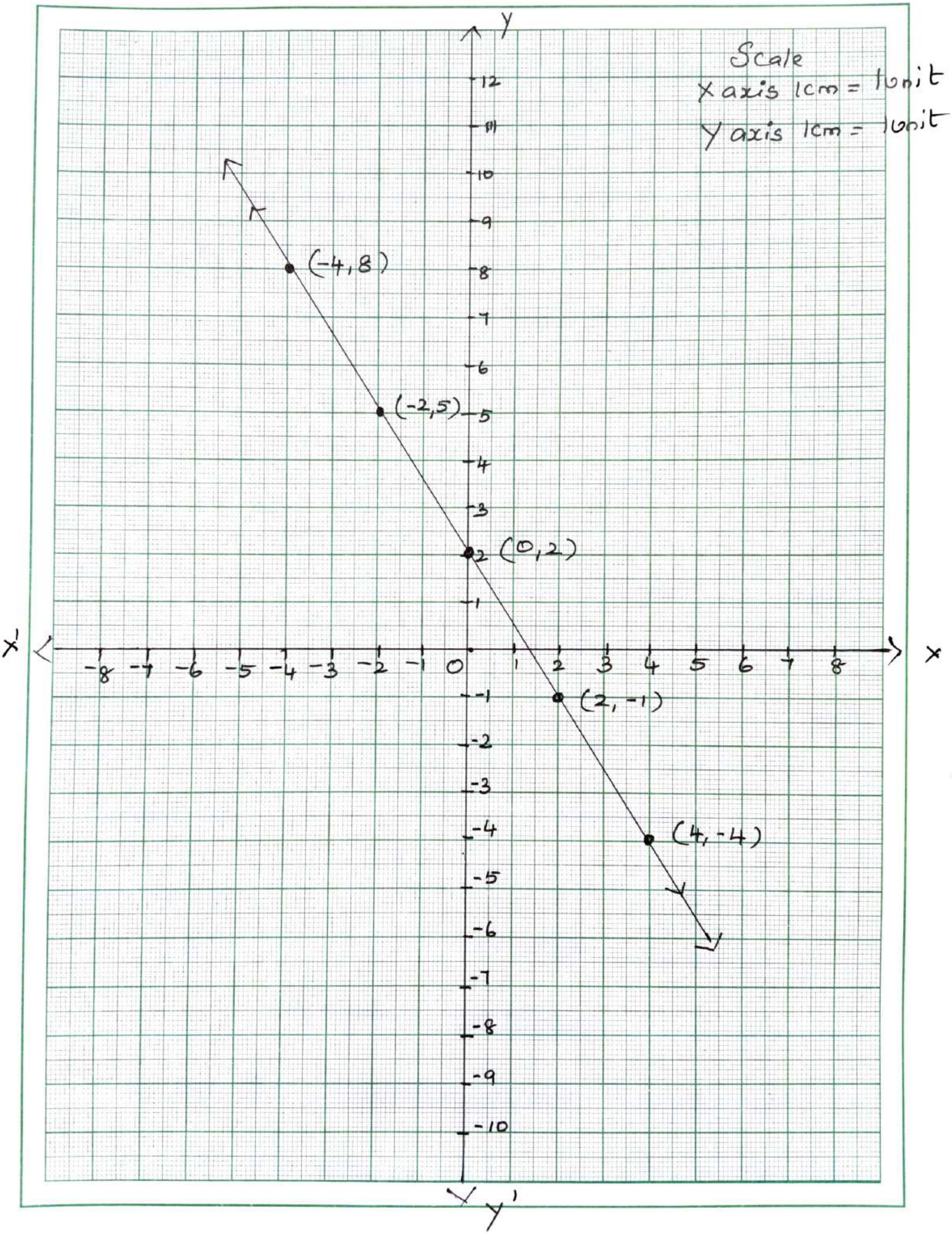
Plot: (-4, 8) (-2, 5) (0, 2) (2, -1) (4, -4)

From (2)

$$3x + 2y = 4$$

$\therefore$ eqn (1) = eqn (2)

It has infinite number of solution


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit
(-4, 8)
(-2, 5)
(0, 2)
(2, -1)
(4, -4)
X
X'
Y
Y'

(iii) $\frac{x}{2} + \frac{y}{4} = 1$; $\frac{x}{2} + \frac{y}{4} = 2$

Sol: -

$\frac{x^{\times 2}}{2} + \frac{y^{\times 1}}{4} = 1$

$\frac{2x + y}{4} = 1$

$2x + y = 4 \longrightarrow (1)$

$\frac{x^{\times 2}}{2} + \frac{y^{\times 1}}{4} = 2$

$\frac{2x + y}{4} = 2$

$2x + y = 8 \longrightarrow (2)$

From (1)

$$2x + y = 4$$

$$y = 4 - 2x$$

x	-2	-1	0	1	2
4	4	4	4	4	4
$-2x$	4	2	0	-2	-4
y	8	6	4	2	0

Plot: (-2, 8), (-1, 6), (0, 4) (1, 2) (2, 0)

From (2)

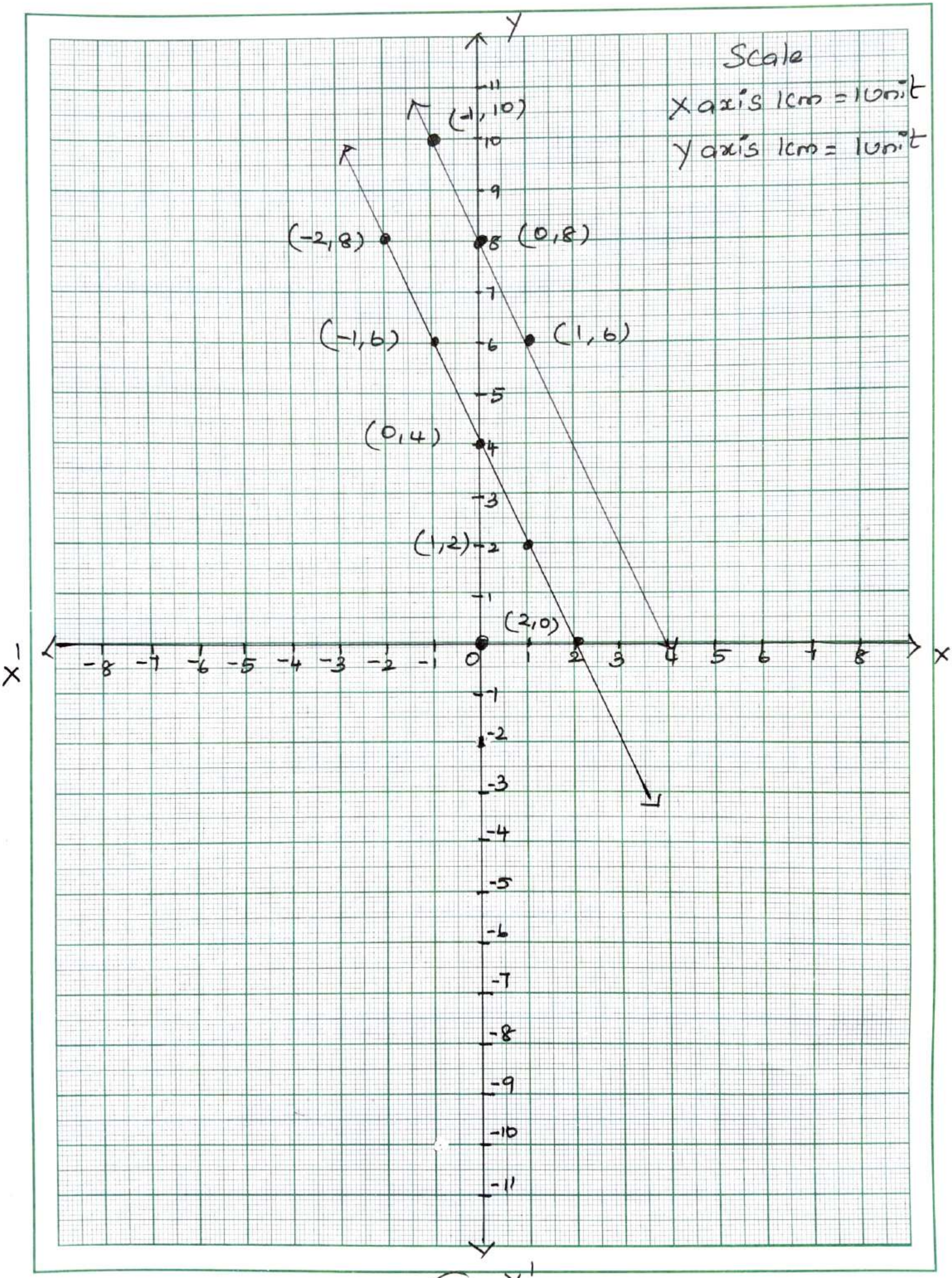
$2x + y = 8$

$y = 8 - 2x$

x	-1	0	1
8	8	8	8
-2	$+2$	0	-2
y	$+10$	8	6

Plot: $(-1, +10)$ $(0, 8)$ $(1, 6)$

The system of solution has No solution


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit
(-1,10)
(-2,8)
(0,8)
(-1,6)
(1,6)
(0,4)
(1,2)
(2,0)
X'
X
Y
Y'

(iv) $x - y = 0$; $y + 3 = 0$

Sol:- $x - y = 0 \rightarrow$ (1)

$y + 3 = 0 \rightarrow$ (2)

From (1)

$x - y = 0$

$\boxed{x = y}$

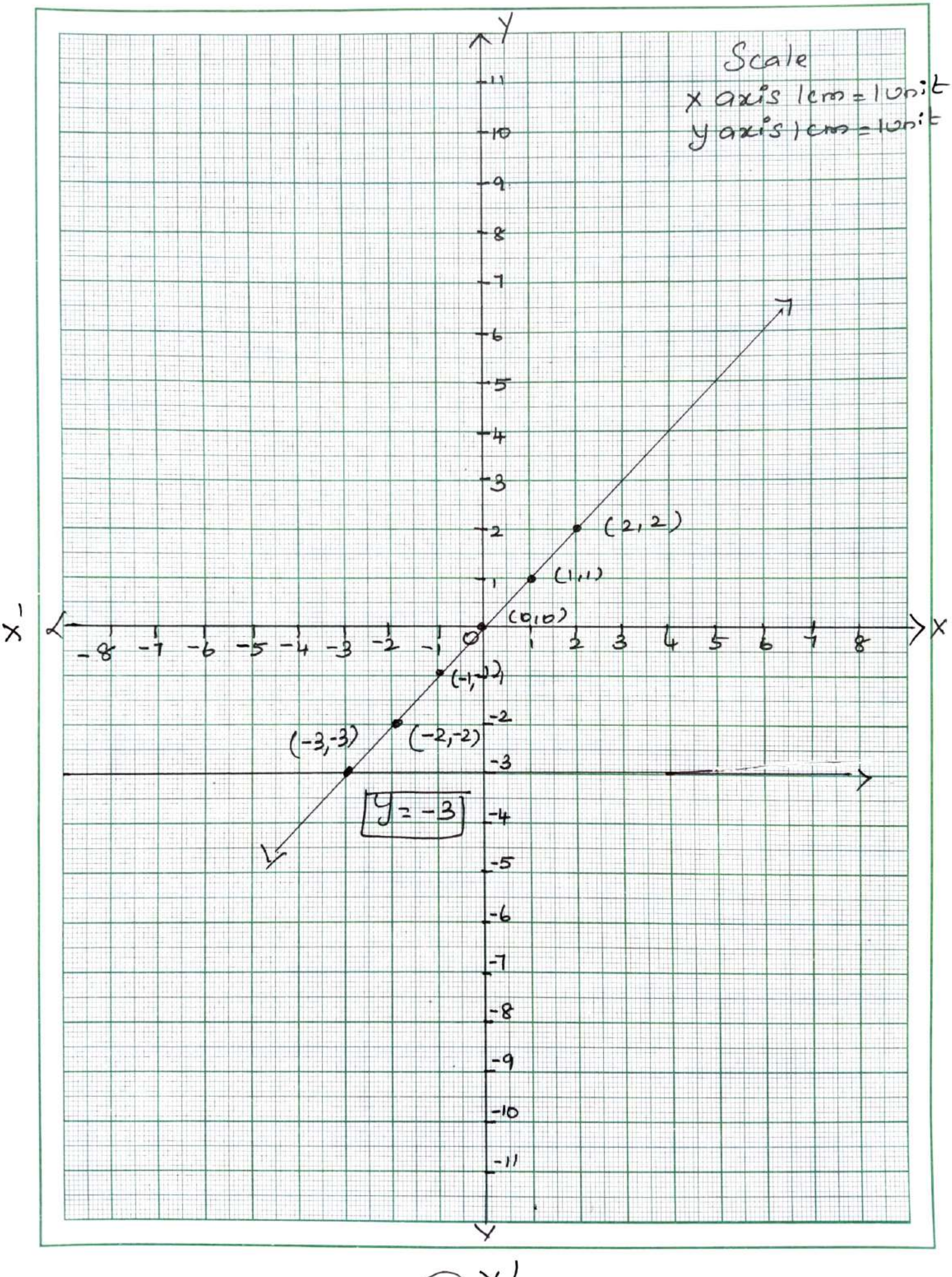
plot:- $(-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2)$

From (2)

$y + 3 = 0$

$\boxed{y = -3}$

Solution $= \{-3, -3\}$


Scale
x axis 1cm = 1unit
y axis 1cm = 1unit
(2, 2)
(1,1)
(0,0)
(-1,-1)
(-2,-2)
(-3,-3)
y = -3
X
X'
Y
Y'

(v) $y = 2x+1$; $y + 3x - 6 = 0$

Sol:- $y = 2x+1 \rightarrow (1)$

$y + 3x - 6 = 0$

$y = -3x + 6 \rightarrow (2)$

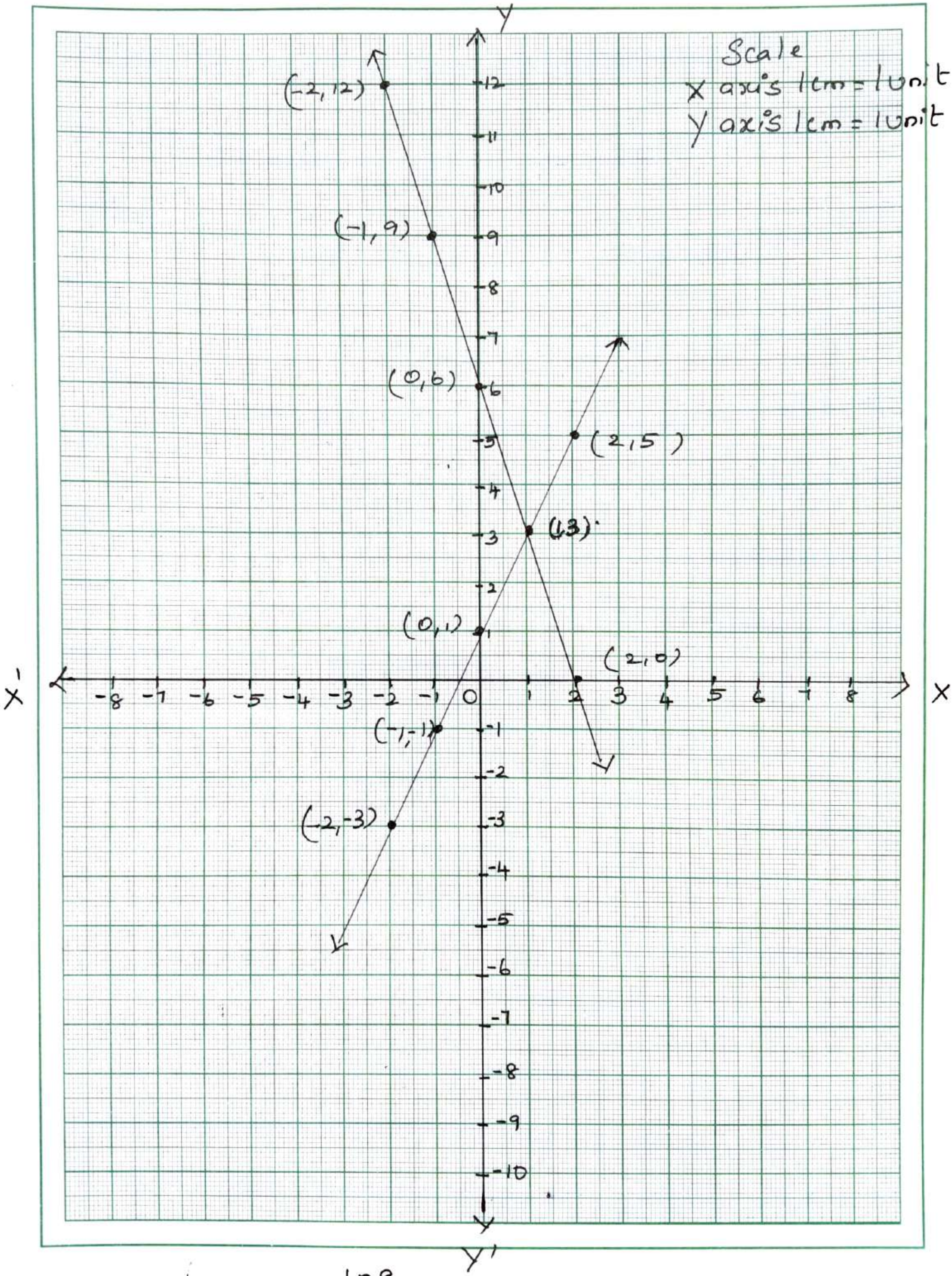
From (1)

x	-2	-1	0	1	2
$2x$	-4	-2	0	2	4
1	1	1	1	1	1
y	-3	-1	1	3	5

plot: $(-2,-3), (-1,-1), (0,1), (1,3), (2,5)$

From (2)

x	-2	-1	0	1	2
$-3x$	6	3	0	-3	-6
6	6	6	6	6	6
y	12	9	6	3	0


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit
(-2, 12)
(-1, 9)
(0, 6)
(2, 5)
(1, 3)
(0, 1)
(2, 0)
(-1, -1)
(-2, -3)
X
X'
Y
Y'

Plot:- $(-2, 12), (-1, 9), (0, 6), (1, 3), (2, 0)$

$\therefore$ Solution Set $= \{1, 3\}$

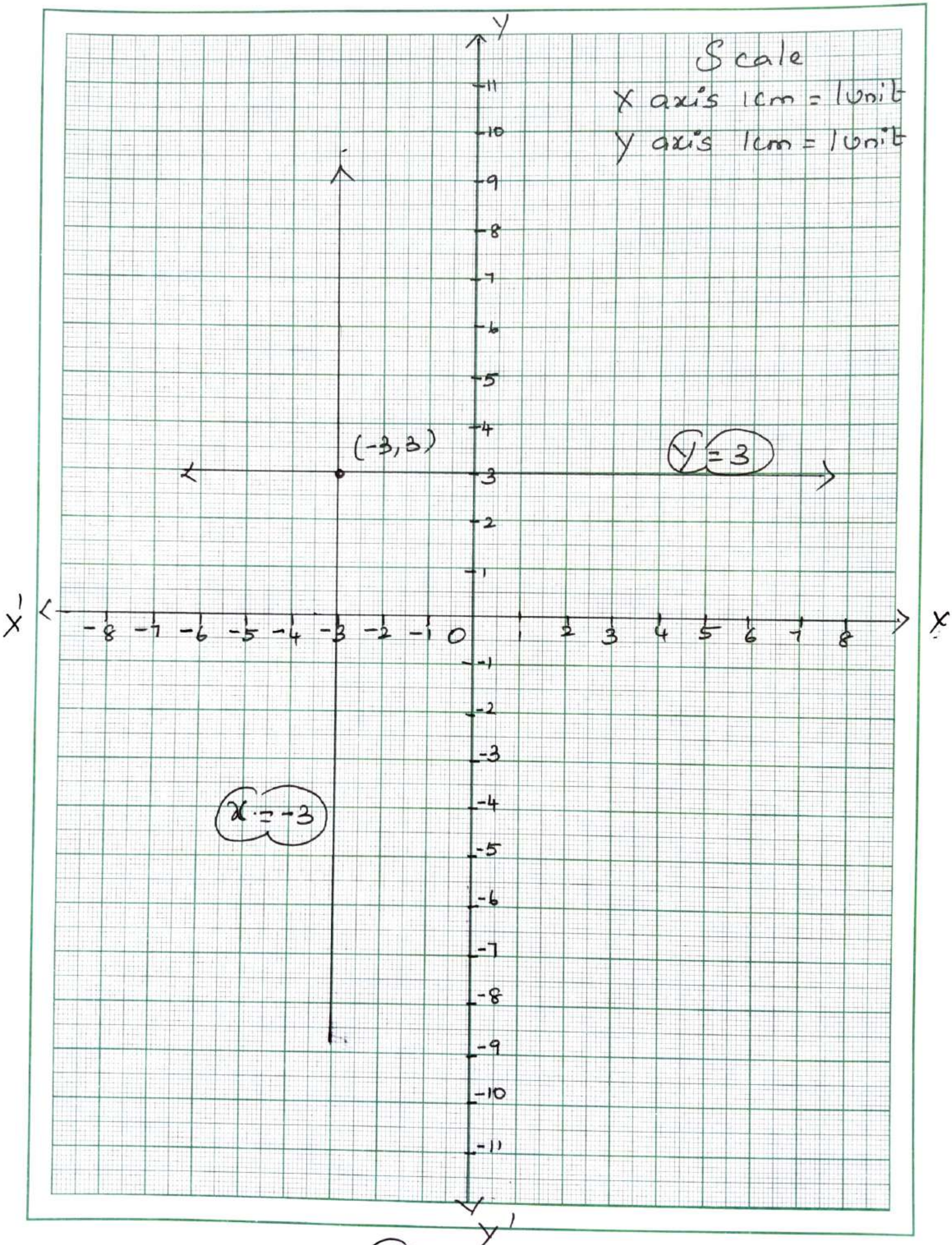
(Vi) $x = -3; \quad y = 3$

Sol

$$x = -3 \longrightarrow (1)$$

$$y = 3 \longrightarrow (2)$$

Solution $= \{-3, 3\}$


Scale
X axis 1cm = 1unit
Y axis 1cm = 1unit
(-3,3)
y = 3
x = -3
X'
X
Y
Y'
O

③ Two Cars are 100 miles apart. If they drive towards each other they will meet in 1 hr. If they drive in the same direction they will meet in 2 hrs. Find their speed by using graphical method.

Sol :-

Driving Toward Each other

Car A $\xrightarrow[S = x]{}$ $\xleftarrow[S = y]{}$ Car B

Speed = $x + y$

T = 1 hr

D = 100 miles

$T = \frac{D}{S}$

$1 = \frac{100}{x+y}$

$\boxed{x + y = 100} \rightarrow$ (1)

Driving in Same Direction

Car A ($S = x$) $\longrightarrow$

Car B $\longrightarrow$ ($S = y$)

$S = x - y$

T = 2 hr

D = 100 miles.

$T = \frac{D}{S}$

$2 = \frac{100}{x-y}$

$2(x - y) = 100$

$x - y = \frac{100}{2}$

$\boxed{x - y = 50} \rightarrow$ (2)

From 1

$$x + y = 100$$

$$y = 100 - x$$

x	-100	0	100
100	100	100	100
$-x$	100	0	-100
y	200	100	0

Plot :- $(-100, 200)$, $(0, 100)$ $(100, 0)$

From 2

$$x - y = 50$$

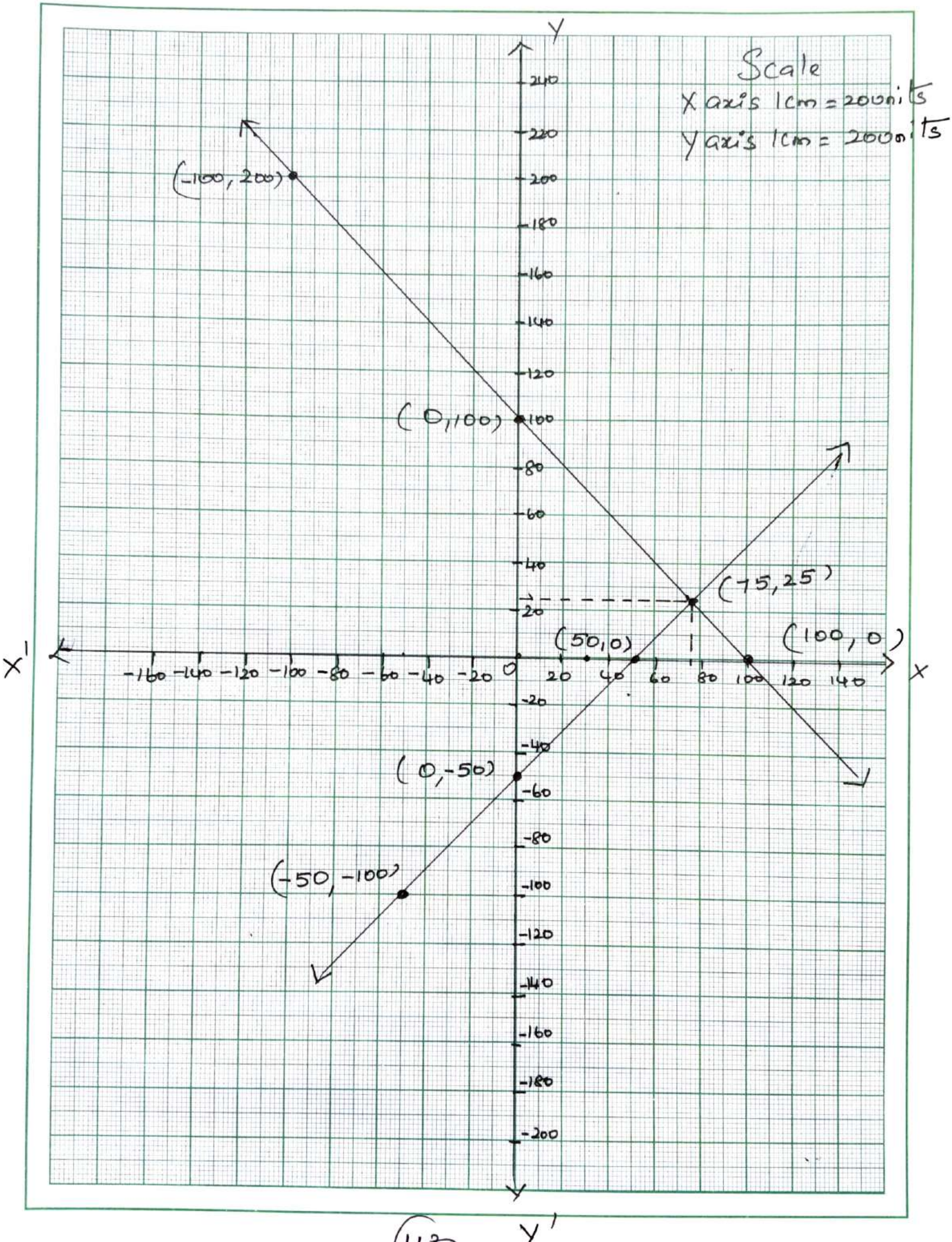
$$x - 50 = y$$

$$y = x - 50$$

x	-50	0	50
-50	-50	-50	-50
y	-100	-50	0

Plot :- $(-50, -100)$, $(0, -50)$, $(50, 0)$

Solution $= \{75, 25\}$



Scale
X axis 1cm = 20units
Y axis 1cm = 20units
Y
X
X'
Y'
(-100, 200)
(0,100)
(75,25)
(50,0)
(100, 0)
(0,-50)
(-50, -100)
O

Exercise - 3.11

(1) Solve, using the method of Substitution.

(i) $2x - 3y = 7$; $5x + y = 9$

Sol:- $2x - 3y = 7 \longrightarrow (1)$

$5x + y = 9 \longrightarrow (2)$

From (2)

$y = 9 - 5x \rightarrow (3)$

Put (3) in (1)

$2x - 3(9 - 5x) = 7$

$2x - 27 + 15x = 7$

$17x = 7 + 27$

$17x = 34$

$x = \frac{34}{17}$

$\boxed{x = 2}$

Put $x=2$ in (3)

$$y = 9-5(2)$$

$$y = 9-10$$

$$\boxed{y=-1}$$

$$\therefore \boxed{\begin{aligned} x &= 2 \\ y &= -1 \end{aligned}}$$

(ii) $1.5x + 0.1y = 6.2$; $3x - 0.4y = 11.2$

Sol :

$$1.5x + 0.1y = 6.2 \rightarrow (1) \times^{ly} \text{ by } 10$$

$$3x + 0.4y = 11.2 \rightarrow (2) \times^{ly} \text{ by } 10$$

$$\Rightarrow \quad 15x + y = 62 \rightarrow (1)$$

$$30x + 4y = 112 \rightarrow (2)$$

From (1)

$$15x + y = 62$$

$$y = 62 - 15x \rightarrow (3)$$

Put (3) in (2)

$$30x - 4(62 - 15x) = 112$$

$$30x - 248 + 60x = 112$$

$$90x = 112 + 248$$

$$90x = 360$$

$$x = \frac{\cancel{360}^{4}}{\cancel{90}}$$

$$\boxed{x = 4}$$

Put $x = 4$ in (3)

$$y = 62 - 15x$$

$$= 62 - 15(4)$$

$$= 62 - 60$$

$$\boxed{y = 2}$$

$$\therefore \boxed{\begin{aligned} x &= 4 \\ y &= 2 \end{aligned}}$$

(iii) 10% of x + 20% of $y = 24$;

$3x - y = 20$

Sol:-

$$10\% \text{ of } x + 20\% \text{ of } y = 24$$

$$\Rightarrow \frac{10}{100}x + \frac{20}{100}y = 24$$

$$\frac{x}{10} + \frac{2y}{10} = 24$$

$$\frac{x+2y}{10} = 24$$

$$x + 2y = 240 \rightarrow (1)$$

$$3x - y = 20 \rightarrow (2)$$

From (1)

$$x + 2y = 240$$

$$x = 240 - 2y \rightarrow (3)$$

Put (3) in (2)

$$3(240-2y) - y = 20$$

$$720 - 6y - y = 20$$

$$720 - 7y = 20$$

$$720 - 20 = 7y$$

$$700 = 7y$$

$$7y = 700$$

$$y = \frac{700}{7}$$

$$\boxed{y = 100}$$

Put $y = 100$ in (3)

$$x = 240 - 2(100)$$

$$= 240 - 200$$

$$\boxed{x = 40}$$

$$\therefore \boxed{\begin{array}{l} x = 40 \\ y = 100 \end{array}}$$

(iv) $\sqrt{2}x - \sqrt{3}y = 1$; $\sqrt{3}x - \sqrt{8}y = 0$

Sol:-

$$\sqrt{2}x - \sqrt{3}y = 1 \longrightarrow (1)$$

$$\sqrt{3}x - \sqrt{8}y = 0 \longrightarrow (2)$$

From (2)

$$\sqrt{3}x - \sqrt{8}y = 0$$

$$\sqrt{3}x = \sqrt{8}y$$

$$x = \frac{\sqrt{8}y}{\sqrt{3}}$$

$\sqrt{8} = \sqrt{2\times2\times2}$
$= 2\sqrt{2}$

$$\boxed{x = \frac{2\sqrt{2}y}{\sqrt{3}}} \rightarrow (3)$$

Put $x = \frac{2\sqrt{2}y}{\sqrt{3}}$ in (1)

$$\sqrt{2}\left(\frac{2\sqrt{2}y}{\sqrt{3}}\right) - \sqrt{3}y = 1$$

$$\frac{2\times2y}{\sqrt{3}} - \sqrt{3}y = 1$$

$$\frac{4y}{\sqrt{3}} - \sqrt{3}y = 1$$

$$4y - 3y = \sqrt{3}$$

$$\boxed{y = \sqrt{3}}$$

Put $y = \sqrt{3}$ in (3)

$$x = \frac{2\sqrt{2}\times\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$$

$$\boxed{x = 2\sqrt{2}}$$

$\therefore$ $\boxed{\begin{array}{l} x = 2\sqrt{2} \\ y = \sqrt{3} \end{array}}$

(2) Raman's age is three times the Sum of the ages of his two Sons. After 5 years his age will be twice the Sum of the ages of his two Sons. Find the age of Raman.

Sol :-

Let Raman's age = x

Sum of age of his two Sons $\}$ = y

$\Rightarrow$ Raman's age = 3 Times Sum of age of his two Sons.

$$x = 3y$$

$$\Rightarrow \boxed{x - 3y = 0} \longrightarrow (1)$$

After 5 years

$\Rightarrow$ Raman's age = Twice the Sum of ages of his two Sons.

$$x + 5 = 2(y + 10)$$

$x+5 = 2y+20$

$x-2y = 20-5$

$x-2y = 15 \longrightarrow (2)$

From (1)

$x-3y = 0$

$x = 3y \longrightarrow (3)$

Put (3) in (2)

$x-2y = 15$

$3y-2y = 15$

$\boxed{y = 15}$

Put $y=15$ in (3)

$x = 3(15)$

$\boxed{x = 45}$

$\therefore$ Age of Raman = 45 yrs.

(3) The middle digit of a number between 100 and 1000 is Zero and the Sum of the other digit is 13. If the digits are reversed, the number so formed exceeds the Original number by 495. Find the number.

Sol:-

Let the 3 digit Number = $x0y$

[Since '0' is the middle Number]

$\Rightarrow$ Sum of other 2 digit is 13

ie, $x + y = 13 \longrightarrow (1)$

$\Rightarrow$ Original Number $= 100x + y$

Reversed digit $= 100y + x$

Reversed digit = Original No + 495

$$100y + x = 100x + y + 495$$

$$100y + x - 100x - y = 495$$

$$99y - 99x = 495 \quad \div \text{ by } 99$$

$$y - x = 5$$

$$y = 5 + x \longrightarrow (2)$$

Put (2) in (1)

$$x + 5 + x = 13$$

$$2x = 13 - 5$$

$$2x = 8$$

$$x = \frac{8}{2}$$

$$\boxed{x = 4}$$

Put $x = 4$ in (2)

$$y = 5 + 4$$

$$\boxed{y = 9}$$

$\therefore$ The number $= x0y$

$= 409$

Exercise - 3.12

1. Solve by the method of Elimination

(i) $2x - y = 3; \quad 3x + y = 7$

Sol:- $2x - y = 3 \rightarrow (1)$

$3x + y = 7 \rightarrow (2)$

Add (1) & (2)

$$2x - \cancel{y} = 3$$
$$3x + \cancel{y} = 7$$
$$\overline{5x \quad = 10}$$

$$x = \frac{10}{5}$$

$$\boxed{x = 2}$$

Put $x = 2$ in (1)

$$2(1) - y = 3$$
$$2 - y = 3$$
$$2 - 3 = y$$
$$\therefore \boxed{y = -1}$$

$$\therefore \boxed{\begin{array}{l} x = 2 \\ y = -1 \end{array}}$$

(ii) $x - y = 5$; $3x + 2y = 25$

Sol :- $x - y = 5 \rightarrow (1)$ $\times^{ly}$ by 2

$3x + 2y = 25 \rightarrow (2)$

$$\Rightarrow \begin{array}{r} 2x - \cancel{2y} = 10 \\ 3x + \cancel{2y} = 25 \\ \hline 5x = 35 \\ \hline \end{array}$$

$$\therefore x = \frac{35}{5}$$

$$\boxed{x = 7}$$

Put $x = 7$ in (1)

$x - y = 5$

$7 - y = 5$

$-y = 5 - 7$

$\cancel{-}y = \cancel{-}2$

$$\boxed{y = 2}$$

$$\therefore \boxed{\begin{array}{l} x = 7 \\ y = 2 \end{array}}$$

(iii) $\frac{x}{10} + \frac{y}{5} = 14$; $\frac{x}{8} + \frac{y}{6} = 15$

Sol :-

$$\frac{x}{10} + \frac{y^{\times 2}}{5} = 14$$

$$\frac{x + 2y}{10} = 14$$

$$\boxed{x + 2y = 140} \rightarrow (1)$$

$$\frac{x^{\times 3}}{8_{\times 3}} + \frac{y^{\times 4}}{6_{\times 4}} = 15$$

$$\frac{3x + 4y}{24} = 15$$

$$\boxed{3x + 4y = 360} \rightarrow (2)$$

$$\begin{array}{c|l} 2 & 10, 5 \\ \hline 5 & 5, 5 \\ \hline & 1, 1 \end{array}$$

$$\begin{array}{c|l} 2 & 8, 6 \\ \hline 2 & 4, 3 \\ \hline 2 & 2, 3 \\ \hline 3 & 1, 3 \\ \hline & 1, 1 \end{array}$$

$$\begin{array}{r} 24 \\ \times 15 \\ \hline 120 \\ 24 \\ \hline 360 \end{array}$$

Solve (1) & (2)

$$x + 2y = 140 \rightarrow \times^{ly} \text{ by } 2$$

$$3x + 4y = 360$$

$$\Rightarrow \quad 2x + \cancel{4y} = 280$$

$$\underset{(-)}{} 3x \underset{(-)}{+} \cancel{4y} \underset{(-)}{=} 360$$

$$\cancel{+}x = + 80$$

$$\boxed{x = 80}$$

$$\begin{array}{r} 360 \\ -280 \\ \hline 80 \end{array}$$

Put $x = 80$ in (1)

$$80 + 2y = 140$$

$$2y = 140 - 80$$

$$2y = 60$$

$$y = \frac{60}{2}$$

$$\boxed{y = 30}$$

$$\therefore \boxed{\begin{matrix} x = 80 \\ y = 30 \end{matrix}}$$

(iv) $3(2x + y) = 7xy$; $3(x + 3y) = 11xy$

Sol:- $3(2x + y) = 7xy \Rightarrow 6x + 3y = 7xy \rightarrow (1)$

$3(x + 3y) = 11xy \Rightarrow 3x + 9y = 11xy \rightarrow (2)$

Solve (1) & (2)

$$6x + 3y = 7xy$$

$$3x + 9y = 11xy \longrightarrow \times^{ly} \text{ by } 2$$

$$\Rightarrow \quad \cancel{6x} + 3y = 7xy$$

$$\cancel{6x} + 18y = 22xy$$

$$(-) \quad (-) \quad (-)$$

$$-15y = -15xy$$

$\therefore \cancel{1}15y = \cancel{1}15xy$

$\therefore 15 = 15x$

$\therefore \frac{15}{15} = x$

$\therefore \boxed{x = 1}$

$\therefore$ Put $x = 1$ in (1)

$6(1) + 3y = 7(1)y$

$6 + 3y = 7y$

$6 = 7y - 3y$

$6 = 4y$

$4y = 6$

$y = \frac{\cancel{6}^{3}}{\cancel{4}_{2}}$

$\boxed{y = \frac{3}{2}}$

$\therefore \boxed{\begin{array}{l} x = 1 \\ y = \frac{3}{2} \end{array}}$

(v) $\frac{4}{x} + 5y = 7$; $\frac{3}{x} + 4y = 5$

Sol:- $\frac{4}{x} + 5y = 7 \rightarrow$ (1) Put $\frac{1}{x} = z$

$\frac{3}{x} + 4y = 5 \rightarrow$ (2)

$\Rightarrow \quad 4z + 5y = 7 \longrightarrow$ (3) x^{ly} by 3

$3z + 4y = 5 \rightarrow$ (4) x^{ly} by 4

$\Rightarrow \quad \cancel{12z} + 15y = 21$

$\underset{(-)}{\cancel{12z}} \underset{(-)}{+} 16y \underset{(-)}{=} 20$

$-y = 1$

$\therefore \boxed{y = -1}$

Put $y = -1$ in (1)

$\frac{4}{x} + 5(-1) = 7$

$\frac{4}{x} - 5 = 7$

$\frac{4}{x} = 7 + 5$

$\frac{4}{x} = 12$

$\frac{\cancel{4}}{\cancel{12}_3} = x \quad \therefore \boxed{x = \frac{1}{3}}$

$\therefore \boxed{\begin{array}{l} x = \frac{1}{3} \\ y = -1 \end{array}}$

(vi) $13x + 11y = 70; \quad 11x + 13y = 74$

Sol:- $13x + 11y = 70 \rightarrow (1)$

$11x + 13y = 74 \rightarrow (2)$

Add (1) & (2)

$$\begin{array}{r} 13x + 11y = 70 \\ + \; 11x + 13y = 74 \\ \hline 24x + 24y = 144 \\ \hline \end{array}$$

÷ by 24

$\Rightarrow \boxed{x + y = 6} \rightarrow (3)$

Subtract (1) & (2)

$$\begin{array}{r} 13x + 11y = 70 \\ \underset{(-)}{} 11x \underset{(-)}{+} 13y \underset{(-)}{=} 74 \\ \hline 2x - 2y = -4 \\ \hline \end{array}$$

÷ by 2

$\Rightarrow \boxed{x - y = -2} \rightarrow (4)$

Solve (3) & (4)

$$\begin{array}{r} x + \cancel{y} = 6 \\ x - \cancel{y} = -2 \\ \hline 2x \quad = 4 \\ \hline \end{array}$$

$x = \frac{4}{2}$

$\boxed{x = 2}$

Put $x = 2$ in (3)

$$2 + y = 6$$

$$y = 6 - 2$$

$$\boxed{y = 4}$$

$$\boxed{\begin{array}{c} x = 2 \\ y = 4 \end{array}}$$

---

(2) The monthly income of A and B are in the ratio 3:4 and their monthly expenditure are in the ratio 5:7. If each Saves ₹ 5,000 per month, find the monthly income of each.

Sol

PERSON	INCOME (x)	EXPENDITURE (y)	SAVINGS
A	3	5	5000
B	4	7	5000

Income - Expenditure = Saving

$$3x - 5y = 5000 \rightarrow (1) \quad \times^{ly} \text{ by } 4$$
$$4x - 7y = 5000 \rightarrow (2) \quad \times^{ly} \text{ by } 3$$

$$\Rightarrow \quad \cancel{12x} - 20y = 20000$$
$$\cancel{12x} - 21y = 15000$$
$$\underset{(-)}{} \qquad \underset{(+)}{} \qquad \underset{(-)}{}$$

$$y = 5000$$

Put $y = 5000$ in (1)

$$3x - 5(5000) = 5000$$
$$3x - 25000 = 5000$$
$$3x = 5000 + 25000$$
$$3x = 30000$$
$$x = \frac{\cancel{30000}^{10000}}{\cancel{3}_{1}}$$
$$\boxed{x = 10000}$$

$\therefore$ Income of $A = 3x = 3(10000) = ₹30000$

Income of $B = 4x = 4(10000) = ₹40000$

(3) Five years ago, a man was Seven times as old as his Son, while five years hence, the man will be four times as old as his Son. Find their present age.

<u>Sol</u>:

Let age of man $= x$

age of Son $= y$

Five years ago '−'

Man = 7 times the Son

$$x - 5 = 7(y - 5)$$

$$x - 5 = 7y - 35$$

$$x - 7y = -35 + 5$$

$$\boxed{x - 7y = -30} \longrightarrow (1)$$

Five years Hence '+'

Man = 4 times the Son

$$x + 5 = 4(y + 5)$$

$$x + 5 = 4y + 20$$

$$x - 4y = 20 - 5$$

$$\boxed{x - 4y = 15} \longrightarrow (2)$$

Solve (1) & (2)

$$\cancel{x} - 7y = -30$$
$$\underset{(-)}{\cancel{x}} \underset{(+)}{-} 4y = \underset{(-)}{15}$$

$$+3y = +45$$

$$y = \frac{45}{3}$$

$$\boxed{y = 15}$$

Put $y = 15$ in (2)

$$x - 4(15) = 15$$

$$x - 60 = 15$$

$$x = 15 + 60$$

$$x = 75$$

$\therefore$ Age of Man = 75 yrs

Age of Son = 15 yrs

Exercise - 3.13

(1) Solve by cross multiplication method:-

(i) $8x - 3y = 12$; $5x = 2y + 7$

Sol:-

$$8x - 3y - 12 = 0$$
$$5x - 2y - 7 = 0$$

$$\begin{array}{cccccc} & x & & y & & 1 \\ -3 & & -12 & & 8 & & -3 \\ -2 & (-) & -7 & (-) & 5 & (-) & -2 \end{array}$$

$$\frac{x}{21 - 24} = \frac{y}{-60 - (-56)} = \frac{1}{-16 - (-15)}$$

$$\frac{x}{-3} = \frac{y}{-60 + 56} = \frac{1}{-16 + 15}$$

$$\frac{x}{-3} = \frac{y}{-4} = \frac{1}{-1}$$

$$\frac{x}{-3} = -1 \qquad \frac{y}{-4} = -1$$

$$\boxed{x = 3} \qquad \boxed{y = 4}$$

(ii) $6x + 7y - 11 = 0$; $5x + 2y = 13$

<u>Sol</u> $6x + 7y - 11 = 0$

$5x + 2y - 13 = 0$

$$\begin{array}{ccccccc} & x & & y & & 1 & \\ 7 & & -11 & & 6 & & 7 \\ 2 & (-) & -13 & (-) & 5 & (-) & 2 \end{array}$$

$$\frac{x}{-91-(-22)} = \frac{y}{-55-(-78)} = \frac{1}{12-35}$$

$$\frac{x}{-91+22} = \frac{y}{-55+78} = \frac{1}{-23}$$

$$\frac{x}{-69} = \frac{y}{23} = \frac{1}{-23}$$

$\frac{x}{-69} = \frac{1}{-23}$

$x = \frac{-69}{-23}$

$\boxed{x = 3}$

$\frac{y}{23} = \frac{1}{-23}$

$y = \frac{23}{-23}$

$\boxed{y = -1}$

(iii) $\frac{2}{x} + \frac{3}{y} = 5 \; ; \; \frac{3}{x} - \frac{1}{y} + 9 = 0$

__Sol__ Let $\boxed{\frac{1}{x} = a}$, $\boxed{\frac{1}{y} = b}$

$\Rightarrow \; 2a + 3b - 5 = 0$

$\quad\;\; 3a - b + 9 = 0$

$$\begin{array}{cccccccc} & a & & b & & 1 & & \\ 3 & & -5 & & 2 & & 3 & \\ -1 & (-) & 9 & (-) & 3 & (-) & -1 & \end{array}$$

$$\frac{a}{27-5} = \frac{b}{-15-18} = \frac{1}{-2-9}$$

$$\frac{a}{22} = \frac{b}{-33} = \frac{1}{-11}$$

$\frac{a}{22} \Rightarrow \frac{1}{-11}$

$a = \frac{22}{-11}$

$\boxed{a = -2}$

$\frac{1}{x} \Rightarrow -2$

$\boxed{\frac{1}{-2} = x}$

$\frac{b}{-33} \Rightarrow \frac{1}{-11}$

$b = \frac{-33}{-11}$

$\boxed{b = 3}$

$\frac{1}{y} \Rightarrow 3$

$\boxed{\frac{1}{3} = y}$

(2) Akshaya has 2 rupee coins and 5 rupee coins in her purse. If in all She has 80 coins totalling ₹220, how many coins of each kind does She have.

<u>Sol</u>:-

Let 2 rupee $= x$

5 rupee $= y$

Total Coins = 80

$$x + y = 80$$

$$x + y - 80 = 0 \longrightarrow (1)$$

Total amount = 220

$$2x + 5y = 220$$

$$2x + 5y - 220 = 0 \rightarrow (2)$$

Solve (1) & (2)

$$\begin{array}{cccccc} & x & & y & & 1 \\ 1 & & -80 & & 1 & & 1 \\ 5 & (-) & -220 & (-) & 2 & (-) & 5 \end{array}$$

$$\frac{x}{-220-(-400)} = \frac{y}{-160-(-220)} = \frac{1}{5-2}$$

$$\frac{x}{-220+400} = \frac{y}{-160+220} = \frac{1}{3}$$

$$\frac{x}{180} = \frac{y}{60} = \frac{1}{3}$$

$$\frac{x}{180} = \frac{1}{3}$$

$$x = \frac{180}{3}$$

$$\boxed{x = 60}$$

$$\frac{y}{60} = \frac{1}{3}$$

$$y = \frac{60}{3}$$

$$\boxed{y = 20}$$

$\therefore$ 2 rupee coins = 60

5 rupee coins = 20

---

(3) It takes 24 hours to fill a Swimming pool using two pipes. If the pipe of Larger diameter is used for 8 hours and the pipe of the Smaller diameter is used for 18

hours. Only half of the pool is filled. How long would each pipe take to fill the swimming pool.

Sol :-

Let Time taken by larger pipe $= x$

For 1 hr $= \frac{1}{x}$

Time taken by Smaller pipe $= y$

For 1 hr $= \frac{1}{y}$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{24} \longrightarrow (1)$$

Larger pipe takes 8 hrs
Smaller pipe takes 18 hrs } = Fills half of the pool

$$\Rightarrow \frac{8}{x} + \frac{18}{y} = \frac{1}{2} \longrightarrow (2)$$

Let $\boxed{\frac{1}{x} = a}$ $\boxed{\frac{1}{y} = b}$

$(1) \Rightarrow a + b = \frac{1}{24}$

$$24(a+b) = 1$$

$$24a + 24b - 1 = 0 \longrightarrow (3)$$

$$(2) \Rightarrow 8a + 18b = \frac{1}{2}$$

$$2(8a + 18b) = 1$$

$$16a + 36b - 1 = 0 \longrightarrow (4)$$

Solve (3) & (4)

$$\begin{array}{cccccccc}
 & a & & b & & 1 & & \\
24 & & -1 & & 24 & & 24 \\
 & \times & & \times & & \times & \\
36 & (-) & -1 & (-) & 16 & (-) & 36
\end{array}$$

$$\begin{array}{r} 24 \\ \times 36 \\ \hline 144 \\ 72 \\ \hline 864 \end{array} \qquad \begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ 24 \\ \hline 384 \end{array}$$

$$\frac{a}{-24-(-36)} = \frac{b}{-16-(-24)} = \frac{1}{864-384}$$

$$\frac{a}{-24+36} = \frac{b}{-16+24} = \frac{1}{480}$$

$$\frac{a}{12} = \frac{b}{8} = \frac{1}{480}$$

$$\frac{a}{12} = \frac{1}{480}$$

$$a = \frac{\cancel{12}^{1}}{\cancel{480}_{40}}$$

$$a = \frac{1}{40}$$

$$\frac{1}{x} = a$$

$$\frac{1}{x} = \frac{1}{40}$$

$$40 = x$$

$$\boxed{x = 40}$$

$$\frac{b}{8} = \frac{1}{480}$$

$$b = \frac{\cancel{8}^{1}}{\cancel{480}_{60}}$$

$$b = \frac{1}{60}$$

$$\frac{1}{y} = b$$

$$\frac{1}{y} = \frac{1}{60}$$

$$60 = y$$

$$\boxed{y = 60}$$

$\therefore$ Larger pipe takes 40 hrs.

Smaller pipe takes 60 hrs.

Exercise - 3.14

(1) The Sum of a two digit number and the number formed by interchanging the digits is 110. If 10 is Subtracted from the first number, the new number is 4 more than 5 times the Sums of the digits of the first number. Find the first number.

Sol:-

Let Two digit Number = x, y

Original Number = $10x + y$

After interchanging the digits

New Number = $10y + x$

$\Rightarrow$ [Sum of 2 digit No.] + [No. after interchanging digit] = 110

ie, $10x + y + 10y + x = 110$

$11x + 11y - 110 = 0$ ÷ by 11

$x + y - 10 = 0 \longrightarrow$ (1)

⇒ 10 Subtracted from first Number = 4 more than 5 times the Sum of digit

ie, $10x + y - 10 = 5(x+y) + 4$

$10x + y - 10 = 5x + 5y + 4$

$10x + y - 10 - 5x - 5y - 4 = 0$

$5x - 4y - 14 = 0 \longrightarrow$ (2)

	x	y	1
1	-10	1	1
-4	-14	5	-4

$$\frac{x}{-14-40} = \frac{y}{-50-(-14)} = \frac{1}{-4-5}$$

$$\frac{x}{-54} = \frac{y}{-50+14} = \frac{1}{-9}$$

$$\frac{x}{-54} = \frac{y}{-36} = \frac{1}{-9}$$

$$\frac{x}{-54} = \frac{+1}{-9}$$

$$x = \frac{-54}{-9}$$

$$\boxed{x = 6}$$

$$\frac{y}{-36} = \frac{1}{-9}$$

$$y = \frac{-36}{-9}$$

$$\boxed{y = 4}$$

First Number $= 10x + y$

$= 10(6) + 4$

$= 60 + 4$

$= 64$

---

(2) The Sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction.

Sol: Let the numerator $= x$

Denominator $= y$

$\Rightarrow$ Sum of Numerator & Denominator $= 12$

$$\text{i.e., } x + y = 12 \longrightarrow (1)$$

$\Rightarrow$ If Denominator increased by 3 $\Big\} =$ Fraction is $\frac{1}{2}$

$$\text{i.e., } \frac{x}{y+3} = \frac{1}{2}$$

$$2x = y + 3$$

$$2x - y = 3 \longrightarrow (2)$$

Solve (1) & (2)

$$\begin{aligned} x + \cancel{y} &= 12 \\ 2x - \cancel{y} &= 3 \\ \hline 3x &= 15 \\ \hline x &= \frac{15}{3} \end{aligned}$$

$$\boxed{x = 5}$$

Put $x = 5$ in (1)

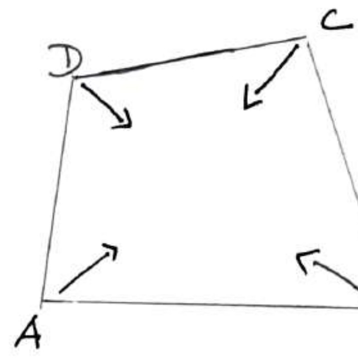
$$5 + y = 12$$

$$y = 12 - 5$$

$$\boxed{y = 7}$$

$$\therefore \boxed{\text{Fraction} = \frac{5}{7}}$$

(3) ABCD is a cyclic quadrilateral such that $\angle A = (4y+20)^\circ$, $\angle B = (3y-5)^\circ$, $\angle C = (4x)^\circ$; $\angle D = (7x+5)^\circ$. Find the four angles.



Sol :- $\angle A = (4y+20)^\circ$

$\angle B = (3y-5)^\circ$

$\angle C = (4x)^\circ$

$\angle D = (7x+5)^\circ$

Sum of opposite angles $= 180^\circ$

$\Rightarrow \quad \angle A + \angle C = 180^\circ$

$(4y+20)^\circ + (4x)^\circ = 180^\circ$

$4y + 4x = 180^\circ - 20^\circ$

$4x + 4y = 160^\circ$ (÷ by 4)

$x + y = 40^\circ \longrightarrow (1)$

$\Rightarrow \quad \angle B + \angle D = 180^\circ$

$3y - \cancel{5} + 7x + \cancel{5} = 180^\circ$

$$7x + 3y = 180^\circ \longrightarrow (2)$$

From (1) & (2)

$$x + y = 40^\circ \longrightarrow x^{ly} \text{ by } 3$$

$$7x + 3y = 180^\circ$$

$$\Rightarrow \quad 3x + \cancel{3y} = 120^\circ$$

$$\begin{array}{r} 7x + \cancel{3y} = 180^\circ \\ (-) \quad (-) \quad (-) \\ \hline \cancel{-}4x = \cancel{-}60 \\ \hline \end{array}$$

$$x = \frac{\cancel{60}^{15}}{\cancel{4}_{1}}$$

$$\boxed{x = 15}$$

Put $x = 15$ in (1)

$$x + y = 40$$

$$15 + y = 40$$

$$y = 40 - 15$$

$$\boxed{y = 25}$$

$\angle A = 4y + 20 = 4(25) + 20 = 100 + 20 = 120^\circ$

$\angle B = 3y - 5 = 3(25) - 5 = 75 - 5 = 70^\circ$

$\angle C = 4x = 4(15) = 60^\circ$

$\angle D = 7x + 5 = 7(15) + 5 = 105^\circ + 5 = 110^\circ$

$$\therefore \boxed{\begin{aligned} \angle A &= 120^\circ \\ \angle B &= 70^\circ \\ \angle C &= 60^\circ \\ \angle D &= 110^\circ \end{aligned}}$$

---

(4) On Selling a T.v at 5% gain and a fridge at 10% gain a Shopkeeper gains ₹ 2000. But if he Sells the T.v at 10% gain and the fridge at 5% loss, he gains Rs 1500 on the transaction. Find the actual price of the T.v and the fridge.

<u>Sol</u> :- Let Price of T.v $= x$

Price of Fridge $= y$

5% gain for T.V + 10% gain for Fridge = Gains ₹2000

ie, $\Rightarrow \frac{5}{100}x + \frac{10}{100}y = 2000$

$$\frac{5x + 10y}{100} = 2000$$

$$5x + 10y = 200000 \quad \div \text{ by } 5$$

$$x + 2y = 40000 \longrightarrow (1)$$

10% gain for T.V + 5% loss for fridge = Gains ₹1500

ie, $\Rightarrow \frac{10}{100}x - \frac{5}{100}y = 1500$

$$\frac{10x - 5y}{100} = 1500$$

$$10x - 5y = 150000 \quad \div \text{ by } 5$$

$$2x - y = 30000 \longrightarrow (2)$$

Solve (1) & (2)

$$x + 2y = 40000$$
$$2x - y = 30000 \longrightarrow \text{×ly by 2}$$

$$\Rightarrow \quad x + \cancel{2y} = 40000$$
$$4x - \cancel{2y} = 60000$$
$$5x = 100000$$

$$x = \frac{100000}{5}$$

$$\boxed{x = 20000}$$

Put $x = 20000$ in (1)

$$20000 + 2y = 40000$$
$$2y = 40000 - 20000$$
$$2y = 20000$$
$$y = \frac{20000}{2}$$
$$\boxed{y = 10000}$$

Price of T.V = ₹ 20000
Price of Fridge = ₹ 10000

(5) Two numbers are in the ratio 5:6. If 8 is subtracted from each of the numbers, the ratio becomes 4:5. Find the numbers.

Sol :-

Let the 2 numbers = x and y.

$\Rightarrow$ Two numbers are in ratio = 5:6

$$\text{ie, } \frac{x}{y} = \frac{5}{6}$$

$$6x = 5y$$

$$6x - 5y = 0 \longrightarrow (1)$$

$\Rightarrow$ $\left.\begin{array}{l}\text{8 Subtracted from} \\ \text{each Number}\end{array}\right\}$ = ratio 4:5

$$\text{ie, } \frac{x-8}{y-8} = \frac{4}{5}$$

$$5(x-8) = 4(y-8)$$

$$5x - 40 = 4y - 32$$

$$5x - 4y - 40 + 32 = 0$$

$$5x - 4y - 8 = 0 \longrightarrow (2)$$

$$\begin{array}{cccccccc} & x & & y & & 1 & & \\ -5 & & 0 & & 6 & & -5 & \\ -4 & & -8 & & 5 & & -4 & \end{array}$$

$$\frac{x}{40-0} = \frac{y}{0-(-48)} = \frac{1}{-24-(-25)}$$

$$\frac{x}{40} = \frac{y}{48} = \frac{1}{-24+25}$$

$$\frac{x}{40} = \frac{y}{48} = \frac{1}{1}$$

$$\frac{x}{40} = 1 \qquad \bigg\| \qquad \frac{y}{48} = 1$$

$$\boxed{x = 40} \qquad \bigg\| \qquad \boxed{y = 48}$$

$\therefore$ Two Numbers are 40 & 48

---

(6) 4 Indians and 4 Chinese can do a piece of Work in 3days. while 2 Indians and 5 Chinese can finish it in 4 days. How long would it take for 1 Indian to do it? How long would it take for 1 Chinese to do it?

Sol :-

Work done by Indians $= x$

$\therefore$ Work done by 1 Indian $= \frac{1}{x}$

Work done by Chinese $= y$

$\therefore$ Work done by 1 Chinese $= \frac{1}{y}$

$$\left.\begin{array}{l}\text{Work done by 4 Indians} \\ \text{and 4 Chinese}\end{array}\right\} = 3 \text{ days}$$

$$\therefore \left.\begin{array}{l}\text{Work done by 1 Indian} \\ \text{and 1 Chinese}\end{array}\right\} = \frac{1}{3} \text{ days}$$

$$\text{ie, } \frac{4^{\times y}}{x} + \frac{4^{\times x}}{y} = \frac{1}{3}$$

$$\frac{4y + 4x}{xy} = \frac{1}{3}$$

$$3(4x + 4y) = xy$$

$$12x + 12y = xy \longrightarrow (1)$$

$$\left.\begin{array}{l}\text{Work done by 2 Indians} \\ \text{and 5 Chinese}\end{array}\right\} = 4 \text{ days.}$$

$\therefore$ Work done by 1 Indian and 1 chinese $\Big\}$ = $\frac{1}{4}$ days

$$\Rightarrow \frac{2}{x}^{\times y} + \frac{5}{y}^{\times x} = \frac{1}{4}$$

$$\frac{2y+5x}{xy} = \frac{1}{4}$$

$$4(5x+2y) = xy$$

$$20x + 8y = xy \longrightarrow (2)$$

Solve (1) & (2)

$$12x + 12y = xy \rightarrow \times^{ly} 8$$
$$20x + 8y = xy \rightarrow \times^{ly} 12$$

$$\Rightarrow \quad 96x + \cancel{96y} = 8xy$$
$$\underset{(-)}{240x} + \underset{(-)}{\cancel{96y}} = \underset{(-)}{12xy}$$

$$\cancel{-}144x = \cancel{-}4xy$$

$$144\cancel{x} = 4\cancel{x}y$$

$$4y = 144$$

$$y = \frac{144}{4}$$

$$\boxed{y = 36}$$

Put $y = 36$ in (1)

$$12x + 12(36) = x(36)$$

$$12x + 432 = 36x$$

$$12x - 36x = -432$$

$$\cancel{-}24x = \cancel{-}432$$

$$x = \frac{432}{24}$$

$$\boxed{x = 18}$$

$$\begin{array}{r} 36 \\ \times\ 12 \\ \hline 72 \\ 36 \\ \hline 432 \end{array}$$

$$\begin{array}{r} 18 \\ 24 \overline{)432} \\ -24 \\ \hline 192 \\ -192 \\ \hline 0 \end{array}$$

$\therefore$ Work done by 1 Indian = 18 days

Work done by 1 Chinese = 36 days.

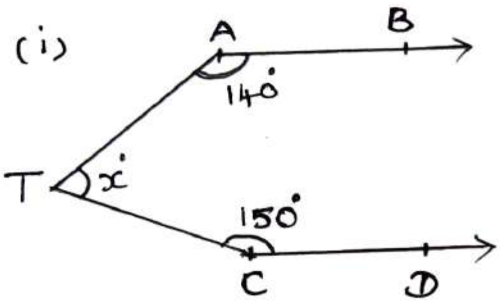
CHAPTER-4

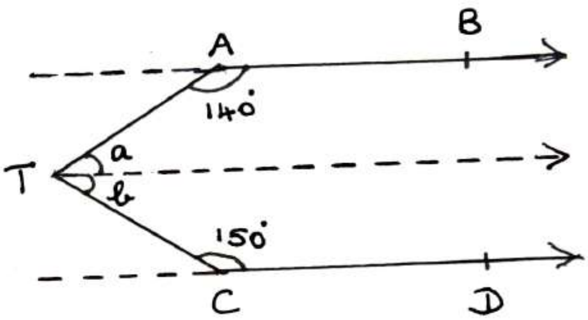
GEOMETRY

EXERCISE 4.1

1) In the figure, AB is parallel to CD, find x.

(i)





Given $\Rightarrow$ $\boxed{AB \parallel CD}$

$\therefore\ a + 140° = 180°$ [co-Interior angles are supplementary]

$a = 180° - 140°$

$a = 40°$

Similarly,

$b + 150° = 180°$ [co-Interior angles are supplementary]

$b = 180° - 150°$

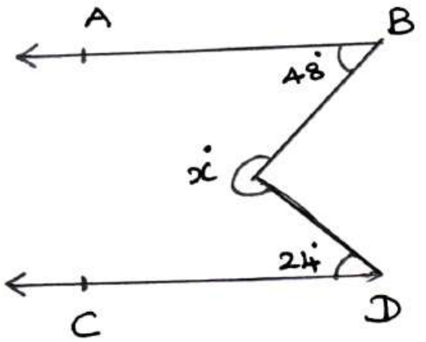
$b = 30°$

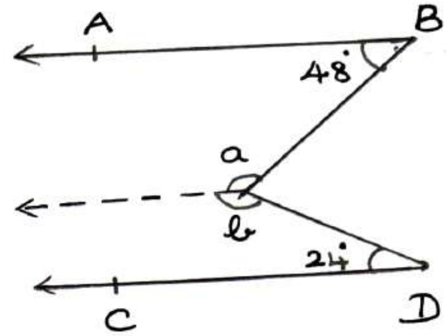
$\Rightarrow\ x° = a + b$

$x° = 40° + 30°$

$x° = 70°$

(ii)





Given $\Rightarrow$ $\boxed{AB \parallel CD}$

$a + 48^\circ = 180^\circ$ [co-Interior angles are supplementary]

$a = 180^\circ - 48^\circ$

$a = 132^\circ$

$b + 24^\circ = 180^\circ$ [co-Interior angles are supplementary]

$b = 180^\circ - 24^\circ$

$b = 156^\circ$

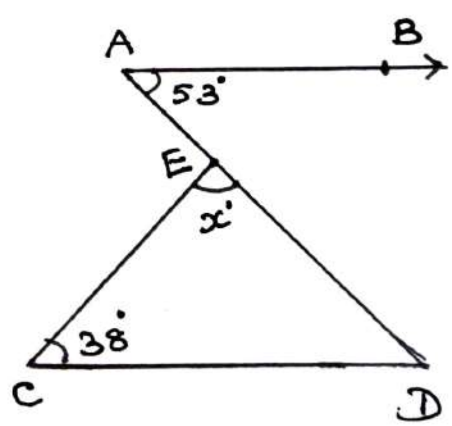
$\Rightarrow x^\circ = a + b$

$= 132^\circ + 156^\circ$

$x^\circ = 288^\circ$

(iii) $AB \parallel CD$

$\angle D = 53^\circ$ (Alternate Interior angles)



In $\triangle$ ECD,

$x^\circ + 38^\circ + 53^\circ = 180^\circ$

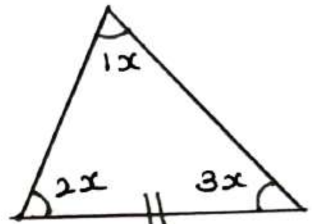
$x^\circ + 91^\circ = 180^\circ$

$x^\circ = 180^\circ - 91^\circ$

$x^\circ = 89^\circ$

2) The angles of a triangle are in the ratio 1:2:3, find the measure of each angle of the triangle.

Ratio $\Rightarrow$ 1:2:3



Sum of three angles of a triangle is 180°

$1x + 2x + 3x = 180^\circ$

$6x = 180^\circ$

$x = \frac{\cancel{180^\circ}\,30^\circ}{\cancel{6}}$

$x = 30^\circ$

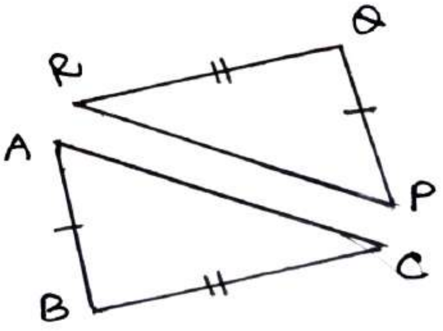
1st angle $\Rightarrow 1x = 1(30^\circ) = 30^\circ$

2nd angle $\Rightarrow 2x = 2(30^\circ) = 60^\circ$

3rd angle $\Rightarrow 3x = 3(30^\circ) = 90^\circ$

$\boxed{180^\circ}$

3) Consider the given pairs of triangles and say whether each pair is that of congruent triangles. If the triangles are congruent, say 'how'; if they are not congruent say 'why' and also say if a small modification would make them congruent.



(i) In Δ ABC and Δ PQR,

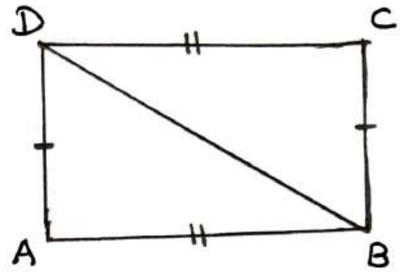
$\left.\begin{array}{l} AB = PQ \\ BC = QR \end{array}\right\}$ (given)

$\Rightarrow \triangle ABC$ is not congruent to $\triangle PQR$

If $AC = PR$ then,

$$\boxed{\triangle ABC \cong \triangle PQR}$$

(ii) In $\triangle ADB$ and $\triangle CBD$,

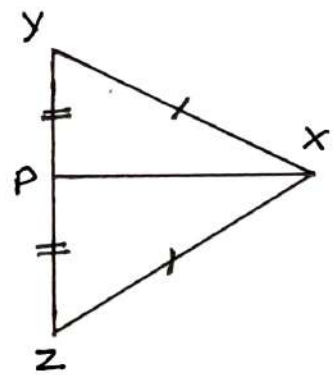


$\left.\begin{array}{l} AD = CB \\ AB = CD \end{array}\right\}$ [given]

Also BD is Common.

$\therefore \triangle ADB \cong \triangle CBD$ (SSS Rule)

(iii) In $\triangle PZX$ and $\triangle PYX$

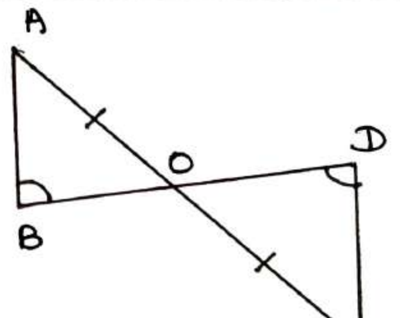


$\left.\begin{array}{l} XY = XZ \\ PY = PZ \end{array}\right\}$ (given)

Also PX is Common

$\therefore \boxed{\triangle PZX \cong \triangle PYX}$ (SSS Rule)

(iv) In $\triangle OAB$ and $\triangle OCD$

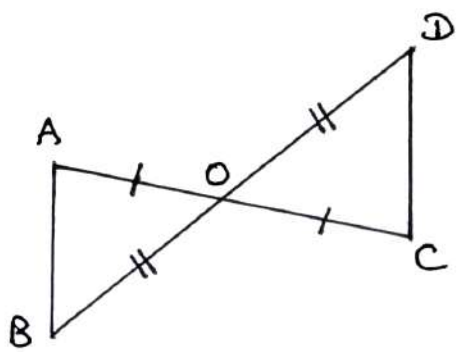


$\left.\begin{array}{l} OA = OC \\ \angle B = \angle D \end{array}\right\}$ given

$\angle AOB = \angle COD$ (vertically opp. angles)

$\therefore \triangle OAB \cong \triangle OCD$ (AAS Rule)

(v) In $\triangle OAB$ and $\triangle OCD$

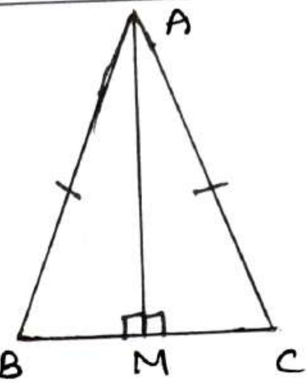


$\left.\begin{array}{l} OA = OC \\ OB = OD \end{array}\right\}$ given

$\angle AOB = \angle COD$ (vertically opp. angles)

$\therefore$ $\triangle OAB \cong \triangle OCD$ (SAS Rule)

(vi) In $\triangle ABM$ and $\triangle ACM$



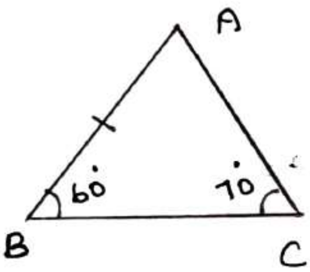
$AB = AC$ (Given) (hypotenuse)

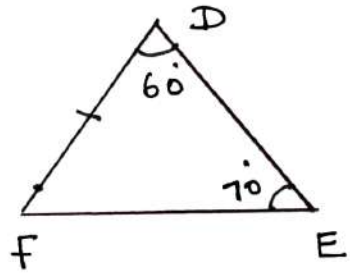
$\angle AMB = \angle AMC = 90^\circ$

AM is Common

$\Rightarrow$ $\triangle ABM \cong \triangle ACM$ (RHS Rule)

4) $\triangle ABC$ and $\triangle DEF$ are two triangles in which $AB = DF$, $\angle ACB = 70^\circ$, $\angle ABC = 60^\circ$, $\angle DEF = 70^\circ$ and $\angle EDF = 60^\circ$. Prove that the triangles are Congruent.





In $\triangle ABC$,

$\angle A + 60^\circ + 70^\circ = 180^\circ$

$\angle A + 130^\circ = 180^\circ$

$\angle A = 180^\circ - 130^\circ$

$\angle A = 50^\circ$

In $\triangle DFE$,

$60^\circ + \angle F + 70^\circ = 180^\circ$

$\angle F + 130^\circ = 180^\circ$

$\angle F = 180^\circ - 130^\circ$

$\angle F = 50^\circ$

Now, In $\triangle ABC$ and $\triangle DFE$

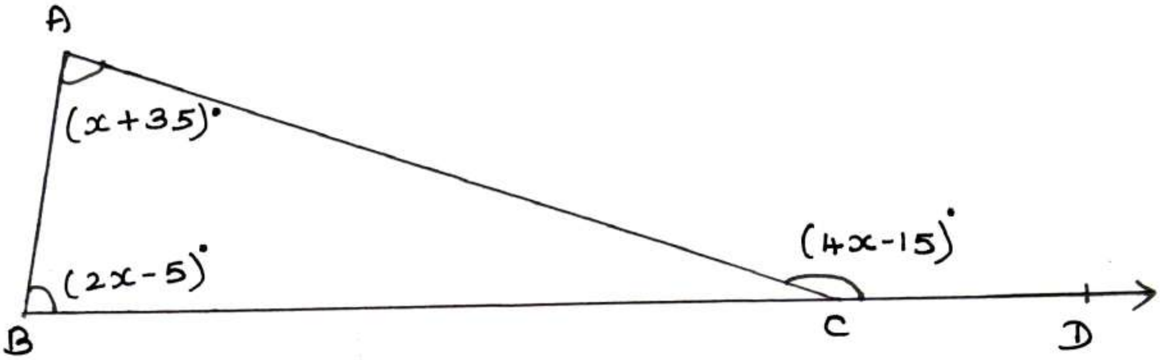
$AB = DF$ (Given)

$\angle ACB = \angle DEF = 70^\circ$ (Given)

$\angle A = \angle F = 50^\circ$

$\therefore$ $\triangle ABC \cong \triangle DFE$ (ASA Rule)

5) Find all the three angles of ΔABC



Exterior angle = sum of two opposite interior angles.

$$4x-15 = x+35+2x-5$$

$$4x-15 = 3x+30$$

$$4x-3x = 30+15$$

$$x = 45^\circ$$

$\Rightarrow \angle A = x+35^\circ = 45^\circ+35^\circ = 80^\circ$

$$\angle A = 80^\circ$$

$\Rightarrow \angle B = 2x-5^\circ = 2(45^\circ)-5^\circ = 90^\circ-5^\circ = 85^\circ$

$$\angle B = 85^\circ$$

$\Rightarrow \angle C = 180^\circ - (4x-15)^\circ$

$$= 180^\circ - [4(45^\circ)-15^\circ]$$

$$= 180^\circ - [180^\circ-15^\circ]$$

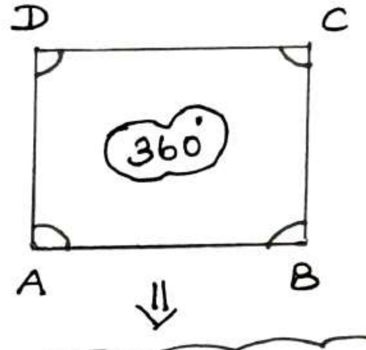
$$= \cancel{180^\circ} - \cancel{180^\circ} + 15^\circ$$

$$\angle C = 15^\circ$$

EXERCISE 4.2

1) The angles of a quadrilateral are in the ratio 2:4:5:7. Find all the angles.

Ratio: $\Rightarrow$ 2:4:5:7



Quadrilateral

$$2x+4x+5x+7x=360°$$

$$18x = 360°$$

$$x = \frac{\cancel{360°}\ 20°}{\cancel{18}}$$

$$x = 20°$$

$\Rightarrow \angle A = 2x = 2(20°) = 40°$

$\angle B = 4x = 4(20°) = 80°$

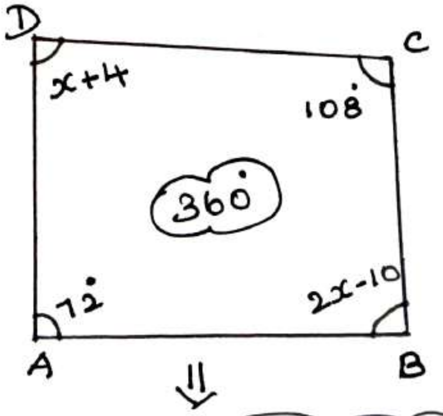
$\angle C = 5x = 5(20°) = 100°$

$\angle D = 7x = 7(20°) = 140°$

$\underline{360°}$

---

2) In a quadrilateral ABCD, $\angle A = 72°$ and $\angle C$ is the supplementary of $\angle A$. The other two angles are $2x-10$ and $x+4$. Find the value of x and the measure of all the angles.



Quadrilateral

$\angle A = 72°$

$\angle C$ is the supplementary of $\angle A$

$\therefore \angle C = 180° - 72°$

$\angle C = 108°$

$\boxed{\angle B = 2x - 10}$ $\boxed{\angle D = x + 4}$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$72^\circ + 2x - 10 + 108^\circ + x + 4 = 360^\circ$$

$$3x + 184^\circ - 10 = 360^\circ$$

$$3x + 174^\circ = 360^\circ$$

$$3x = 360^\circ - 174^\circ$$

$$3x = 186^\circ$$

$$x = \frac{\cancel{186}^\circ \; 62^\circ}{\cancel{3}}$$

$$x = 62^\circ$$

$\Rightarrow$ $\angle A = 72^\circ$

$\Rightarrow$ $\angle B = 2x - 10 = 2(62^\circ) - 10 = 124^\circ - 10 = 114^\circ$

$\angle B = 114^\circ$

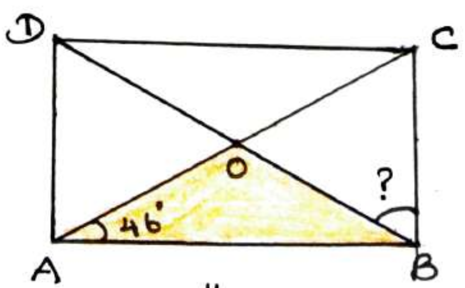
$\Rightarrow$ $\angle C = 108^\circ$

$\Rightarrow$ $\angle D = x + 4 = 62^\circ + 4 = 66^\circ$

$\angle D = 66^\circ$

---

3) ABCD is a rectangle whose diagonals AC and BD intersect at O. If $\angle OAB = 46^\circ$ Find $\angle OBC$.



Given

$\angle OAB = 46^\circ$

Similarly $\angle OBA = 46^\circ$

Rectangle $\Rightarrow$ $\angle B = 90^\circ$

$\Downarrow$

Rectangle

$$\Rightarrow \angle OBA + \angle OBC = 90^\circ$$

$$46^\circ + \angle OBC = 90^\circ$$

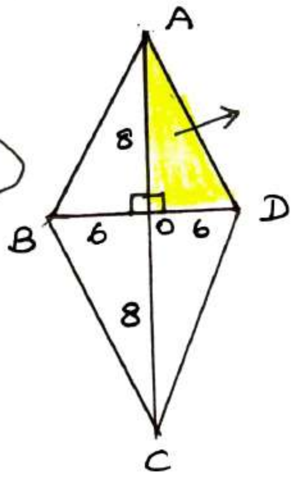
$$\angle OBC = 90^\circ - 46^\circ$$

$$\angle OBC = 44^\circ$$

4) The lengths of the diagonals of a Rhombus are 12 cm and 16 cm. Find the side of the Rhombus.

Rhombus $\Rightarrow$ All sides are equal

Diagonals $\Rightarrow$ 12 cm and 16 cm



In Rhombus, diagonals bisect at 90°

In Δ AOD,

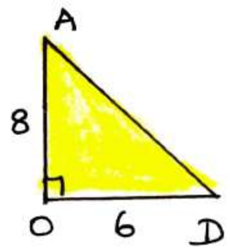
$$AD^2 = AO^2 + OD^2$$

$$AD^2 = 8^2 + 6^2$$

$$AD^2 = 64 + 36$$

$$AD^2 = 100$$

$$AD^{\cancel{2}} = 10^{\cancel{2}}$$



$$AD = 10\text{ cm}$$

$\Rightarrow$ The side of the Rhombus is 10 cm

5) Show that the bisectors of angles of a parallelogram form a Rectangle.

$\angle A = \angle C = x^\circ$

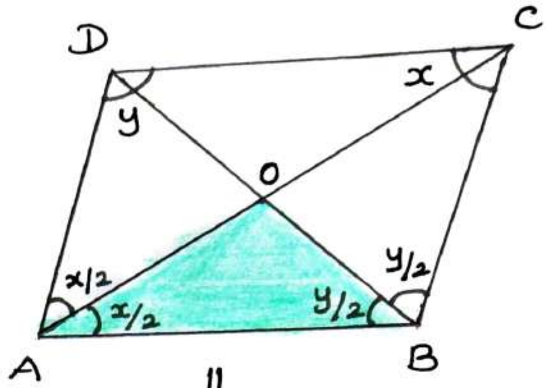
$\angle B = \angle D = y^\circ$

We Know,

$\angle A + \angle B = 180^\circ$

[adjacent angles are supplementary]

$\Rightarrow x + y = 180^\circ \rightarrow (1)$



⇓

Parallelogram

⇓

opp. angles are equal

In ΔAOB

$\angle A + \angle B + \angle O = 180^\circ$

$\frac{x}{2} + \frac{y}{2} + \angle O = 180^\circ$

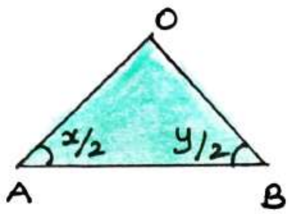
$\frac{x+y}{2} + \angle O = 180^\circ$

$\frac{180^\circ}{2} + \angle O = 180^\circ$

$90^\circ + \angle O = 180^\circ$

$\angle O = 180^\circ - 90^\circ$

$\angle O = 90^\circ$



$\Rightarrow$ The bisectors of angles of a parallelogram form a Rectangle.

---

6) If a triangle and a parallelogram lie on the same base and between the same parallels, then prove that the area of the triangle is equal to half of the area of parallelogram.

Area of a Parallelogram

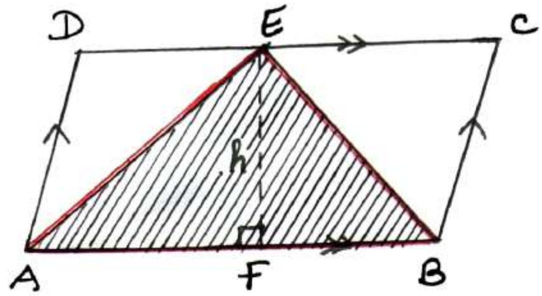
ABCD $= \boxed{b \times h} = \boxed{AB \times EF} \rightarrow (1)$

Area of a triangle ABE

$= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times AB \times EF$

$= \frac{1}{2} \times$ Area of a parallelogram (from ①)

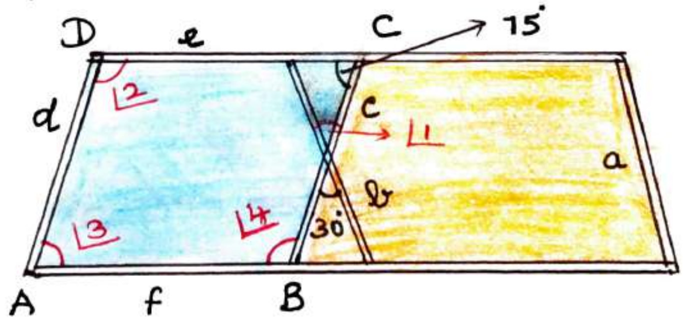


Same base

$\Rightarrow$ Area of a triangle $= \frac{1}{2}$ (Area of a Parallelogram)

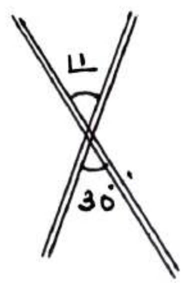
---

7) Iron rods a, b, c, d, e, and f are making a design in a bridge as shown in the figure. If $a \parallel b$, $c \parallel d$, $e \parallel f$, find the marked angles between (i) b and c (ii) d and e (iii) d and f (iv) c and f



(i) b and c:

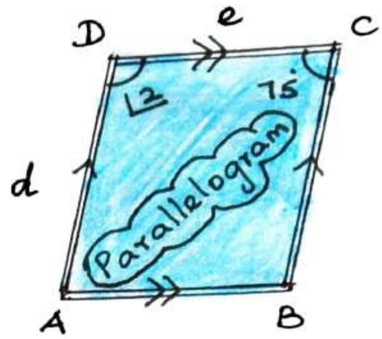
$\angle 1 = 30°$ (vertically opposite angles)



(ii) d and e:

ABCD is a parallelogram

$\angle 2 + 75^\circ = 180^\circ$ (adjacent angles)

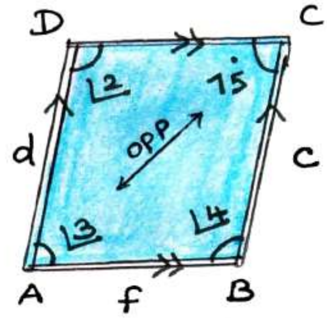


$\angle 2 = 180^\circ - 75^\circ$

$\angle 2 = 105^\circ$

(iii) d and f:

$\angle 3 = 75^\circ$ (opposite angles)



(iv) c and f:

$\angle 4 + 75^\circ = 180^\circ$ (adjacent angles)

$\angle 4 = 180^\circ - 75^\circ$

$\angle 4 = 105^\circ$

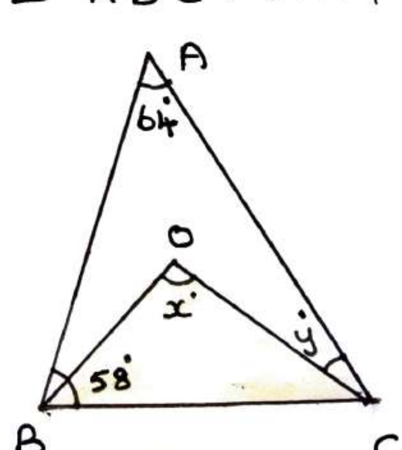
(or)

$\angle 4 = 105^\circ$ [$\because$ $\angle 2$ and $\angle 4$ are opp. angles]

$\Rightarrow$ (i) 30° (ii) 105° (iii) 75° (iv) 105°

---

8) In the given Fig 4.39, $\angle A = 64^\circ$, $\angle ABC = 58^\circ$. If BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$ respectively of ΔABC. find x° and y°.



In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

$64^\circ + 58^\circ + \angle C = 180^\circ$

$122^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 122^\circ$

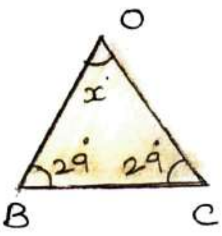
$\angle OBC = \dfrac{58^\circ}{2} = 29^\circ$

$\boxed{\angle C = 58^\circ}$

$y^\circ = \frac{\angle C}{2} = \frac{58^\circ}{2} = 29^\circ$ [CO is the bisector]

$\Rightarrow \boxed{y^\circ = 29^\circ}$

In Δ OBC,



$x^\circ + 29^\circ + 29^\circ = 180^\circ$

$x^\circ + 58^\circ = 180^\circ$

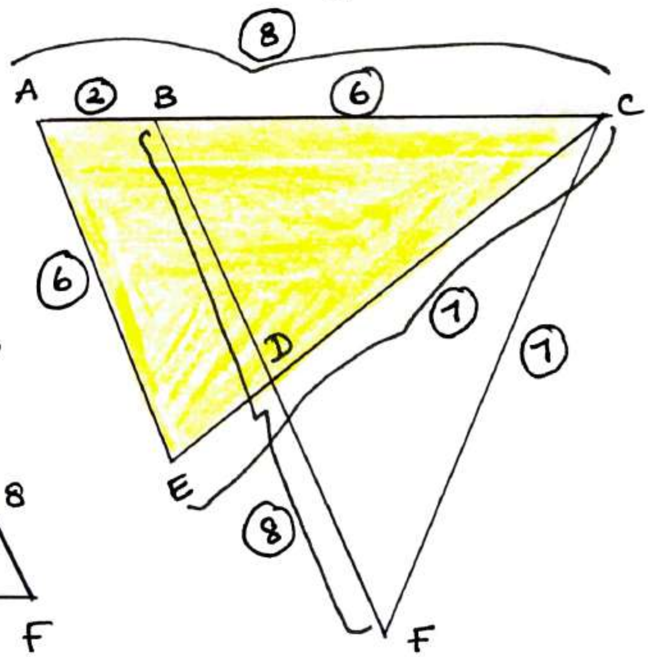
$x^\circ = 180^\circ - 58^\circ$

$\boxed{x^\circ = 122^\circ}$

---

9) In the given Fig. if AB = 2, BC = 6, AE = 6, BF = 8, CE = 7 and CF = 7, Compute the ratio of the area of quadrilateral ABDE to the area of Δ CDF (Use congruent property of triangles)

Given:

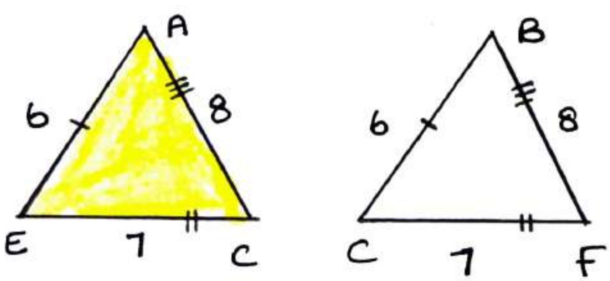


AB = 2, BC = 6,

AE = 6, BF = 8

CE = 7, CF = 7

In Δ AEC and Δ BCF,



AE = BC, EC = CF, AC = BF

$\Rightarrow \boxed{\Delta AEC \cong \Delta BCF}$

$\Rightarrow$ Area of Δ AEC = Area of Δ BCF

[Subtract Δ BDC on both sides]

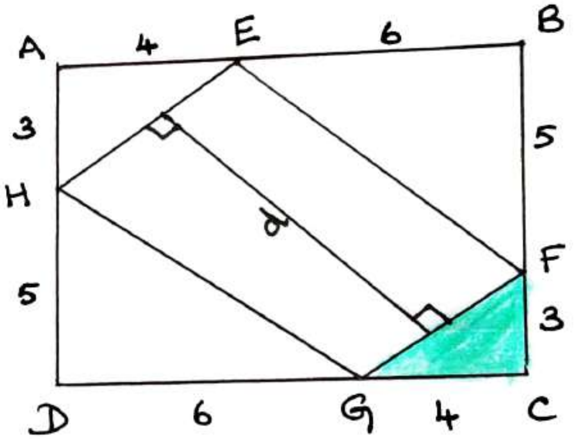
Area of Δ AEC − Area of Δ BDC = Area of Δ BCF − Area of Δ BDC

$\Rightarrow$ Area of a quadrilateral ABDE = Area of Δ CDF.

$\Rightarrow$ Ratios are equal

---

10) In the figure, ABCD is a rectangle and EFGH is a parallelogram. Using the measurements given in the figure, what is the length 'd' of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



$\Rightarrow$ Area of a Rectangle ABCD

= $l \times b$

= 10×8

= $80\ cm^2$

$AB = 4+6 = 10\,cm\ (l)$

$AD = 3+5 = 8\,cm\ (b)$

Area of Δ AHE = Area of Δ CGF

Area of Δ BEF = Area of Δ DHG

Area of the parallelogram EFGH

= Area of a Rectangle ABCD − 2(Area of Δ AHE) − 2(Area of Δ BEF)

$= 80 - 2\left[\frac{1}{\cancel{2}} \times \cancel{4}^{2} \times 3\right] - 2\left[\frac{1}{\cancel{2}} \times \cancel{6}^{3} \times 5\right]$

$= 80 - 2(2 \times 3) - 2(3 \times 5)$

$= 80 - 2(6) - 2(15)$

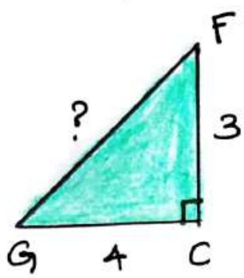
$= 80 - 12 - 30$

$= 80 - 42$

$= 38\ cm^2$

$$\begin{array}{r} 7.6 \\ 5\overline{)38} \\ \underline{35} \\ 30 \\ \underline{30} \\ \underline{0} \end{array}$$

In Δ GCF



$\boxed{GF^2 = GC^2 + CF^2}$

$GF^2 = 4^2 + 3^2$

$GF^2 = 16 + 9$

$GF^2 = 25$

$GF^{\cancel{2}} = 5^{\cancel{2}}$

$GF = 5\ cm \Leftarrow b \Leftarrow$ base of a parallelogram

$\therefore$ Area of a Parallelogram EFGH $= 38\ cm^2$

base $\times$ height $= 38\ cm^2$

$5 \times d = 38$

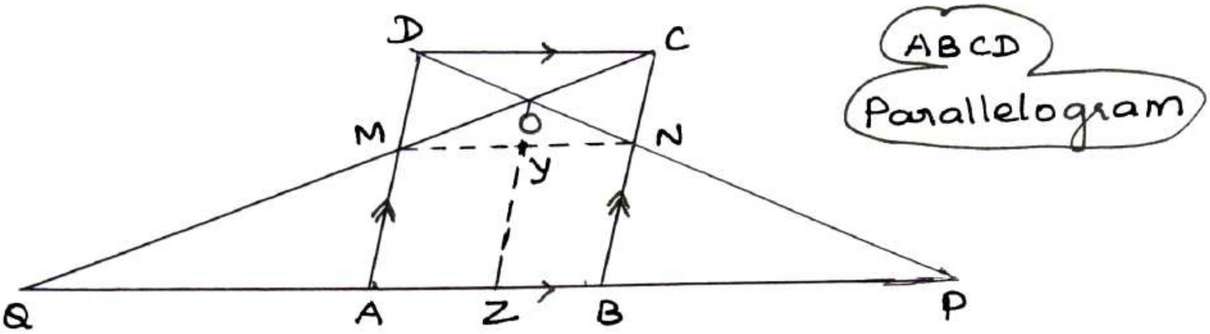
$d = \frac{38}{5}$

$d = 7.6\ cm$

---

11) In parallelogram ABCD of the accompanying diagram, line DP is drawn bisecting BC at N and meeting AB (extended) at P. From vertex C, line CQ is drawn bisecting side

AD at M and meeting AB (extended) at Q. Lines DP and CQ meet at O. Show that the area of triangle QPO is $\frac{9}{8}$ of the area of the parallelogram ABCD.



In $\triangle$ BCQ,

$$\boxed{MN = \frac{1}{2}(QB) \rightarrow (1)}$$

In $\triangle$ ADP

$$\boxed{MN = \frac{1}{2}(AP) \rightarrow (2)}$$

From (1) and (2)

$$\cancel{\frac{1}{2}}(QB) = \cancel{\frac{1}{2}}(AP)$$

$$\boxed{QB = AP}$$

$$\Downarrow$$

$$QB - AB = AP - AB$$

$$\boxed{QA = BP = AB}$$

Area of $\triangle$ QOP $= \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times QP \times OZ$$

$$= \frac{1}{2} \times (QA + AB + BP) \times OZ$$

$$= \frac{1}{2} \times (AB + AB + AB) \times OZ$$

$$\Rightarrow \boxed{\text{Area of } \triangle QOP = \frac{1}{2} \times (3AB) \times OZ} \rightarrow (3)$$

Now, $OZ = OY + YZ$

$= OY + BN$

$= OY + \frac{1}{2}(BC)$

$= \frac{1}{2}(NC) + \frac{1}{2}(BC)$

$= \frac{1}{2}\left(\frac{1}{2}BC\right) + \frac{1}{2}(BC)$

$= \frac{1}{4}BC + \frac{1}{2}BC$

$= BC\left[\frac{1}{4} + \frac{1}{2}\right]$

$= BC\left[\frac{1+2}{4}\right]$

$= BC\left[\frac{3}{4}\right]$

$\Rightarrow \boxed{OZ = \frac{3}{4}(BC)}$

Put $OZ = \frac{3}{4}(BC)$ in (3)

$\Rightarrow$ Area of $\Delta QOP = \frac{1}{2}(3AB) \times OZ$

$= \frac{1}{2}(3AB) \times \frac{3}{4}(BC)$

$= \frac{9}{8}(AB \times BC)$

Area of $\Delta QOP = \frac{9}{8}$(Area of Parallelogram ABCD)

Hence Proved.

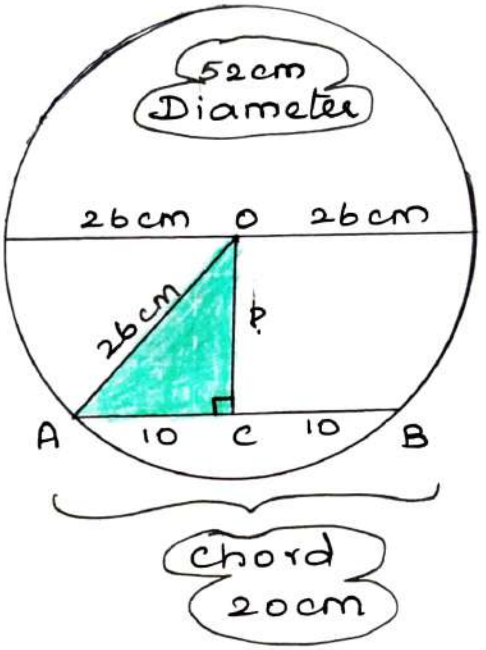
EXERCISE 4.3

1) The diameter of the circle is 52 cm and the length of one of its chord is 20 cm. Find the distance of the chord from the centre.

Diameter = 52 cm

⇓

Radius $= \frac{52}{2} = 26$ cm



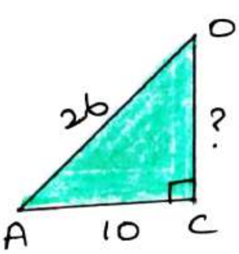
$\Rightarrow$ $\boxed{\begin{array}{l} OA = 26\text{cm} \\ AC = 10\text{ cm} \end{array}}$

In Δ OAC,

$OA^2 = OC^2 + AC^2$

$26^2 = OC^2 + 10^2$

$676 = OC^2 + 100$



$OC^2 + 100 = 676$

$OC^2 = 676 - 100$

$OC^2 = 576$

$\cancel{OC^2} = \cancel{24^2}$

$\boxed{OC = 24\text{ cm}}$

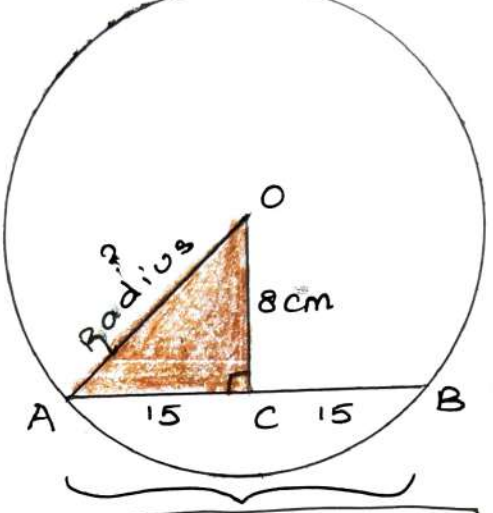
$\Rightarrow$ The distance of the chord from the centre is 24 cm.

---

2) The chord of length 30 cm is drawn

at the distance of 8 cm from the centre of the circle. Find the radius of the circle.

Chord → 30cm
Distance → 8cm



In Δ OAC,

$OA^2 = OC^2 + AC^2$

$OA^2 = 8^2 + 15^2$

$OA^2 = 64 + 225$

$OA^2 = 289$

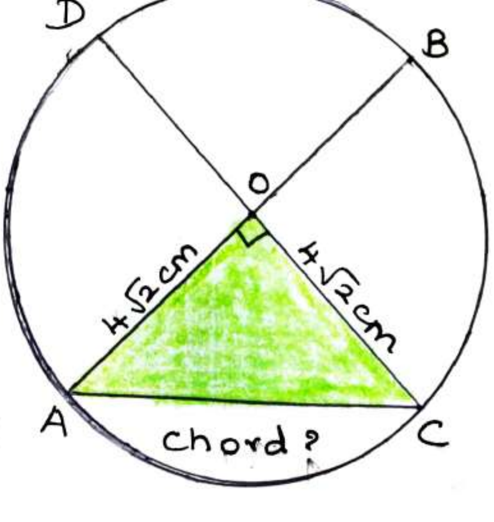
$OA^{\cancel{2}} = 17^{\cancel{2}}$

$\boxed{OA = 17cm}$

⇒ Radius of the circle = 17cm.

3) Find the length of the chord AC where AB and CD are the two diameters perpendicular to each other of a circle with radius $4\sqrt{2}$ cm and also find $\underline{\angle OAC}$ and $\underline{\angle OCA}$.

Radius = $4\sqrt{2}$ cm



In Δ OAC,

$AC^2 = OA^2 + OC^2$

$AC^2 = (4\sqrt{2})^2 + (4\sqrt{2})^2$

$AC^2 = (4 \times 4 \times 2) + (4 \times 4 \times 2)$

$AC^2 = 32 + 32$

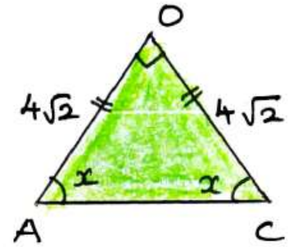
$AC^2 = 64$

$AC^{\cancel{2}} = 8^{\cancel{2}}$

$\boxed{AC = 8cm}$

$\Rightarrow$ The length of the chord AC = 8cm

In Δ OAC,



$90^\circ + x + x = 180^\circ$

$90^\circ + 2x = 180^\circ$

$2x = 180^\circ - 90^\circ$

$2x = 90^\circ$

$x = \frac{\cancel{90^\circ}\ 45^\circ}{\cancel{2}}$

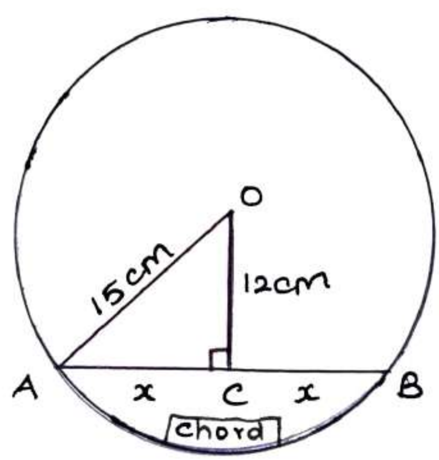
$\boxed{x = 45^\circ}$

$\Rightarrow$ $\angle OAC = 45^\circ$, $\angle OCA = 45^\circ$

---

4) A chord is 12cm away from the centre of the circle of radius 15cm. Find the length of the chord.

$\boxed{\begin{array}{l} \text{Radius} = 15cm \\ \text{distance} = 12cm \end{array}}$



In Δ OAC

$OA^2 = OC^2 + AC^2$

$15^2 = 12^2 + AC^2$

$225 = 144 + x^2$

$225 - 144 = x^2$

$81 = x^2$

$x^2 = 81$

$x^{\cancel{2}} = 9^{\cancel{2}}$

$\boxed{x = 9\,cm}$

$\Rightarrow$ chord $AB = x + x$

$= 9 + 9$

$AB = 18\,cm$

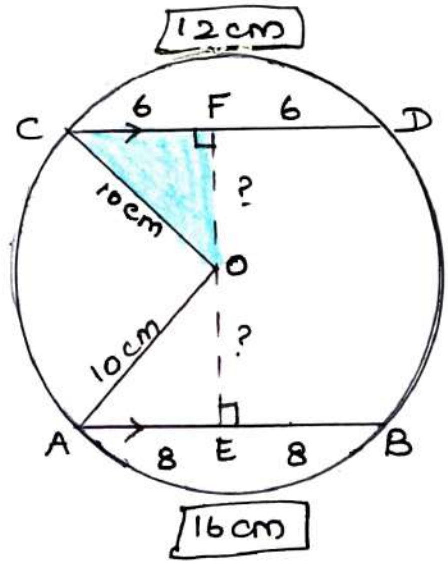
---

5) In a circle, AB and CD are two parallel chords with centre O and radius 10 cm such that AB = 16cm and CD = 12cm. determine the distance between the two chords.

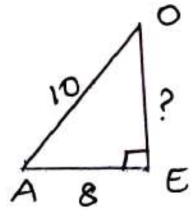
Radius = 10cm

AB = 16cm

CD = 12cm



In Δ OAE,



$OA^2 = OE^2 + AE^2$

$10^2 = OE^2 + 8^2$

$100 = OE^2 + 64$

$100 - 64 = OE^2$

$36 = OE^2$

$OE^2 = 36$

$\cancel{OE^2} = \cancel{6^2}$

$OE = 6\,cm$

In Δ OFC

$OC^2 = FC^2 + FO^2$

$10^2 = 6^2 + FO^2$

$100 = 36 + FO^2$

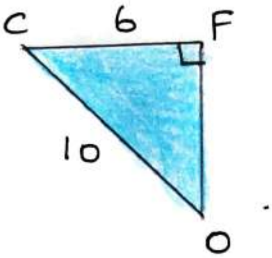
$100 - 36 = FO^2$

$64 = FO^2$

$FO^2 = 64$

$\cancel{FO^2} = \cancel{8^2}$

$FO = 8\,cm$



$\Rightarrow$ Distance between two chords

$EF = OE + FO$

$EF = 6 + 8$

$EF = 14\,cm$

---

6) Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Radius $\Rightarrow$ 5 cm, 3 cm

Distance between their centres is 4 cm.

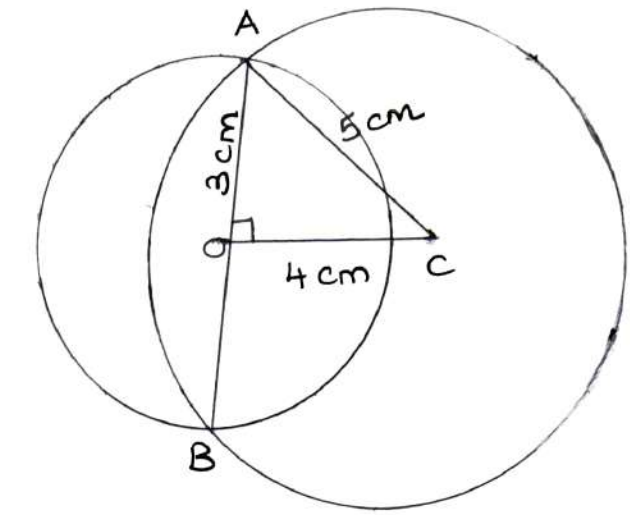
In ΔOAC,

$AC^2 = OA^2 + OC^2$

$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

$\boxed{25 = 25}$



AB => Common chord

$AB = AO + OB$

$= 3 + 3$

$AB = 6\,cm$

---

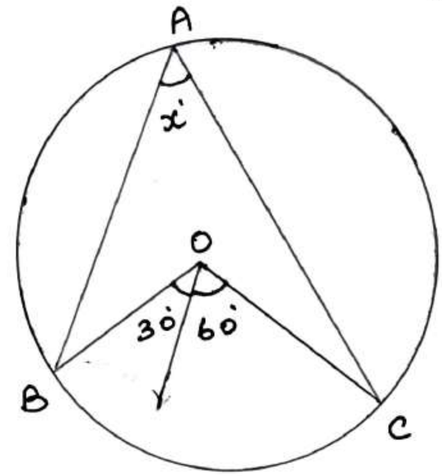
7) Find the value of x° in the following figures.

(i) $\angle O = 30^\circ + 60^\circ$

$\angle O = 90^\circ$

$\Rightarrow x = \frac{90^\circ}{2} = 45^\circ$

$\therefore \boxed{x = 45^\circ}$



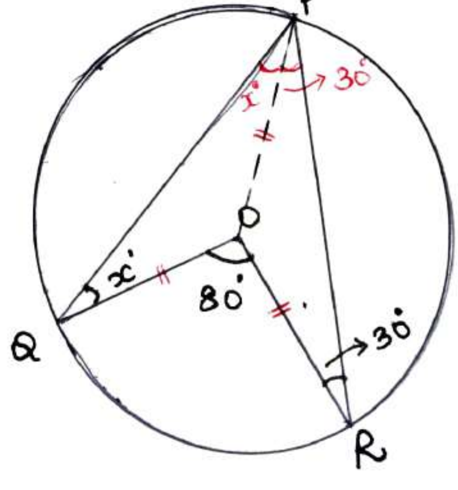
(ii) $\angle P = \frac{80^\circ}{2} = 40^\circ$

$\angle P = 40^\circ$

Δ OPR is an Isosceles triangle.

$\angle OPR = 30^\circ = \angle ORP$

(OR, OP radius)

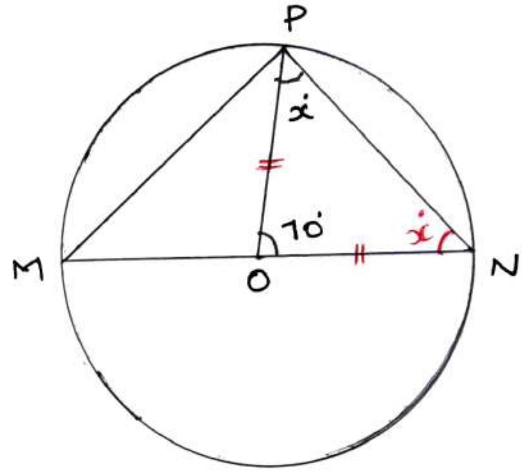


In $\triangle$ OPQ,

$\angle OPQ = 10^\circ$ $(40^\circ - 30^\circ)$

$\Rightarrow$ $x = 10^\circ$ (Isoceles triangle Property)

(iii) ON, OP $\Rightarrow$ radius



$70^\circ + x^\circ + x^\circ = 180^\circ$

$70^\circ + 2x^\circ = 180^\circ$

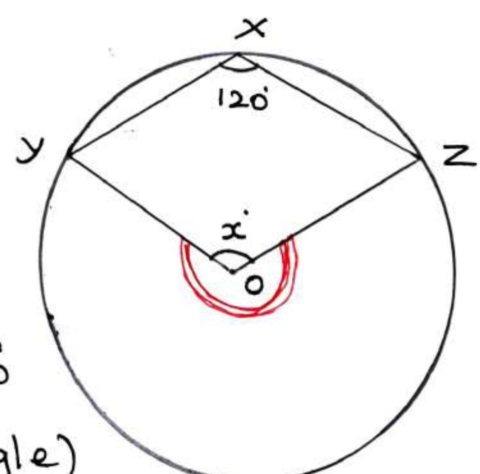
$2x^\circ = 180^\circ - 70^\circ$

$2x = 110^\circ$

$x = \frac{110^\circ}{2}$

$x = 55^\circ$

(iv) exterior of $\angle YOZ$



$= 2(120^\circ)$

$= 240^\circ$

$\Rightarrow$ Interior of $\angle YOZ$

+ Exterior of $\angle YOZ = 360^\circ$

(whole angle)

$x + 240^\circ = 360^\circ$

$x = 360^\circ - 240^\circ$

$x = 120^\circ$

(v) Exterior of $\angle BOC$ $+ 140^\circ + 100^\circ = 360^\circ$

(whole angle)

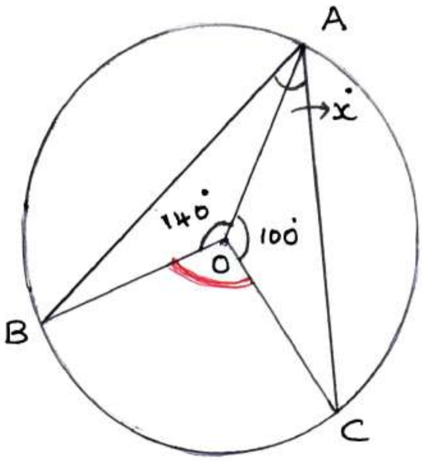
Ext $\angle BOC + 240^\circ = 360^\circ$

Ext $\angle BOC = 360^\circ - 240^\circ$

$\boxed{\text{Ext } \angle BOC = 120^\circ}$

$\Rightarrow x^\circ = \frac{120^\circ}{2} = 60^\circ$

$x = 60^\circ$



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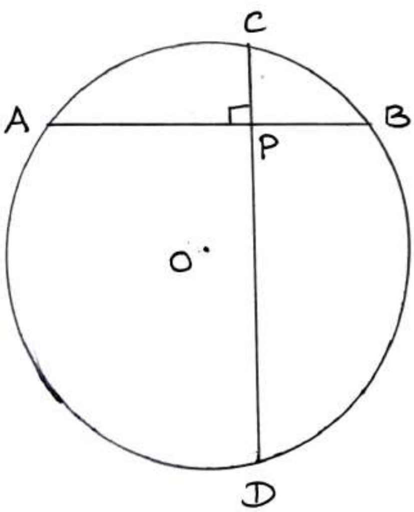
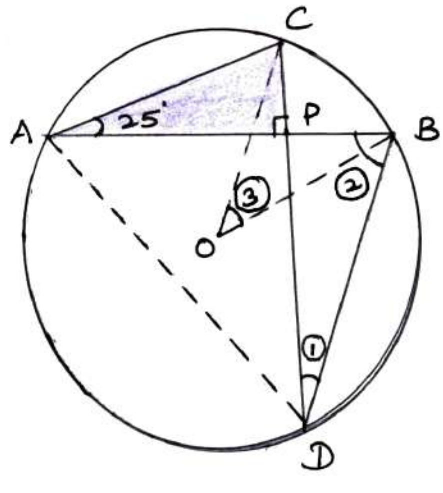
8) In the given figure, $\angle CAB = 25^\circ$, Find $\angle BDC$, $\angle DBA$ and $\angle COB$





Given $\Rightarrow \angle CAB = 25^\circ$

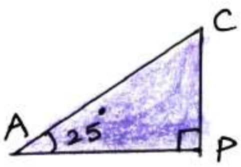
① $\Rightarrow \therefore \boxed{\angle BDC = 25^\circ}$ (Angle in a same segment)

In Δ ACP,

$25^\circ + \angle C + 90^\circ = 180^\circ$

$115^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 115^\circ$



$\angle C = 65^\circ$

$\therefore$ (2) $\Rightarrow$ $\boxed{\angle DBA = 65^\circ}$ $[\angle DBA = \angle DCA]$

(Angle in a same segment]

$\angle COB = 2\angle CAB$

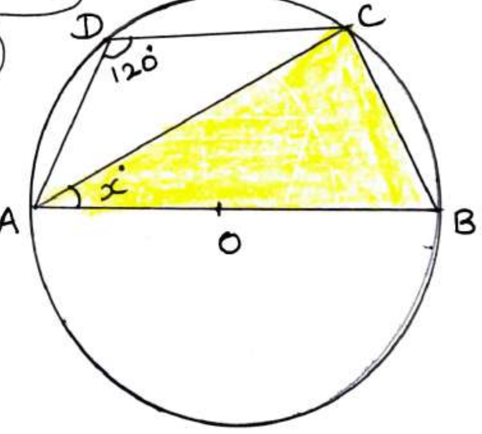
$= 2(25^\circ)$

(3) $\Rightarrow$ $\boxed{\angle COB = 50^\circ}$

EXERCISE 4.4

1) Find the value of x in the given figure.

In a cyclic quadrilatoral, sum of opp angles are supplementary



$\angle B + \angle D = 180^\circ$

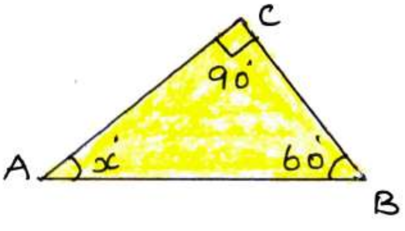
$\angle B + 120^\circ = 180^\circ$

$\angle B = 180^\circ - 120^\circ$

$\boxed{\angle B = 60^\circ}$

$\angle C = 90^\circ$ [Angle in a semicircle is 90°]

In Δ ACB

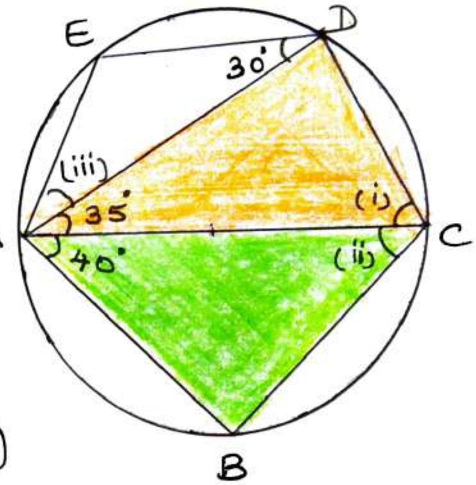


$x^\circ + 90^\circ + 60^\circ = 180^\circ$

$x^\circ + 150^\circ = 180^\circ$

$x^\circ = 180^\circ - 150^\circ$ $\Rightarrow$ $x = 30^\circ$

2) In the given figure, AC is the diameter of the circle with centre O. If $\angle ADE = 30^\circ$; $\angle DAC = 35^\circ$ and $\angle CAB = 40^\circ$. Find (i) $\angle ACD$ (ii) $\angle ACB$ (iii) $\angle DAE$



AC ⇒ Diameter

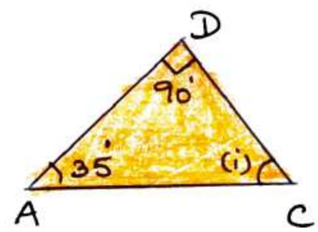
(i) $\angle ACD$?

In $\triangle ACD$,

$\angle D = 90^\circ$ (Angle in a Semicircle)

$90^\circ + 35^\circ + (i) = 180^\circ$

$125^\circ + (i) = 180^\circ$



$\angle ACD = 180^\circ - 125^\circ$

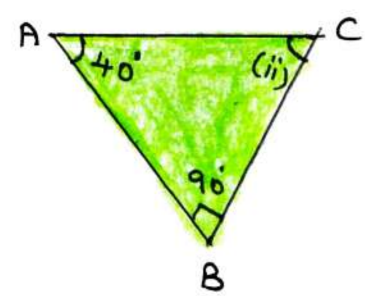
$\angle ACD = 55^\circ$

(ii) $\angle ACB$?

In $\triangle ABC$,

$\angle B = 90^\circ$ (Angle in a Semicircle)

$40^\circ + 90^\circ + (ii) = 180^\circ$



$130^\circ + \angle ACB = 180^\circ$

$\angle ACB = 180^\circ - 130^\circ$

$\angle ACB = 50^\circ$

(iii) $\angle DAE$?

In a cyclic quadrilateral ACDE,

$$\angle A + \angle D = 180^\circ$$

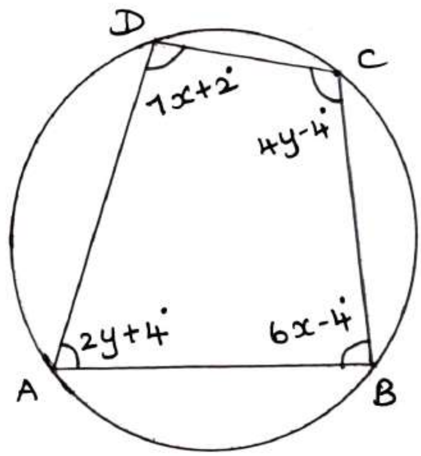
$$35^\circ + \angle DAE + 90^\circ + 30^\circ = 180^\circ$$

$$\angle DAE + 155^\circ = 180^\circ$$

$$\angle DAE = 180^\circ - 155^\circ$$

$$\angle DAE = 25^\circ$$

3) Find all the angles of the given cyclic quadrilateral ABCD in the figure.



$$\boxed{\angle A + \angle C = 180^\circ}$$

$$2y + \cancel{4^\circ} + 4y - \cancel{4^\circ} = 180^\circ$$

$$6y = 180^\circ$$

$$y = \frac{\cancel{180^\circ}\ 30^\circ}{\cancel{6}}$$

$$\boxed{y = 30^\circ}$$

$$\boxed{\angle B + \angle D = 180^\circ}$$

$$6x - 4^\circ + 7x + 2^\circ = 180^\circ$$

$$13x - 2^\circ = 180^\circ$$

$$13x = 180^\circ + 2^\circ$$

$$13x = 182^\circ$$

$$x = \frac{\cancel{182^\circ}\ 14^\circ}{\cancel{13}}$$

$$\boxed{x = 14^\circ}$$

$\angle A = 2y + 4 = 2(30^\circ) + 4 = 60^\circ + 4 = 64^\circ$

$\angle B = 6x - 4 = 6(14^\circ) - 4 = 84^\circ - 4 = 80^\circ$

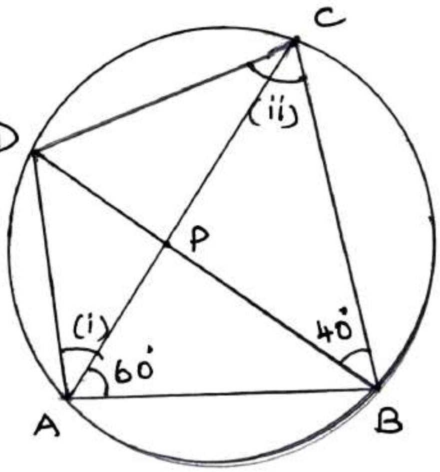
$\angle C = 4y - 4 = 4(30^\circ) - 4 = 120^\circ - 4 = 116^\circ$

$\angle D = 7x + 2 = 7(14^\circ) + 2 = 98^\circ + 2 = 100^\circ$

$\overline{360^\circ}$

---

4) In the given figure, ABCD is a cyclic quadrilateral where diagonals intersect at P such that $\angle DBC = 40^\circ$ and $\angle BAC = 60^\circ$ Find (i) $\angle CAD$ (ii) $\angle BCD$



(i) $\angle CAD$?

$\angle DBC = 40^\circ$ (given)

$\Rightarrow \boxed{\angle CAD = 40^\circ}$ (Angle in a same Segment)

(ii) $\angle BCD$?

$\angle A = (i) + 60^\circ$

$= 40^\circ + 60^\circ$

$\boxed{\angle A = 100^\circ}$

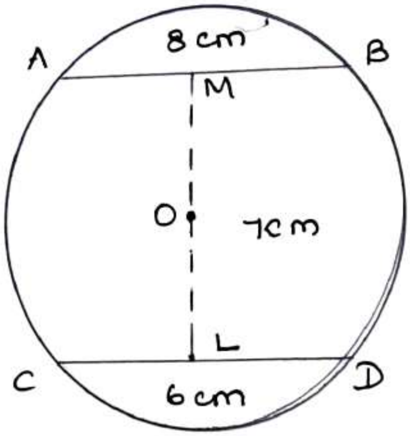
$\Rightarrow \angle A + \angle C = 180^\circ$

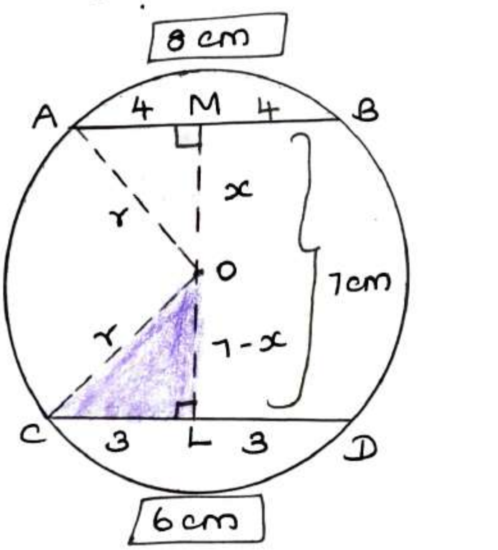
$100^\circ + \angle BCD = 180^\circ$

$\angle BCD = 180^\circ - 100^\circ$

$\Rightarrow \angle BCD = 80^\circ$

5) In the given figure, AB and CD are the parallel chords of a circle with centre O such that $AB = 8\,cm$ and $CD = 6\,cm$. If $OM \perp AB$ and $OL \perp CD$ distance between LM is 7cm. Find the radius of the circle?

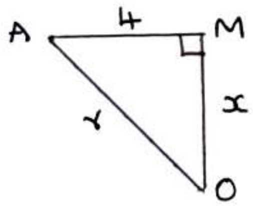




Given

$\boxed{OM \perp AB}$ $\boxed{OL \perp CD}$

In Δ AMO,

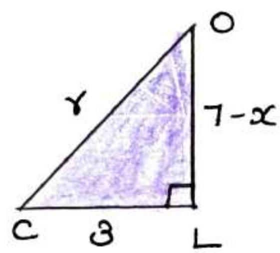


$$OA^2 = OM^2 + MA^2$$

$$r^2 = x^2 + 4^2$$

$$\boxed{r^2 = x^2 + 16} \longrightarrow (1)$$

In Δ CLO,



$$OC^2 = OL^2 + CL^2$$

$$r^2 = (7-x)^2 + 3^2$$

$$r^2 = (7)^2 + (x)^2 - 2(7)(x) + 9$$

$$r^2 = 49 + x^2 - 14x + 9$$

$$\boxed{r^2 = x^2 - 14x + 58} \longrightarrow (2)$$

from (1) and (2)

$$\cancel{x^2} + 16 = \cancel{x^2} - 14x + 58$$

$$14x = 58 - 16$$

$$14x = 42$$

$$x = \frac{\cancel{42}\,3}{\cancel{14}}$$

$$\boxed{x = 3}$$

Put $x = 3$ in (1)

$$r^2 = x^2 + 16$$

$$r^2 = (3)^2 + 16$$

$$r^2 = 9 + 16$$

$$r^2 = 25$$

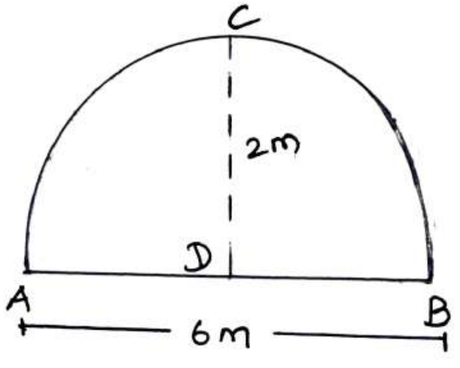
$$r^{\cancel{2}} = 5^{\cancel{2}}$$

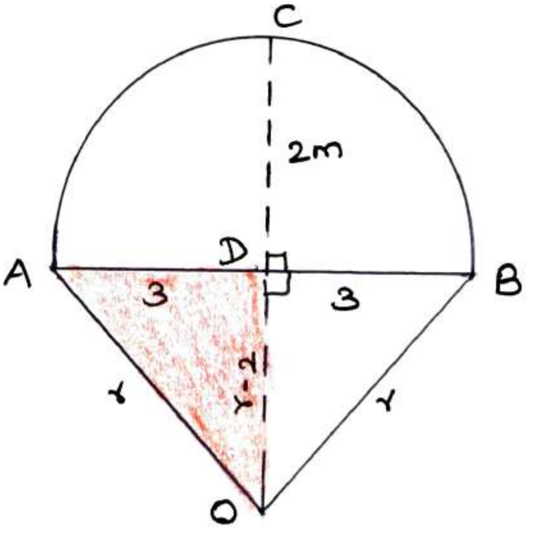
$$r = 5\,cm$$

$\Rightarrow$ Radius of the circle is 5cm

---

6) The arch of a bridge has dimensions as shown, where the arch measure 2m at its highest point and its width is 6m. What is the radius of the circle that contains the arch?

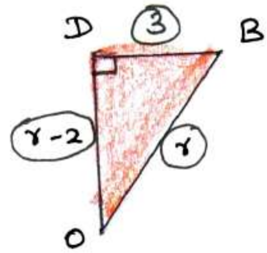




OA, OB, OC ⇒ radius ⇒ r

$OD = OC - CD$

$\boxed{OD = r - 2}$



In Δ OBD

$OB^2 = BD^2 + OD^2$

$r^2 = 3^2 + (r-2)^2$

$r^2 = 9 + (r)^2 + (2)^2 - 2(r)(2)$

$\cancel{r^2} = 9 + \cancel{r^2} + 4 - 4r$

$4r = 13$

$r = \dfrac{13}{4}$

$\boxed{r = 3.25\,m}$

$$\begin{array}{r} 3.25 \\ 4\overline{)13} \\ 12 \\ \hline 10 \\ 8 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

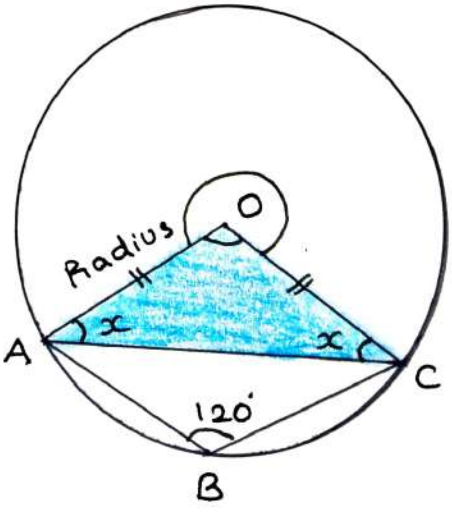
⇒ Radius of the circle = 3.25 m

---

7) In figure, $\angle ABC = 120°$, where A, B and C are points on the circle with centre O. Find $\angle OAC$?

OA, OC ⇒ Radius

$\boxed{\angle OAC = \angle OCA = x}$



Reflex $\angle AOC$

$= 2(120°) = 240°$

$\therefore \angle AOC = 360° - 240° = 120°$

⇒ $\angle AOC = 120°$

In Δ AOC,

$$x + x + 120^\circ = 180^\circ$$

$$2x + 120^\circ = 180^\circ$$

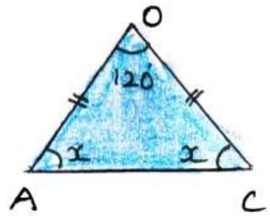
$$2x = 180^\circ - 120^\circ$$

$$2x = 60^\circ$$

$$x = \frac{60^\circ}{2}$$

$$\boxed{x = 30^\circ}$$

$$\Rightarrow \angle OAC = 30^\circ$$



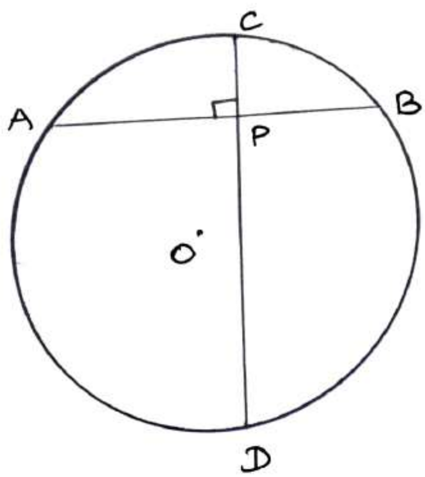
8) A school wants to conduct tree plantation programme. For this a teacher allotted a circle of radius 6m ground to nineth standard students for planting sapplings. Four students plant trees at the points A, B, C and D as shown in figure. Here AB = 8 m, CD = 10 m and AB ⊥ CD. If another student places a flower pot at the point P, the intersection of AB and CD, then find the distance from the centre to P.

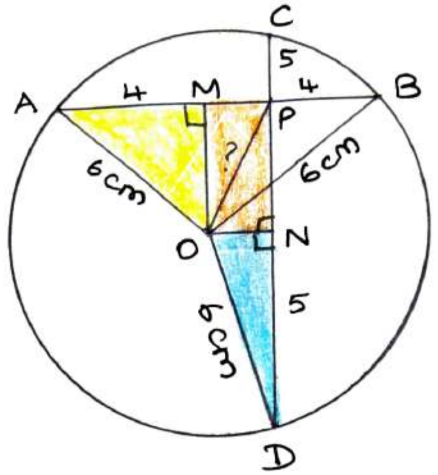
Radius = 6 m, AB = 8 m, CD = 10 m

$$\Downarrow$$

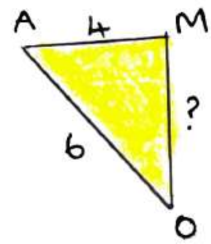
$\boxed{AB \perp CD}$ $\boxed{\text{Chords}}$

OP ?





In Δ AMO



$AO^2 = AM^2 + MO^2$

$6^2 = 4^2 + MO^2$

$36 = 16 + MO^2$

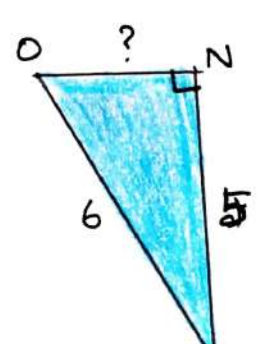
$36 - 16 = MO^2$

$20 = MO^2$

$MO^2 = 20$

$\boxed{MO = \sqrt{20}}$

In Δ OND,



$OD^2 = ON^2 + ND^2$

$6^2 = ON^2 + 5^2$

$36 = ON^2 + 25$

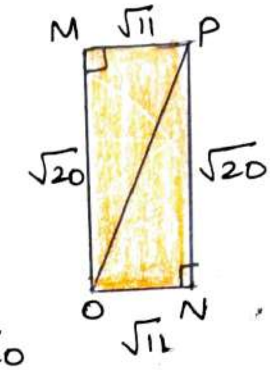
$36 - 25 = ON^2$

$11 = ON^2$

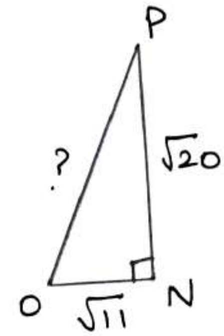
$ON^2 = 11$

$\boxed{ON = \sqrt{11}}$

ONPM is a Rectangle,
↳ (opp sides equal)



In Δ ONP



$OP^2 = ON^2 + PN^2$

$OP^2 = (\sqrt{11})^{\cancel{2}} + (\sqrt{20})^{\cancel{2}}$

$OP^2 = 11 + 20$

$OP^2 = 31$

$OP = \sqrt{31}$

$OP = 5.56$

$\Rightarrow \boxed{OP \simeq 5.6\,m}$

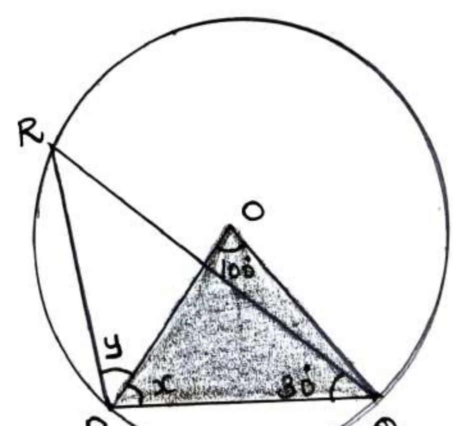
5.56
5 | 31.0000
25
105 | 600
525
1106 | 7500
6636
864

---

9) In the given figure, $\angle POQ = 100^\circ$ and $\angle PQR = 30^\circ$, then find $\angle RPO$

Given

$\angle PQR = 30^\circ$, $\angle POQ = 100^\circ$



$\angle PRQ = \frac{100^\circ}{2} = 50^\circ$

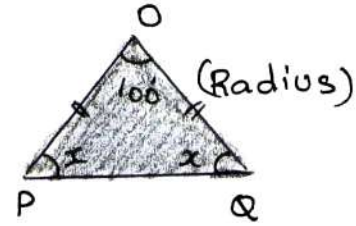
In Δ OPQ

$100^\circ + x + x = 180^\circ$

$100^\circ + 2x = 180^\circ$

$2x = 180^\circ - 100^\circ$

$2x = 80^\circ$

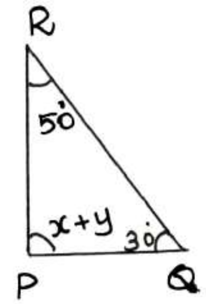


$$x = \frac{80^\circ}{2}$$

$$\boxed{x = 40^\circ}$$

$$\Rightarrow \angle OPQ = 40^\circ$$

In Δ PRQ,



$$50^\circ + x + y + 30^\circ = 180^\circ$$

$$80^\circ + 40^\circ + y = 180^\circ$$

$$120^\circ + y = 180^\circ$$

$$y = 180^\circ - 120^\circ$$

$$\boxed{y = 60^\circ}$$

$$\Rightarrow \angle RPO = 60^\circ$$

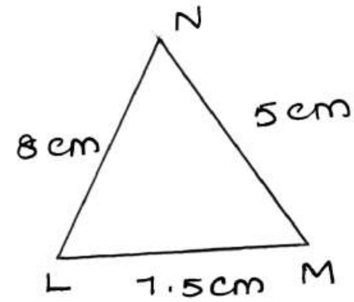
EXERCISE 4.5

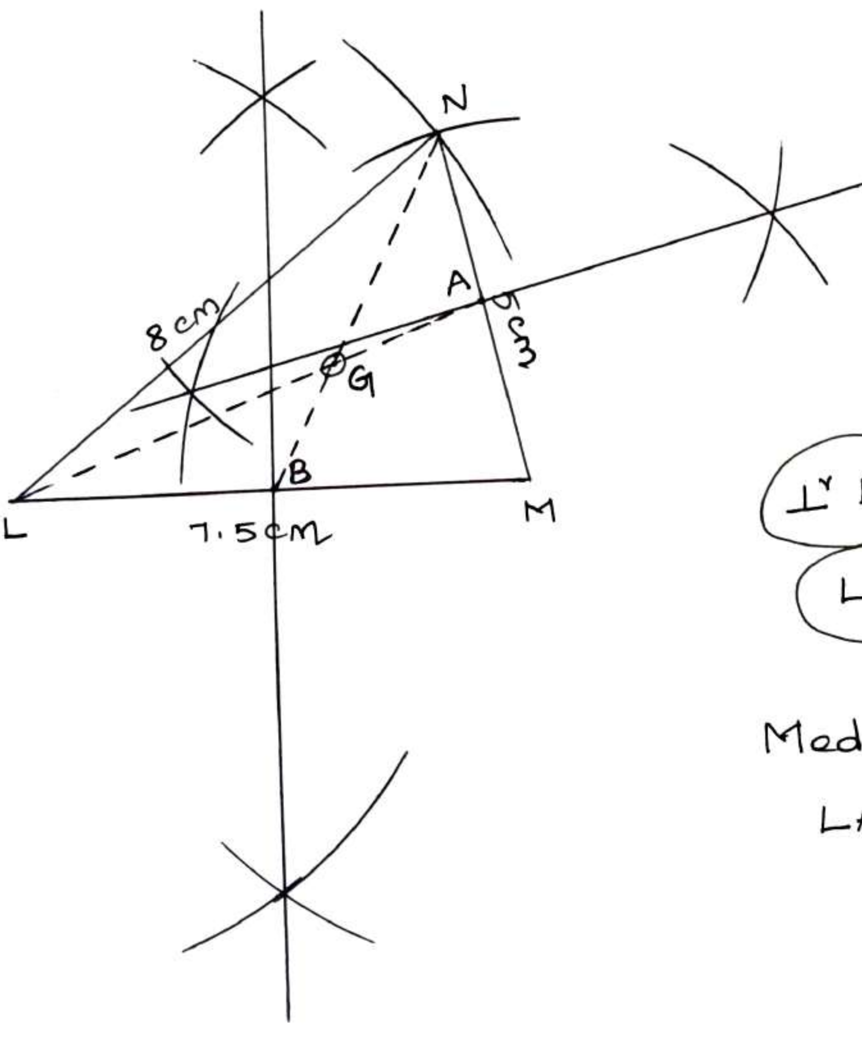
PRACTICAL GEOMETRY

CENTROID

1) Construct Δ LMN, where LM = 7.5cm, MN = 5cm, LN = 8cm. Locate its centroid.

ROUGH DIAGRAM





$\perp^r$ bisectors

LM, MN

Medians

LA, NB

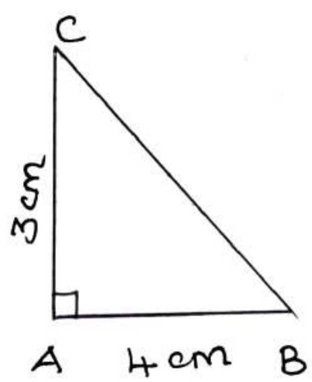
1) CONSTRUCTION:

* Draw LM = 7.5cm
* With L and M as centre draw two arcs of radius 8cm and 5cm to meet at N.
* Join LN and MN
* Thus Δ LMN is the required triangle.
* Draw the perpendicular bisector of any two sides [LM and MN] to meet at A and B.
* Join the medians LA and NB to meet at 'G'.
* 'G' is the centroid of Δ LMN.

---

2) Draw and locate the centroid of triangle ABC where right angled at A where AB = 4cm and AC = 3cm.

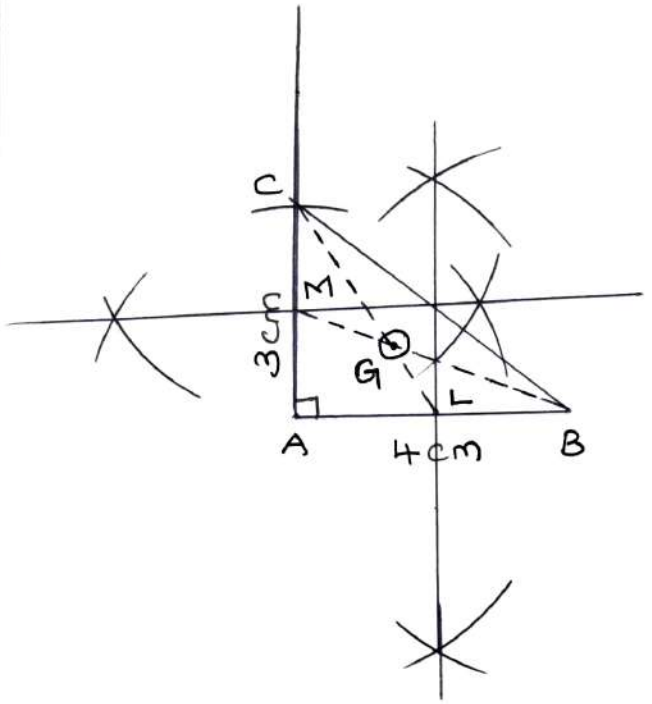
ROUGH DIAGRAM



Right angled triangle

$\perp^r$ bisectors => AB, AC

Medians CL, BM

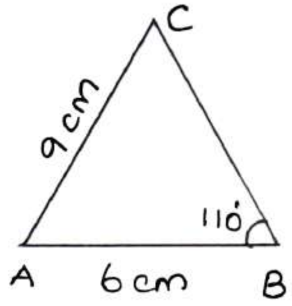


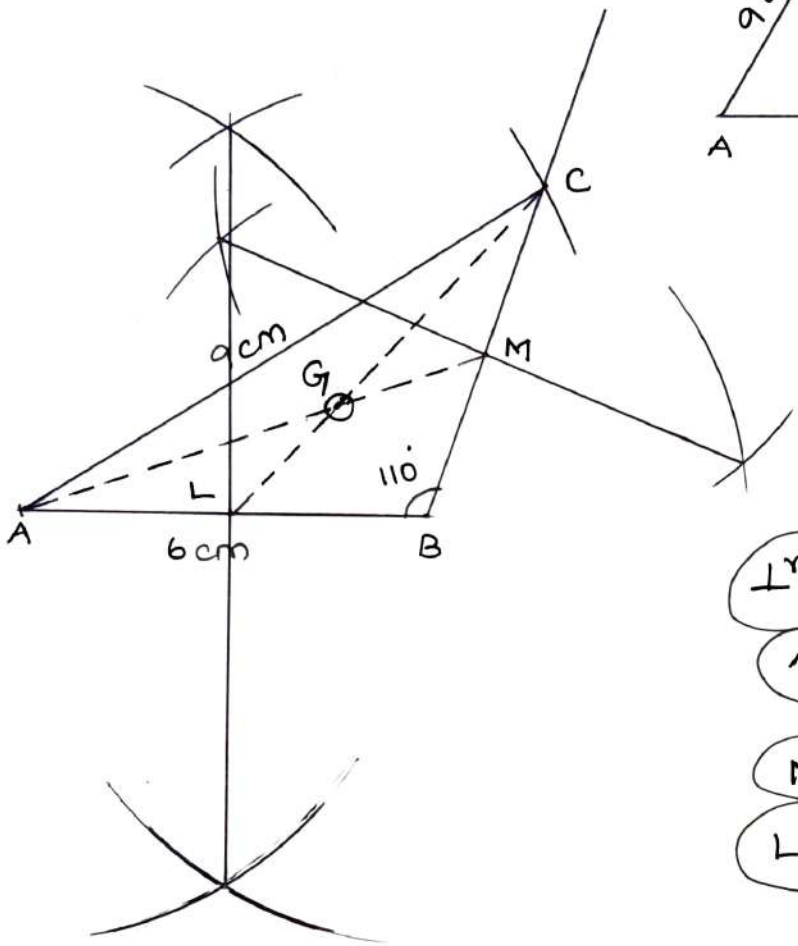
2) CONSTRUCTION:

* Draw AB = 4 cm
* With A as centre make an angle 90°
* With A as centre draw an arc of radius 3 cm to meet at C.
* Join BC
* Thus Δ ABC is the required triangle.
* Draw the perpendicular bisector of any two sides [AB and AC] to meet at L and M.
* Join the medians BM and CL to meet at G.
* 'G' is the centroid of Δ ABC.

3) Draw Δ ABC, where AB = 6 cm, $\angle B = 110°$ AC = 9 cm and construct the centroid.

Rough Diagram





$\perp^r$ Bisectors AB, BC

Medians LC and AM

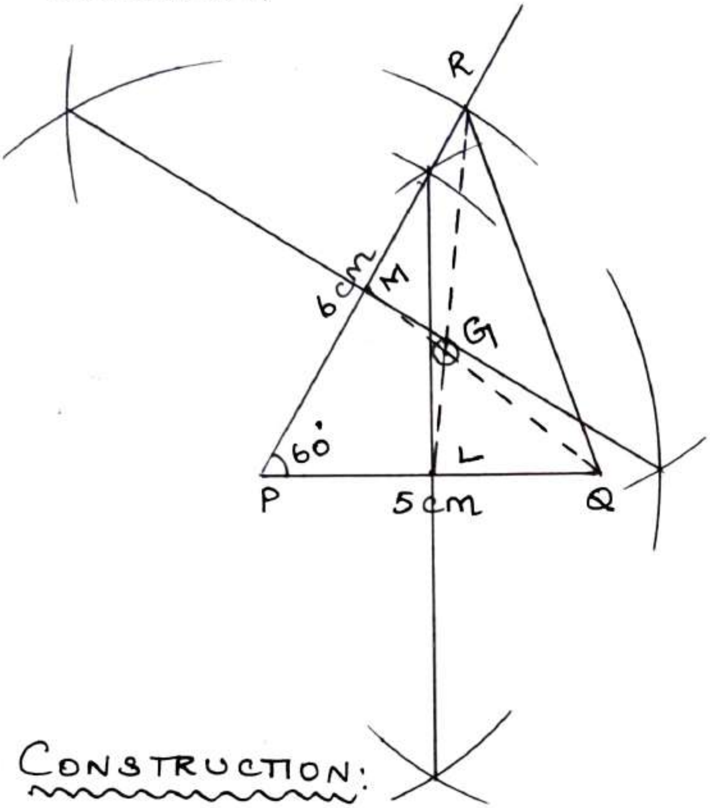
3) CONSTRUCTION:

* Draw AB = 6cm
* With B as centre make an angle 110°
* With A as centre draw an arc of radius 9cm to meet at C.
* Join AC
* Thus Δ ABC is the required triangle.
* Draw the perpendicular bisector of any two sides [AB and BC] to meet at L and M.
* Join the medians CL and AM to

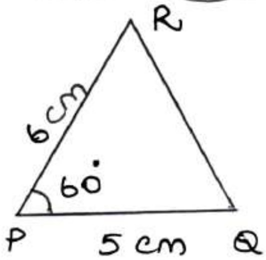
meet at 'G'.

* G is the centroid of Δ ABC

4) Construct Δ PQR such that PQ = 5 cm, PR = 6 cm, $\angle QPR = 60^\circ$ and locate its centroid.



Rough Diagram



$\perp^r$ bisectors PQ, PR

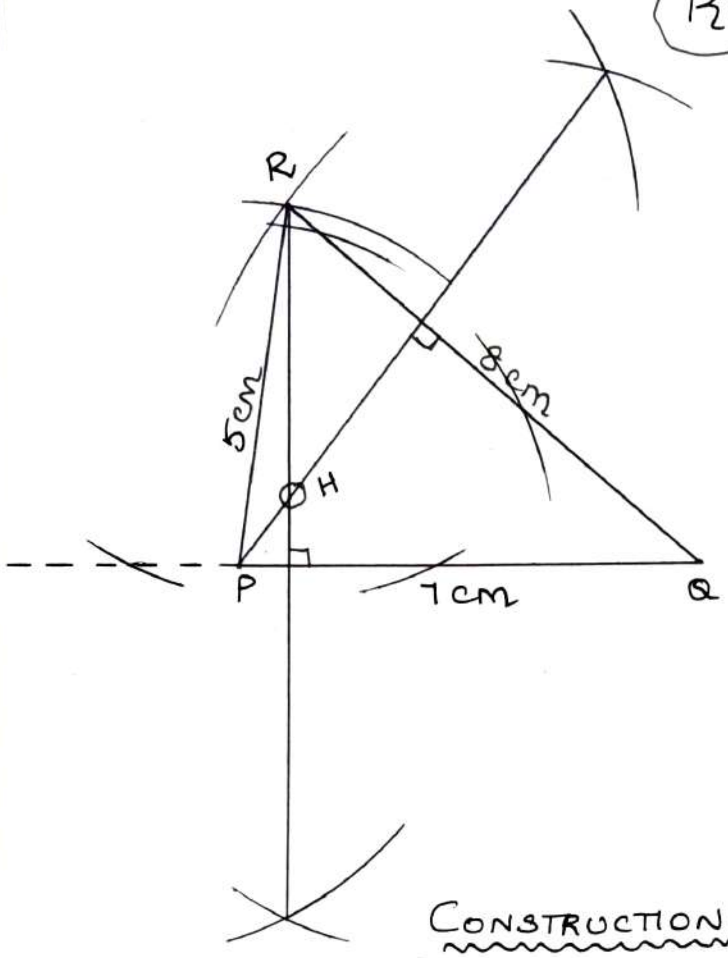
Medians LR, QM

4) CONSTRUCTION:

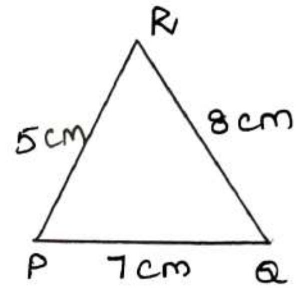
* Draw PQ = 5 cm
* With P as centre make an angle 60°
* With P as centre draw an arc of radius 6 cm to meet at R.
* Join QR
* Thus Δ PQR is the required triangle.
* Draw the perpendicular bisector of any two sides [PQ and PR] to meet at L and M.
* Join the medians RL and QM to meet at G.
* G is the centroid of Δ PQR.

ORTHOCENTRE

5) Draw Δ PQR with sides PQ = 7cm, QR = 8cm and PR = 5cm and construct its orthocentre.



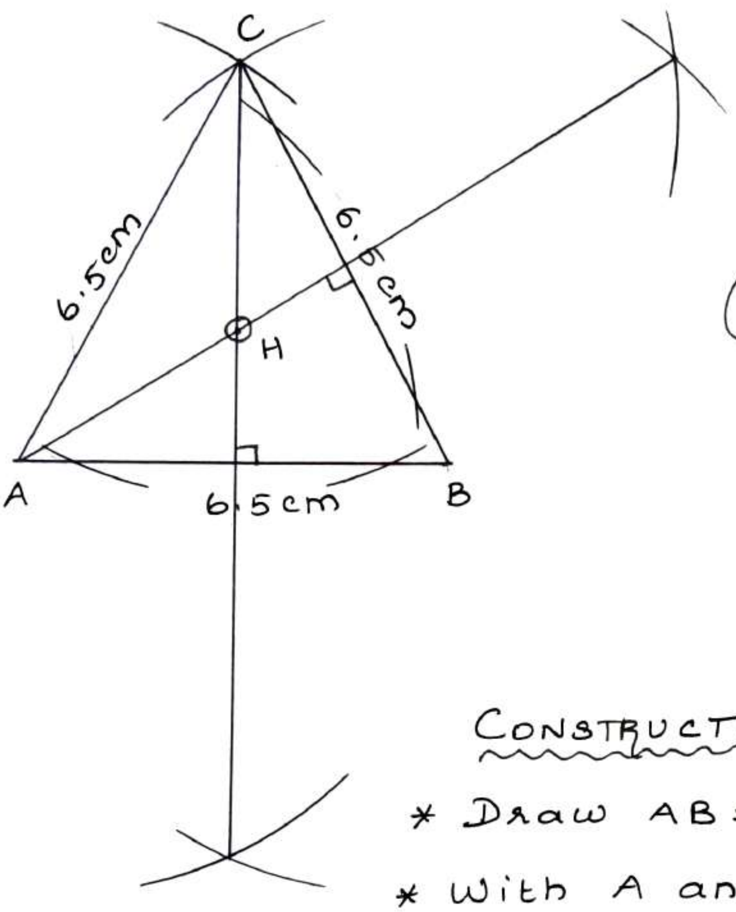
Rough Diagram



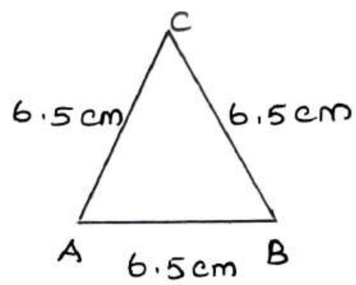
CONSTRUCTION:

* Draw PQ = 7cm
* With P and Q as centre, 5cm and 8cm radius, draw two arcs to meet at R
* Join PR and QR
* Thus Δ PQR is formed.
* Draw altitudes from any two vertices to their opposite sides to meet at 'H'.
* 'H' is the Orthocentre of Δ PQR

6) Draw an equilateral triangle of sides 6.5cm and locate its orthocentre.



Rough Diagram

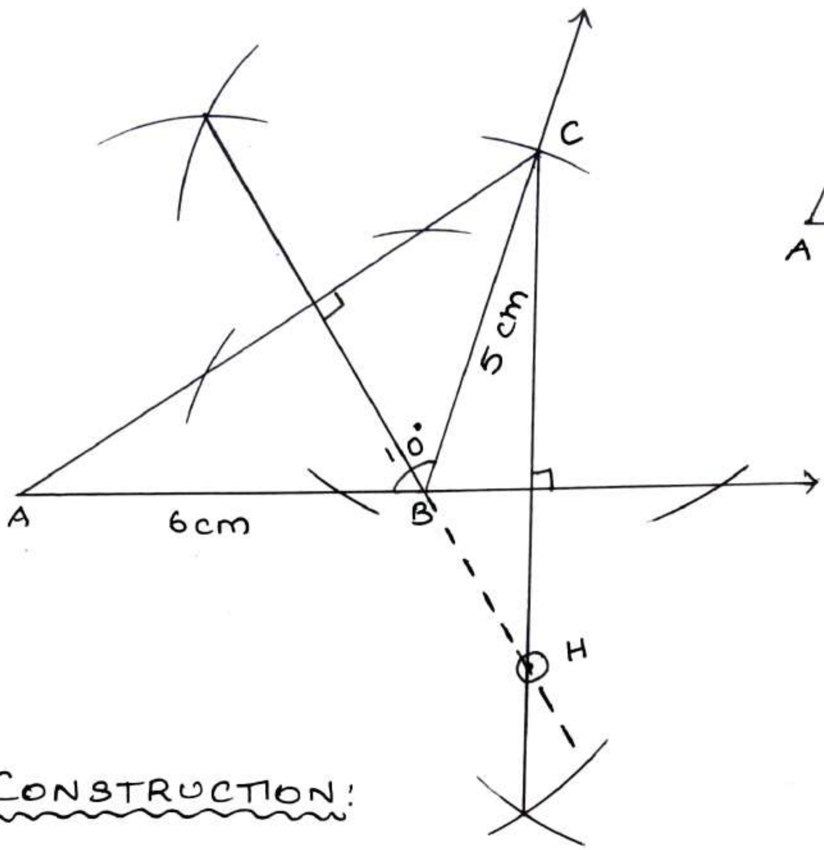


Equilateral triangle

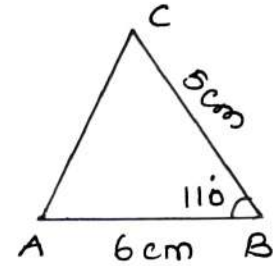
CONSTRUCTION:

* Draw AB = 6.5cm
* With A and B as centre, draw two arcs of radius 6.5cm to meet at C.
* Join AC and BC
* Thus Δ ABC is formed.
* Draw Altitudes from any two vertices to their opposite sides to meet at H.
* H is the orthocentre of Δ ABC.

7) Draw Δ ABC, where AB = 6 cm, $\angle B = 110°$, and BC = 5 cm and construct its orthocentre.



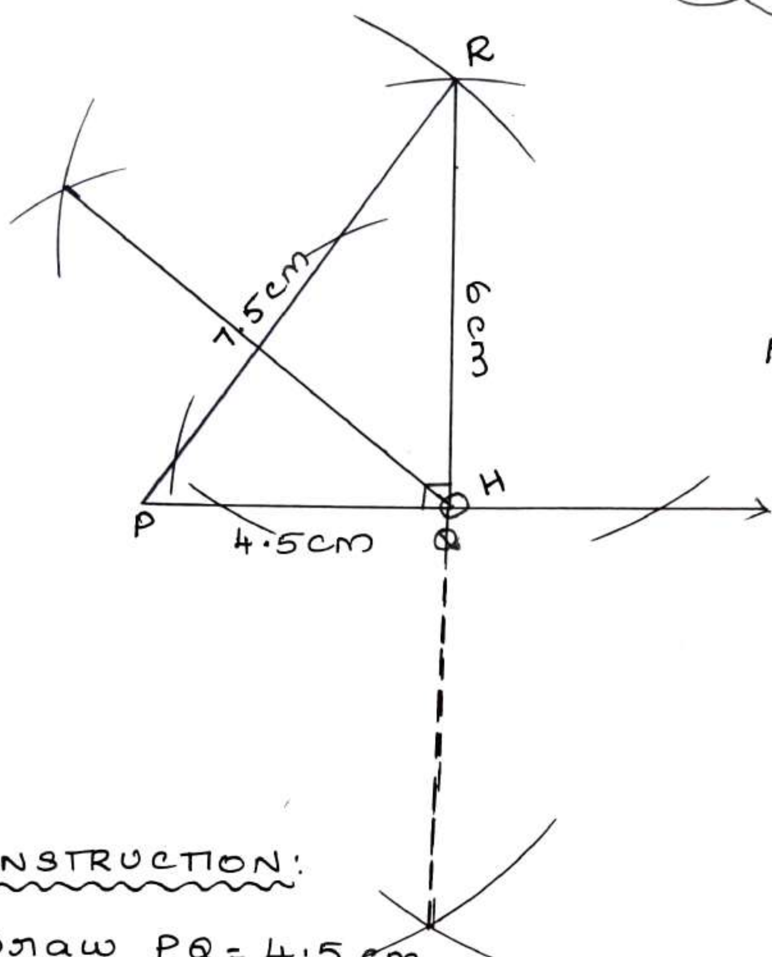
Rough Diagram



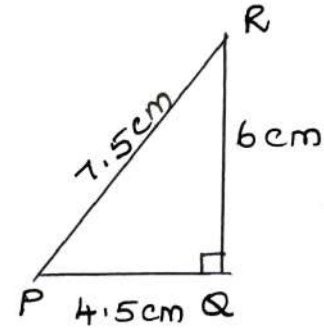
CONSTRUCTION:

* Draw AB = 6 cm.
* With B as centre make an angle $110°$
* With B as centre, 5 cm radius, draw an arc to meet at C.
* Join AC
* Thus Δ ABC is formed
* Draw altitudes from any two vertices to their opposite sides to meet at H.
* H is the orthocentre of Δ ABC.

8) Draw and locate the orthocentre of a right triangle PQR where PQ = 4.5 cm, QR = 6 cm, PR = 7.5 cm.



Rough Diagram



CONSTRUCTION:

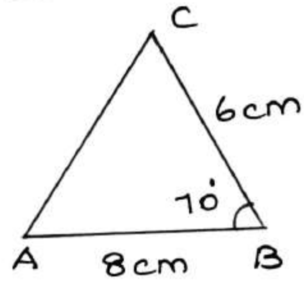
* Draw PQ = 4.5 cm
* With P and Q as centre 7.5 cm and 6 cm radius, draw two arcs to meet at R.
* Join PR and QR
* Thus Δ PQR is formed.
* Draw altitudes from any two vertices to their opposite sides to meet at H.
* H is the orthocentre of Δ PQR.

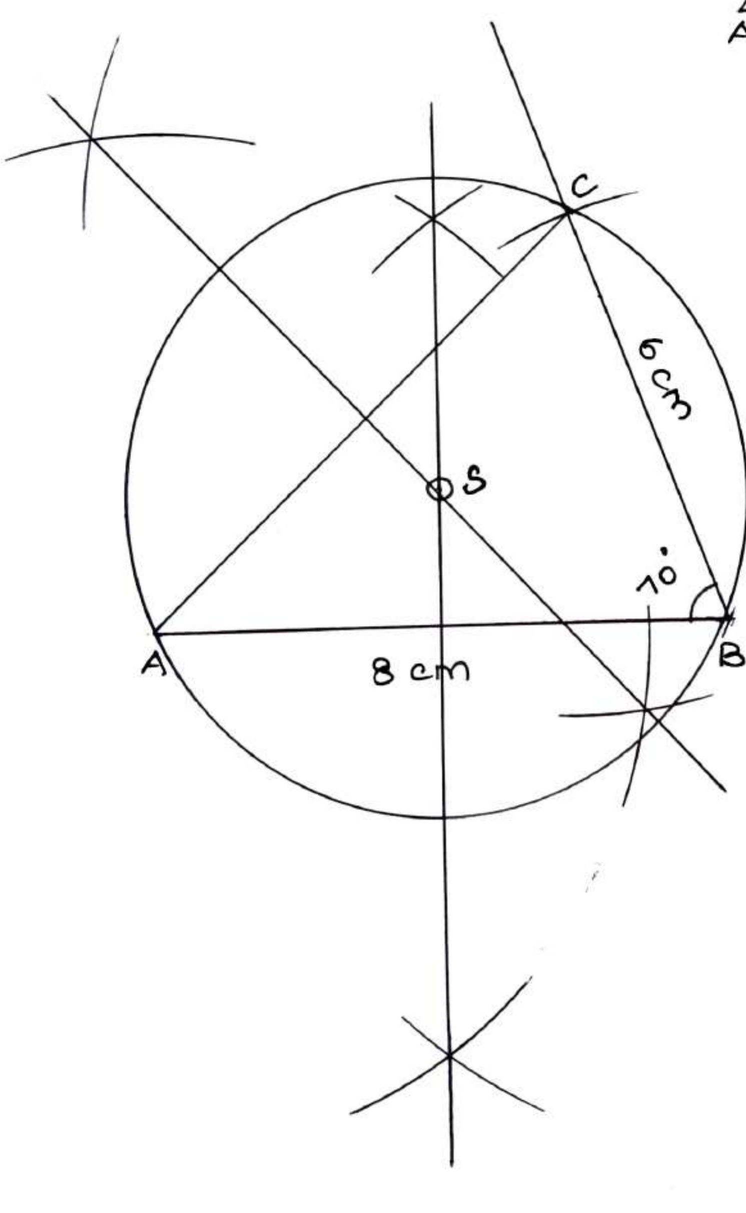
EXERCISE 4.6

CIRCUMCIRCLE

1) Draw a triangle ABC, where AB = 8 cm, BC = 6 cm and $\angle B = 70^\circ$ and locate its circumcentre and draw the circumcircle.

Rough Diagram





$\perp^r$ bisector AB, AC

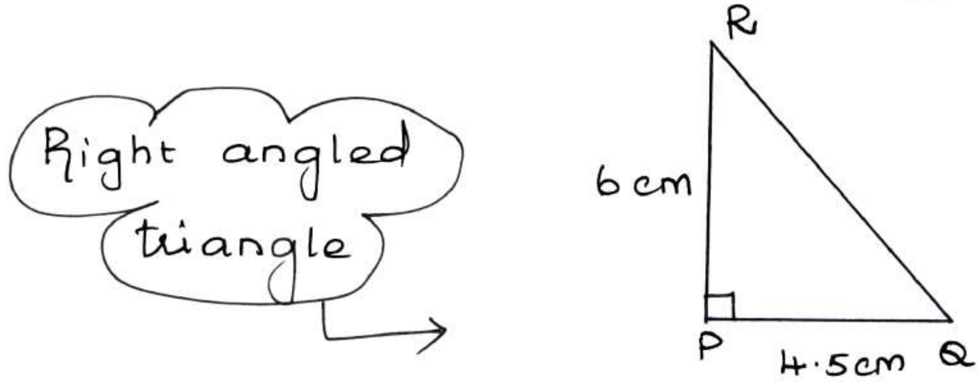
1) CONSTRUCTION:

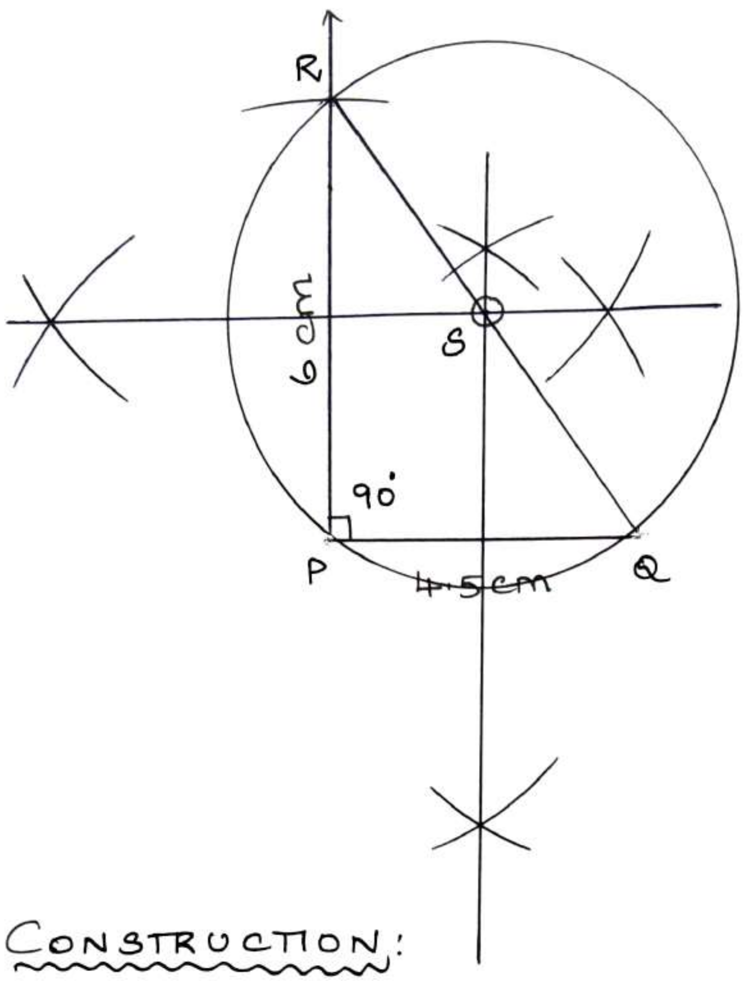
* Draw AB = 8 cm
* With 'B' as centre make an angle 70°
* With 'B' as centre draw an arc of radius 6 cm to meet at C.
* Join AC
* Thus Δ ABC is formed.
* Draw the perpendicular bisectors of any two sides (AB and AC) to meet at 'S'.
* 'S' is the circumcentre of the triangle.
* With 'S' as centre, SA, SB and SC as radius draw the circumcircle.

---

2) Construct the right angled triangle PQR whose perpendicular sides are 4.5 cm and 6 cm. Also locate its circumcentre and draw circumcircle.

Rough Diagram





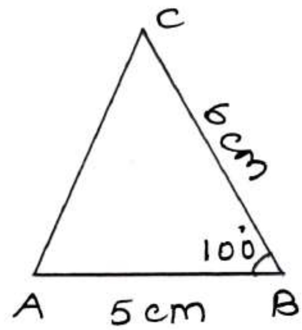
$\perp^r$ bisector
PQ, PR

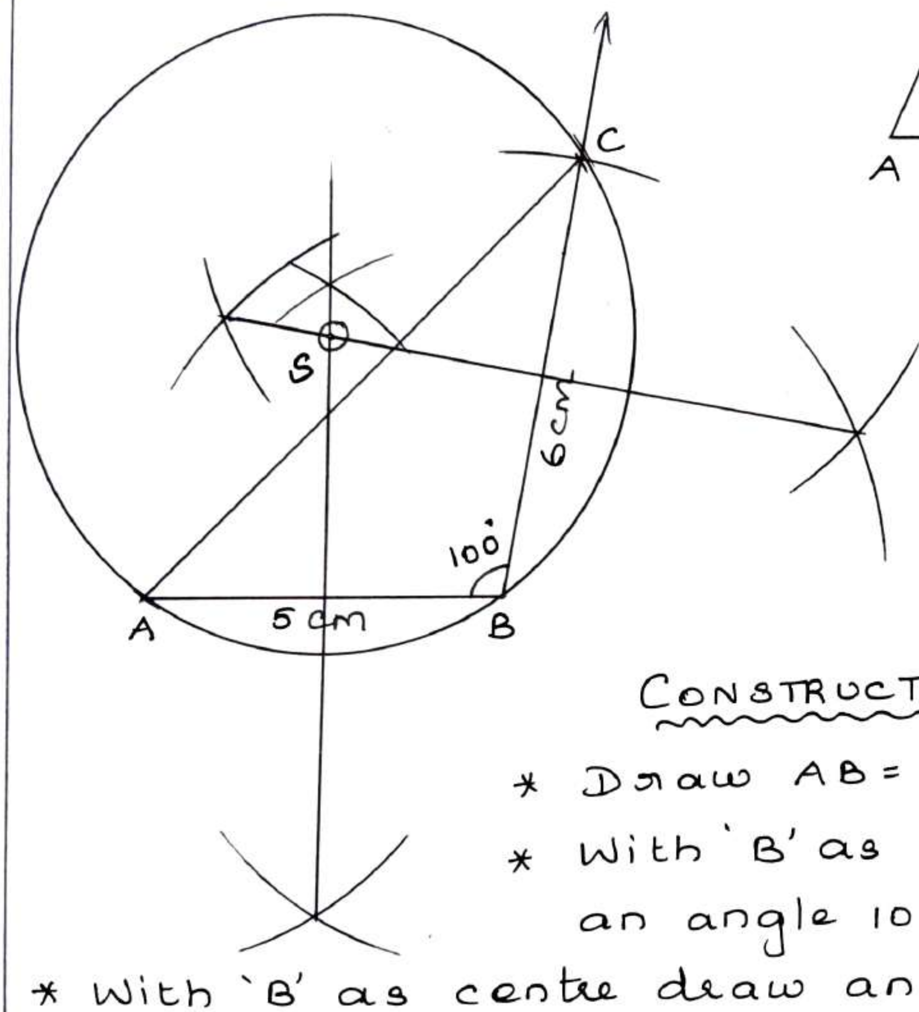
2) CONSTRUCTION:

* Draw PQ = 4.5cm
* With 'P' as centre make an angle 90°
* With 'P' as centre draw an arc of radius 6cm to meet at R.
* Join QR
* Thus Δ PQR is formed.
* Draw the perpendicular bisectors of any two sides (PQ and PR) to meet at 'S'.
* 'S' is the circumcentre of the triangle.
* With 'S' as centre SP, SQ, SR as radius draw the circumcircle.

3) Construct Δ ABC with AB = 5 cm, $\angle B = 100°$ and BC = 6 cm. Also locate its circumcentre and draw circum-circle.

Rough Diagram





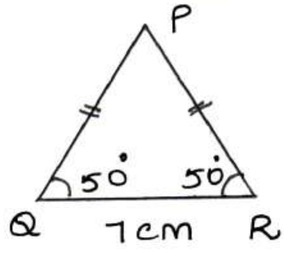
$\perp^r$ bisector AB, BC

CONSTRUCTION:

* Draw AB = 5 cm
* With 'B' as centre make an angle 100°
* With 'B' as centre draw an arc of radius 6 cm to meet at C.
* Join AC
* Thus Δ ABC is formed
* Draw the perpendicular bisectors of any two sides (AB, BC) to meet at S.
* 'S' is the circumcentre of the triangle.
* With 'S' as centre, SA, SB, SC as radius draw the circumcircle.

4) Construct an Isosceles triangle PQR where PQ = PR and $\angle Q = 50^\circ$, QR = 7cm. Also draw its circumcircle.

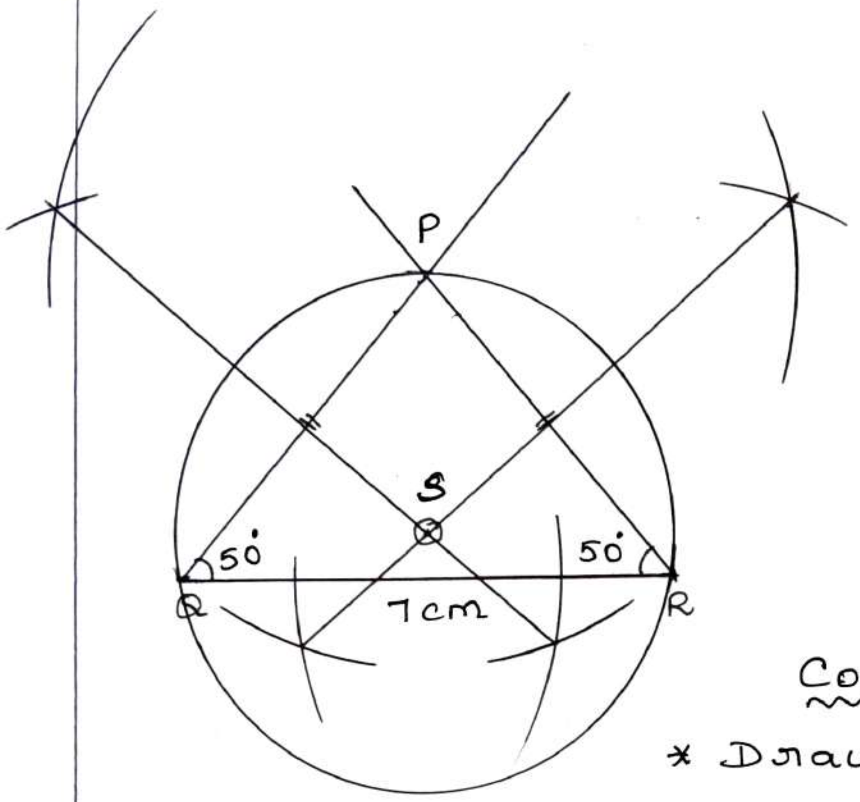
Rough Diagram



PQ = PR

Isosceles triangle

$\perp^r$ bisector PQ, PR



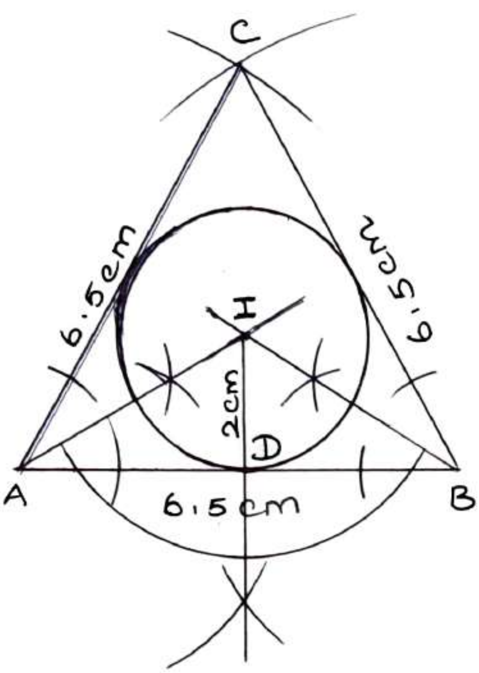
CONSTRUCTION:

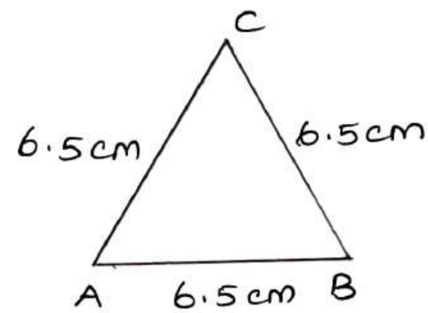
* Draw QR = 7cm
* With Q and R as centre make an angle 50° to meet at P.
* Thus Δ PQR is formed.
* Draw the perpendicular bisectors of any two sides [PQ and PR] to meet at S.
* 'S' is the circumcentre of the triangle.
* With 'S' as centre SP, SQ and SR as radius draw the circumcircle.

Incentre

5) Draw an equilateral triangle of sides 6.5 cm and locate its incentre. Also draw the incircle.

Rough Diagram





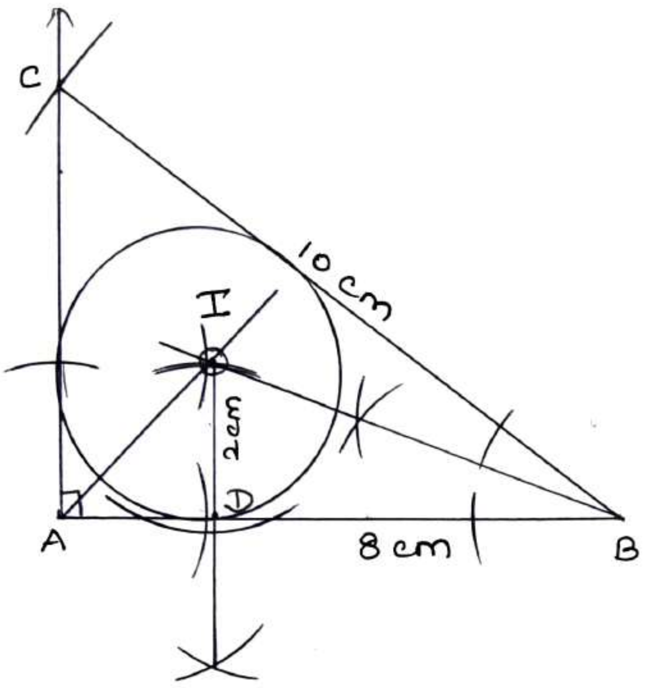
ID → Inradius 2cm

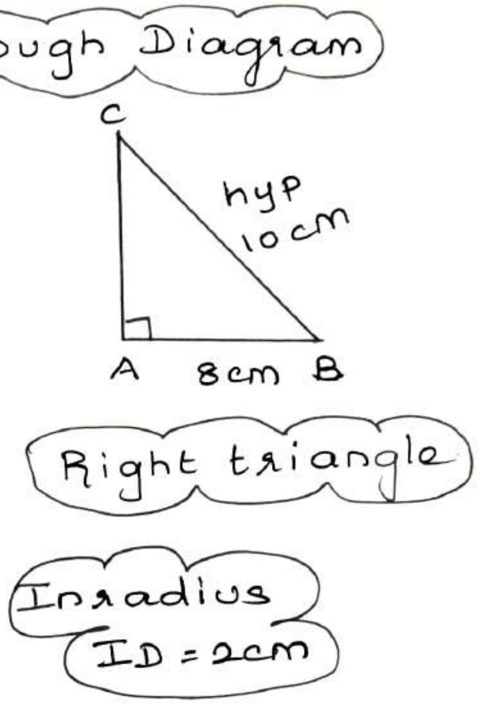
Equilateral triangle

CONSTRUCTION:

* Draw AB = 6.5 cm
* With A and B as centre, draw two arcs of radius 6.5 cm to meet at C.
* Join AC and BC
* Thus Δ ABC is formed.
* Draw the angle bisectors of A and B to meet at I
* 'I' is the incentre of the triangle.
* Draw perpendicular from I to AB to meet at 'D'
* With I as centre and ID as radius draw the incircle that touches all sides of the triangle.
* Inradius ID = 2 cm.

6) Draw a right triangle whose hypotenuse is 10cm and one of the legs is 8 cm. Locate its incentre and also draw incircle.



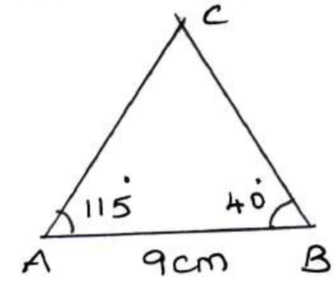


CONSTRUCTION:

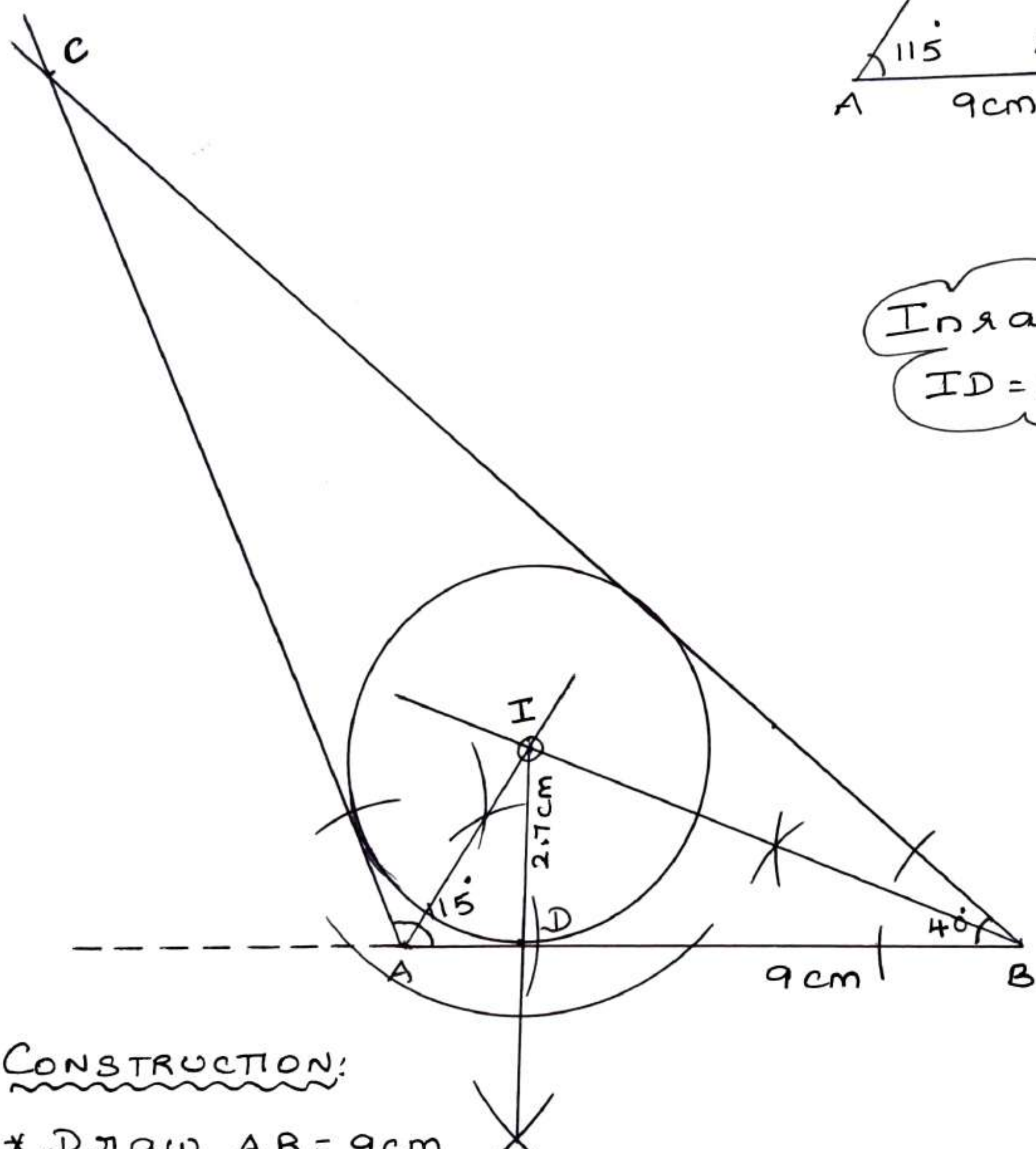
* Draw AB = 8 cm
* With 'A' as centre make an angle 90°
* With 'B' as centre draw an arc of radius 10cm too meet at C.
* Join BC, Thus Δ ABC is formed.
* Draw the angle bisectors of A and B to meet at I.
* I is the incentre of the triangle.
* Draw the perpendicular from I to AB to meet at D.
* With I as centre and ID as radius draw the incircle that touches all sides of the triangle.
* Inradius ID = 2cm

7) Draw ΔABC, given $AB = 9cm$, $\underline{|CAB} = 115^\circ$ and $\underline{|ABC} = 40^\circ$. Locate its incentre and also draw the incircle.

Rough Diagram



Inradius
ID = 2.7cm



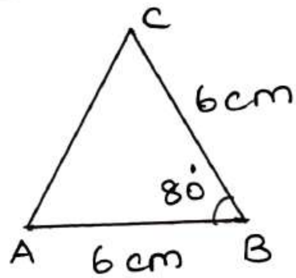
CONSTRUCTION:

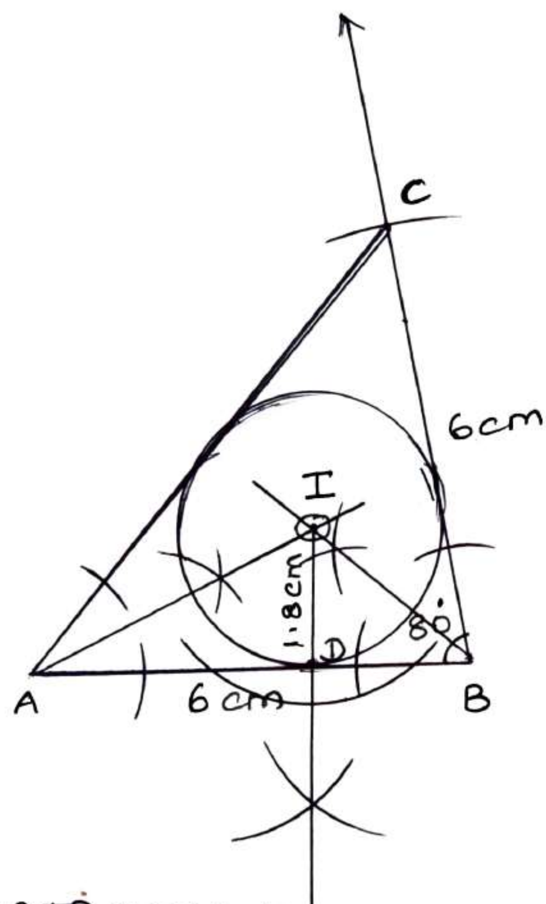
* Draw AB = 9cm
* With A and B as centre make angles 115° and 40° to meet at C.
* Thus ΔABC is formed.
* Draw the angle bisectors of A and B to meet at I.

* I is the incentre of the triangle.
* Draw perpendicular from I to AB to meet at D.
* With I as centre and ID as radius draw the incircle that touches all sides of the triangle.
* Inradius, ID = 2.7 cm.

8) Construct ΔABC in which $AB = BC = 6$ cm, $\angle B = 80^\circ$. Locate its incentre and draw the incircle.

Rough Diagram





Inradius
ID = 1.8 cm

CONSTRUCTION:

* Draw AB = 6 cm
* With B as centre make an angle 80°

* With 'B' as centre draw an arc of radius 6 cm to meet at C.
* Join AC
* Thus Δ ABC is formed.
* Draw the angle bisectors of A and B to meet at I.
* I is the incentre of the triangle.
* Draw perpendicular from I to AB to meet at D.
* With I as centre and ID as radius draw the incircle that touches all sides of the triangle.
* Inradius ID = 1.8 cm.

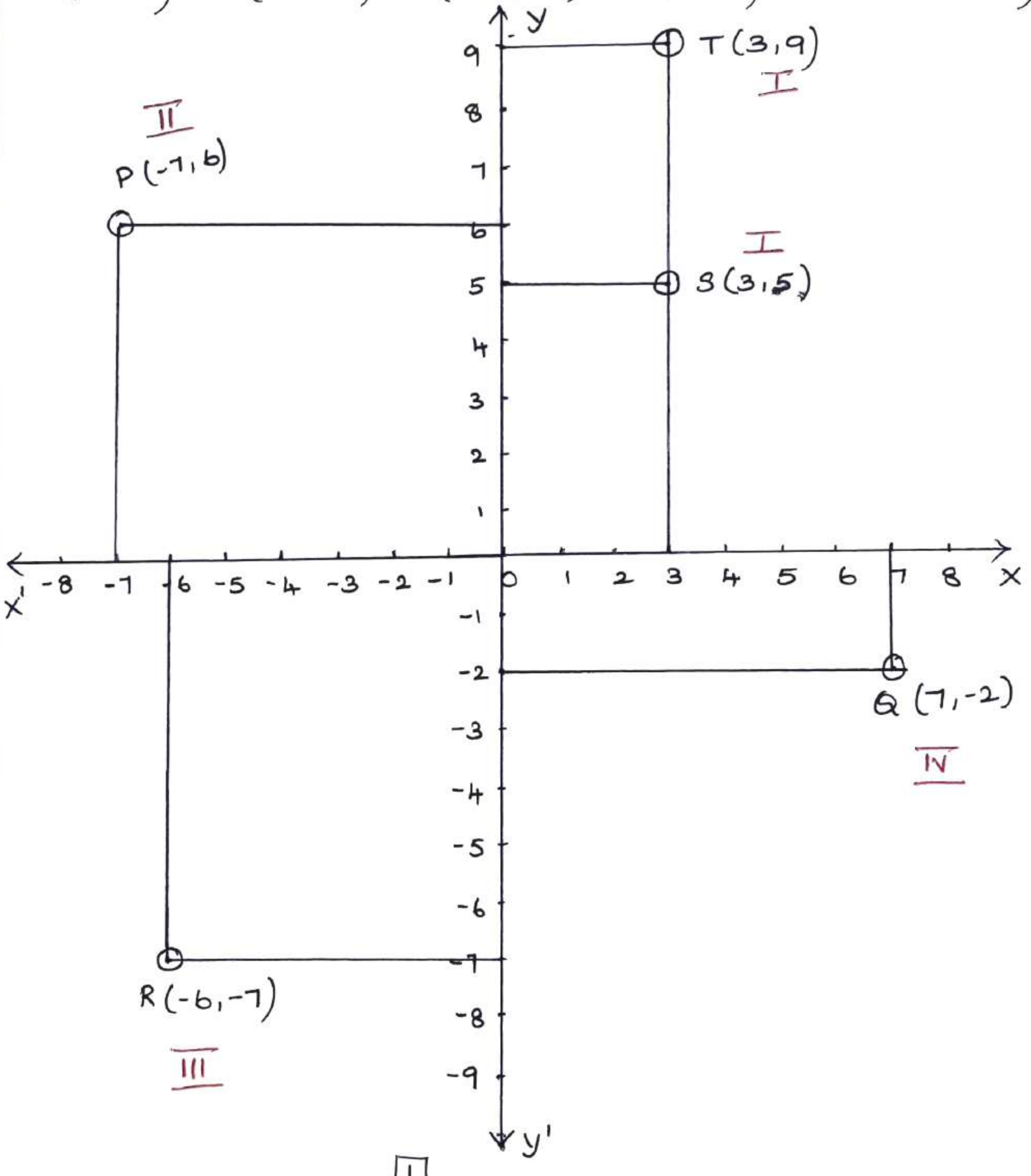
CHAPTER - 5

CO-ORDINATE GEOMETRY

Ex 5.1

1) Plot the following points in the co-ordinate system and identify the quadrants.

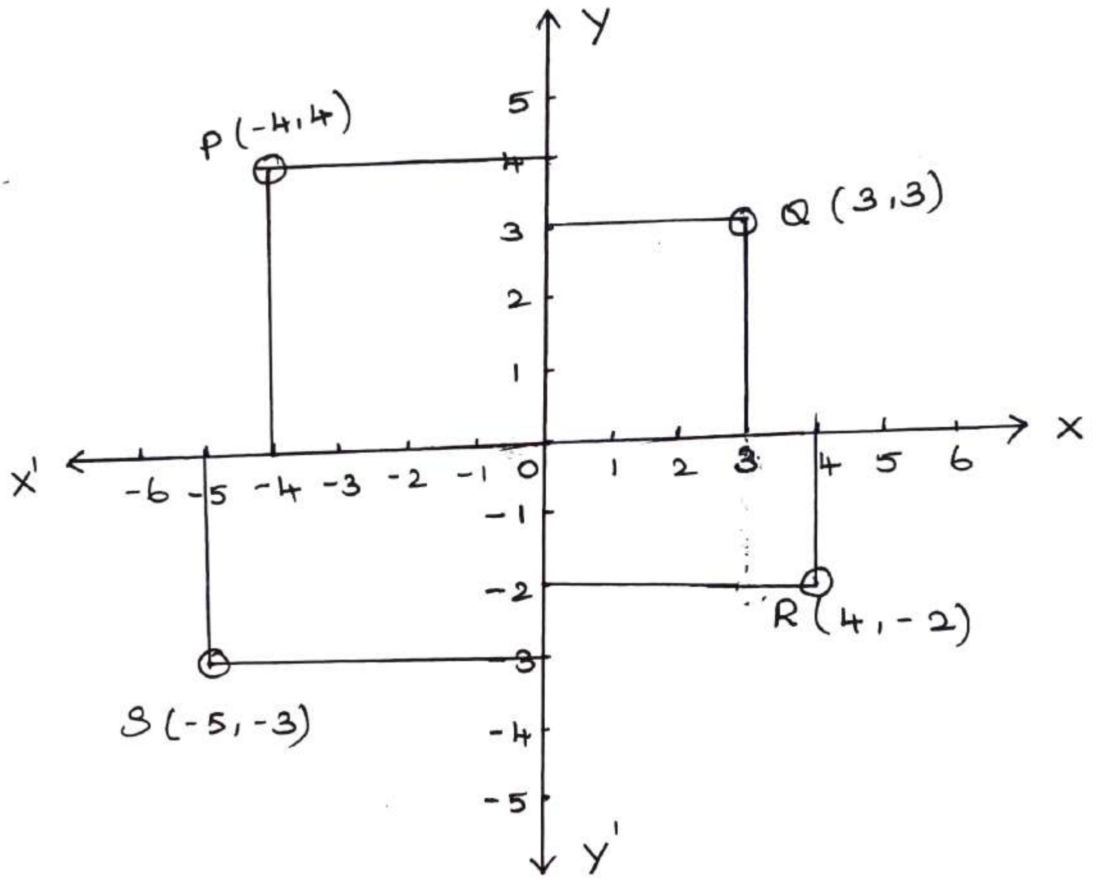
P(-7, 6) Q(7, -2) R(-6, -7) S(3, 5) and T(3, 9)



POINTS	QUADRANTS
P (-7,6)	II
Q (7,-2)	IV
R (-6,-7)	III
S (3,5)	I
T (3,9)	I

2) Write down the abscissa and ordinate of the following from figure.

(i) P (ii) Q (iii) R (iv) S

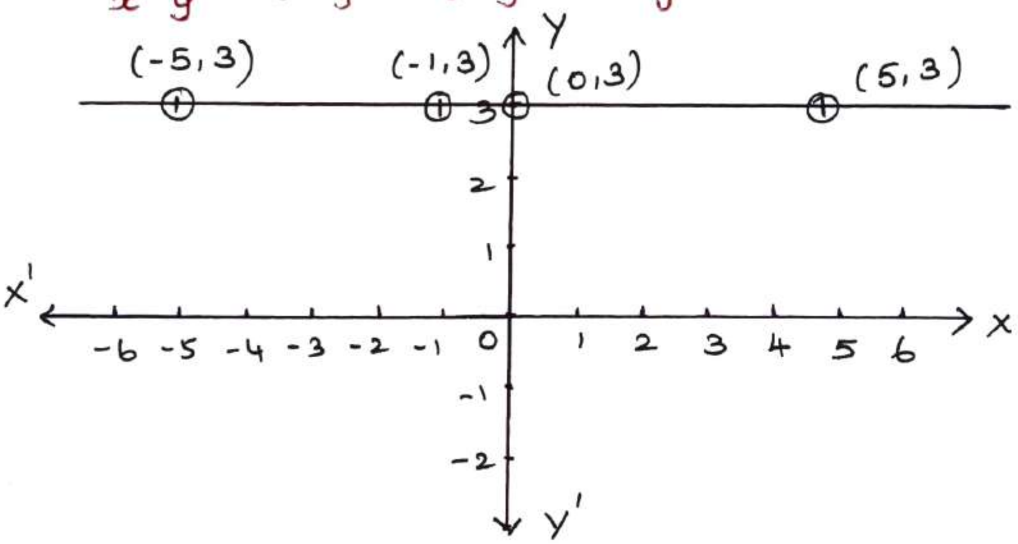


$\Rightarrow$ P(-4,4) Q(3,3) R(4,-2) S(-5,-3)

POINTS	abscissa	Ordinate
(i) P(-4, 4)	-4	4
(ii) Q (3, 3)	3	3
(iii) R (4, -2)	4	-2
(iv) S (-5, -3)	-5	-3

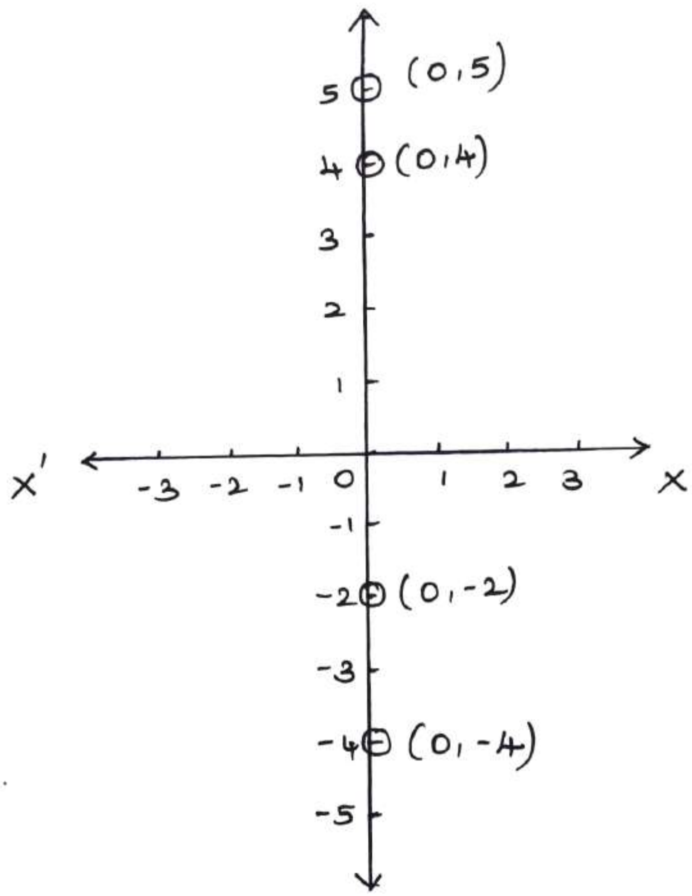
3) Plot the following points in the co-ordinate plane and join them. What is your conclusion about the resulting figure?

(i) (-5, 3) (-1, 3) (0, 3) (5, 3)
x y x y x y x y



⇒ The line is parallel to X-axis.

(ii) (0, -4) (0, -2) (0, 4) (0, 5)
x y x y x y x y



⇒ The points lie on Y-axis

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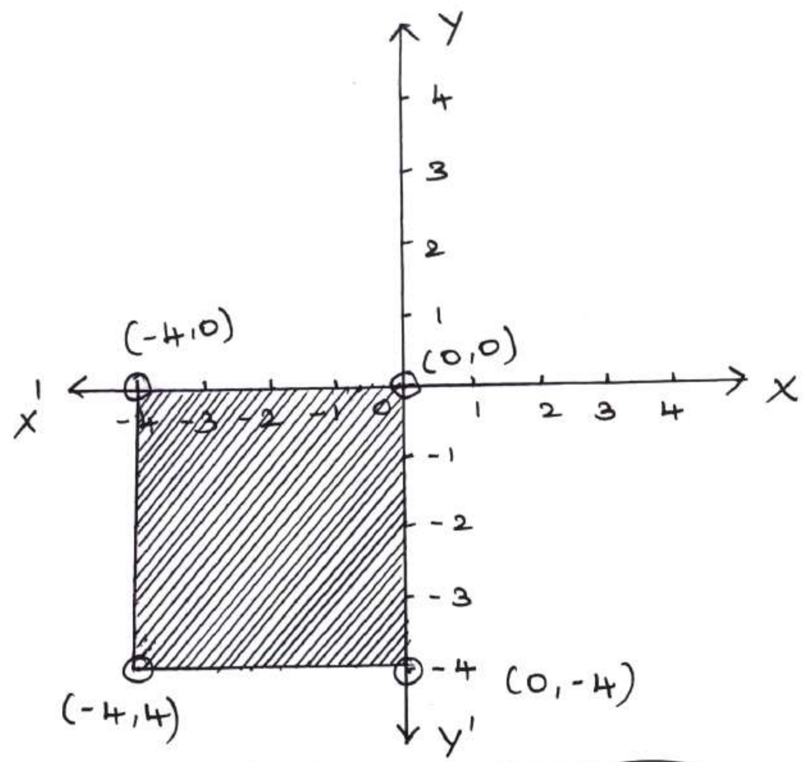
4) Plot the following points in the co-ordinate plane. Join them in order. What type of geometrical shape is formed?

(i) (0,0) (-4,0) (-4,4) (0,-4)

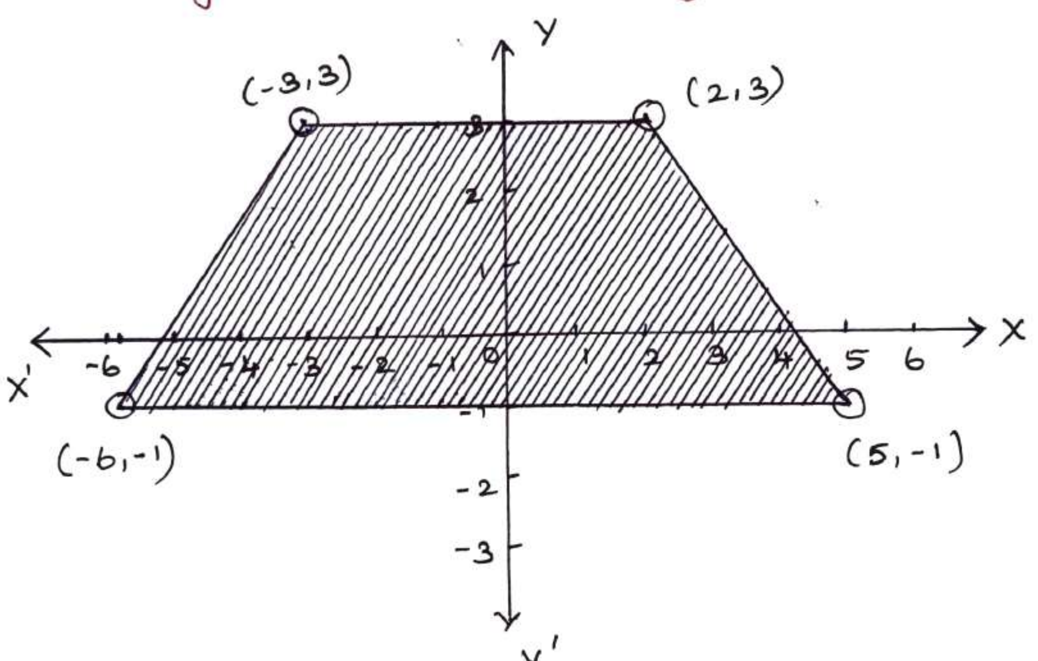
x y   x y   x y   x y



⇒ The Points form a Square

(ii) (-3,3) (2,3) (-6,-1) (5,-1)

x y x y x y x y



⇒ The points form a trapezium

EXERCISE 5.2

1. Find the distance between the following pair of points.

(i) (1,2) and (4,3)

x_1, y_1 $\quad x_2\, y_2$

Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-1)^2+(3-2)^2}$

$= \sqrt{(3)^2+(1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ units

(ii) (3,4) and (-7,2)

$x_1\, y_1$ $\quad x_2\, y_2$

Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-7-3)^2+(2-4)^2}$

$= \sqrt{(-10)^2+(-2)^2}$

$= \sqrt{100+4}$

$= \sqrt{104}$ units

(iii) (a, b) and (c, b)

$x_1\, y_1$ $\quad x_2\, y_2$

Distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(c-a)^2+(b-b)^2}$

$$= \sqrt{(c-a)^2 + (0)^2}$$

$$= \sqrt{(c-a)^{\cancel{2}}}$$

$$= c - a \text{ units}$$

(iv) $(3,-9)$ $(-2,3)$

x_1 y_1 x_2 y_2

$$\text{Distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-2-3)^2 + (3+9)^2}$$

$$= \sqrt{(-5)^2 + (12)^2}$$

$$= \sqrt{25+144}$$

$$= \sqrt{169}$$

$$= \sqrt{(13)^{\cancel{2}}}$$

$$= 13 \text{ units.}$$

2) Determine whether the given set of points in each case are Collinear or not?

(i) $(7,-2)$ $(5,1)$ $(3,4)$

$A(7,-2)$ $B(5,1)$ $C(3,4)$

$A(7,-2)$ $B(5,1)$

x_1 y_1 x_2 y_2

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(5-7)^2 + (1+2)^2}$$

$B(5,1)$ $C(3,4)$

x_1 y_1 x_2 y_2

$$BC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(3-5)^2 + (4-1)^2}$$

$= \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{4+9}$

$AB = \sqrt{13}$ units

$= \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{4+9}$

$BC = \sqrt{13}$ units

$A(7,-2)$ $C(3,4)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$AC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(3-7)^2 + (4+2)^2}$$

$$= \sqrt{(-4)^2 + (6)^2}$$

$$= \sqrt{16+36}$$

$$= \sqrt{52}$$

$$= \sqrt{\underbrace{2\times 2}\times 13}$$

$$\begin{array}{r|l} 2 & 52 \\ 2 & 26 \\ & 13 \end{array}$$

$AC = 2\sqrt{13}$ units

$\Rightarrow \boxed{AB + BC = AC}$

$\sqrt{13} + \sqrt{13} = 2\sqrt{13}$

$2\sqrt{13} = 2\sqrt{13}$

$\therefore$ The points are Collinear

(ii) $(a,-2)$ $(a,3)$ $(a,0)$

$A(a,-2)$ $B(a,3)$ $C(a,0)$

$A(a,-2)$ $B(a,3)$
$x_1\ y_1 \quad x_2\ y_2$

$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(a-a)^2+(3+2)^2}$

$= \sqrt{(0)^2+(5)^2}$

$= \sqrt{(5)^{\cancel{2}}}$

$AB = 5$ units

$B(a,3)$ $C(a,0)$
$x_1\ y_1 \quad x_2\ y_2$

$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(a-a)^2+(0-3)^2}$

$= \sqrt{(0)^2+(-3)^2}$

$= \sqrt{(-3)^2}$

$BC = 3$ units

$A(a,-2)$ $C(a,0)$
$x_1\ y_1 \quad x_2\ y_2$

$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(a-a)^2+(0+2)^2}$

$= \sqrt{(0)^2+(2)^2}$

$= \sqrt{(2)^{\cancel{2}}}$

$AC = 2$ units

$\Rightarrow \quad AB = BC + AC$

$5 = 3 + 2$

$5 = 5$

$\therefore$ The points are Collinear.

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3) Show that the following points taken in order form an Isosceles triangle.

(i) $A(5,4)$ $B(2,0)$ $C(-2,3)$

$A(5,4)$ $B(2,0)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(2-5)^2+(0-4)^2}$$
$$= \sqrt{(-3)^2+(-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= \sqrt{(5)^{\cancel{2}}}$$

$AB = 5$ units

$B(2,0)$ $C(-2,3)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-2)^2+(3-0)^2}$$
$$= \sqrt{(-4)^2+(3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= \sqrt{(5)^{\cancel{2}}}$$

$BC = 5$ units

$A(5,4)$ $C(-2,3)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-5)^2+(3-4)^2}$$
$$= \sqrt{(-7)^2+(-1)^2}$$
$$= \sqrt{49+1}$$
$$= \sqrt{50}$$
$$= \sqrt{2\times \underbrace{5\times 5}}$$

$AC = 5\sqrt{2}$ units

$$\begin{array}{r|l} 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$\Rightarrow$ $\boxed{AB = BC}$

$\therefore$ It is an Isosceles triangle.

(ii) $A(6,-4)$ $B(-2,-4)$ $C(2,10)$

$A(6,-4)$ $B(-2,-4)$

$x_1\ y_1 \quad x_2\ y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-6)^2+(-4+4)^2}$$
$$= \sqrt{(-8)^2+(0)^2}$$
$$= \sqrt{(-8)^2}$$

$AB = 8$ units

$B(-2,-4)$ $C(2,10)$

$x_1\ y_1 \quad x_2\ y_2$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(2+2)^2+(10+4)^2}$$
$$= \sqrt{(4)^2+(14)^2}$$
$$= \sqrt{16+196}$$
$$= \sqrt{212}$$
$$= \sqrt{2\times2\times53}$$

$BC = 2\sqrt{53}$ units

$A(6,-4)$ $C(2,10)$

$x_1\ y_1 \quad x_2\ y_2$

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(2-6)^2+(10+4)^2}$$
$$= \sqrt{(-4)^2+(14)^2}$$
$$= \sqrt{16+196}$$
$$= \sqrt{212}$$
$$= \sqrt{2\times2\times53}$$

$AC = 2\sqrt{53}$ units

$$\begin{array}{r|l} 2 & 212 \\ 2 & 106 \\ & 53 \end{array}$$

$\Rightarrow \boxed{BC = AC}$

$\therefore$ It is an Isosceles triangle.

4) Show that the points taken in order form an equilateral triangle in each case.

(i) $A(2,2)$ $B(-2,-2)$ $C(-2\sqrt{3}, 2\sqrt{3})$

$A(2,2)$ $B(-2,-2)$
$x_1\ y_1$ $\quad x_2\ y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-2)^2+(-2-2)^2}$$
$$= \sqrt{(-4)^2+(-4)^2}$$
$$= \sqrt{16+16}$$
$$AB = \sqrt{32} \text{ units}$$

$B(-2,-2)$ $C(-2\sqrt{3}, 2\sqrt{3})$
$x_1\ y_1$ $\quad x_2\ y_2$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2\sqrt{3}+2)^2+(2\sqrt{3}+2)^2}$$
$$= \sqrt{(-2\sqrt{3})^2+(2)^2+2(-2\sqrt{3})(2)+(2\sqrt{3})^2+(2)^2+2(2\sqrt{3})(2)}$$
$$= \sqrt{(4\times 3)+4-\cancel{8\sqrt{3}}+(4\times 3)+4+\cancel{8\sqrt{3}}}$$
$$= \sqrt{12+4+12+4}$$
$$BC = \sqrt{32} \text{ units}$$

$A(2,2)$ $C(-2\sqrt{3}, 2\sqrt{3})$
$x_1\ y_1$ $\quad x_2\ y_2$

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2\sqrt{3}-2)^2+(2\sqrt{3}-2)^2}$$

$$= \sqrt{(-2\sqrt{3})^2 + (2)^2 - 2(-2\sqrt{3})(2) + (2\sqrt{3})^2 + (2)^2 - 2(2\sqrt{3})(2)}$$

$$= \sqrt{(4\times 3) + 4 + \cancel{8\sqrt{3}} + (4\times 3) + 4 - \cancel{8\sqrt{3}}}$$

$$= \sqrt{12+4+12+4}$$

$$AC = \sqrt{32} \text{ units}$$

$$\Rightarrow \boxed{AB = BC = AC}$$

$\therefore$ It is an equilateral triangle

(ii) $A(\sqrt{3},2)$ $B(0,1)$ $C(0,3)$

$A(\sqrt{3},2)$ $B(0,1)$

$x_1\ y_1 \qquad x_2\ y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(0-\sqrt{3})^2+(1-2)^2}$$

$$= \sqrt{(-\sqrt{3})^2+(-1)^2}$$

$$= \sqrt{3+1}$$

$$= \sqrt{4}$$

$$= \sqrt{(2)^{\cancel{2}}}$$

$AB = 2$ units

$B(0,1)$ $C(0,3)$

$x_1\ y_1 \qquad x_2\ y_2$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(0-0)^2+(3-1)^2}$$

$$= \sqrt{(0)^2+(2)^2}$$

$$= \sqrt{(2)^{\cancel{2}}}$$

$BC = 2$ units

$A(\sqrt{3},2)$ $C(0,3)$

$x_1\ y_1 \qquad x_2\ y_2$

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(0-\sqrt{3})^2+(3-2)^2}$$

$= \sqrt{(-\sqrt{3})^2 + (1)^2}$

$= \sqrt{3+1}$

$= \sqrt{4}$

$= \sqrt{(2)^{\cancel{2}}}$

$AC = 2$ units

$\Rightarrow$ $\boxed{AB = BC = AC}$

$\therefore$ It is an equilateral triangle.

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5) Show that the following points taken in order form the vertices of a parallelogram.

(i) $A(-3,1)$ $B(-6,-7)$ $C(3,-9)$ $D(6,-1)$

$A(-3,1)$ $B(-6,-7)$

$x_1\ y_1$ $\quad x_2\ y_2$

$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(-6+3)^2 + (-7-1)^2}$

$= \sqrt{(-3)^2 + (-8)^2}$

$= \sqrt{9+64}$

$AB = \sqrt{73}$ units

$B(-6,-7)$ $C(3,-9)$

$x_1\ y_1$ $\quad x_2\ y_2$

$BC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(3+6)^2 + (-9+7)^2}$

$= \sqrt{(9)^2 + (-2)^2}$

$= \sqrt{81+4}$

$BC = \sqrt{85}$ units

$C(3,-9)$ $D(6,-1)$

$x_1\ y_1$ $\quad x_2\ y_2$

$CD = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$A(-3,1)$ $D(6,-1)$

$x_1\ y_1$ $\quad x_2\ y_2$

$AD = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$$= \sqrt{(6-3)^2+(-1+9)^2}$$
$$= \sqrt{(3)^2+(8)^2}$$
$$= \sqrt{9+64}$$
$$CD = \sqrt{73} \text{ units}$$

$$= \sqrt{(6+3)^2+(-1-1)^2}$$
$$= \sqrt{(9)^2+(-2)^2}$$
$$= \sqrt{81+4}$$
$$AD = \sqrt{85} \text{ units}$$

$\boxed{AB = CD}$ $\boxed{BC = AD}$

$\Rightarrow$ It is a parallelogram

(ii) $A(-7,-3)$ $B(5,10)$ $C(15,8)$ $D(3,-5)$

$A(-7,-3)$ $B(5,10)$

$x_1\ y_1 \quad x_2\ y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(5+7)^2+(10+3)^2}$$
$$= \sqrt{(12)^2+(13)^2}$$
$$= \sqrt{144+169}$$
$$AB = \sqrt{313} \text{ units}$$

$C(15,8)$ $D(3,-5)$

$x_1\ y_1 \quad x_2\ y_2$

$$CD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(3-15)^2+(-5-8)^2}$$
$$= \sqrt{(-12)^2+(-13)^2}$$
$$= \sqrt{144+169}$$
$$CD = \sqrt{313} \text{ units}$$

$B(5,10)$ $C(15,8)$

$x_1\ y_1 \quad x_2\ y_2$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(15-5)^2+(8-10)^2}$$
$$= \sqrt{(10)^2+(-2)^2}$$
$$= \sqrt{100+4}$$
$$BC = \sqrt{104} \text{ units}$$

$A(-7,-3)$ $D(3,-5)$

$x_1\ y_1 \quad x_2\ y_2$

$$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(3+7)^2+(-5+3)^2}$$
$$= \sqrt{(10)^2+(-2)^2}$$
$$= \sqrt{100+4}$$
$$AD = \sqrt{104} \text{ units}$$

$\Rightarrow$ $\boxed{AB = CD}$ $\boxed{BC = AD}$

$\therefore$ It is a parallelogram

6) Verify that the following points taken in order form the vertices of a Rhombus.

(i) $A(3,-2)$ $B(7,6)$ $C(-1,2)$ $D(-5,-6)$

$A(3,-2)$ $B(7,6)$
$x_1\ y_1 \quad x_2\ y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(7-3)^2+(6+2)^2}$$
$$= \sqrt{(4)^2+(8)^2}$$
$$= \sqrt{16+64}$$
$$AB = \sqrt{80} \text{ units}$$

$B(7,6)$ $C(-1,2)$
$x_1\ y_1 \quad x_2\ y_2$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-1-7)^2+(2-6)^2}$$
$$= \sqrt{(-8)^2+(-4)^2}$$
$$= \sqrt{64+16}$$
$$BC = \sqrt{80} \text{ units}$$

$C(-1,2)$ $D(-5,-6)$
$x_1\ y_1 \quad x_2\ y_2$

$$CD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-5+1)^2+(-6-2)^2}$$
$$= \sqrt{(-4)^2+(-8)^2}$$
$$= \sqrt{16+64}$$
$$CD = \sqrt{80} \text{ units}$$

$A(3,-2)$ $D(-5,-6)$
$x_1\ y_1 \quad x_2\ y_2$

$$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-5-3)^2+(-6+2)^2}$$
$$= \sqrt{(-8)^2+(-4)^2}$$
$$= \sqrt{64+16}$$
$$AD = \sqrt{80} \text{ units}$$

$\Rightarrow$ $\boxed{AB = BC = CD = AD}$

$\therefore$ It is a Rhombus.

(ii) A(1,1) B(2,1) C(2,2) D(1,2)

A(1,1) B(2,1)

$x_1 y_1$ $x_2 y_2$

$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-1)^2+(1-1)^2}$

$= \sqrt{(1)^2+(0)^2}$

$= \sqrt{\cancel{(1)^2}}$

AB = 1 units

B(2,1) C(2,2)

$x_1 y_1$ $x_2 y_2$

$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-2)^2+(2-1)^2}$

$= \sqrt{(0)^2+(1)^2}$

$= \sqrt{\cancel{(1)^2}}$

BC = 1 units

C(2,2) D(1,2)

$x_1 y_1$ $x_2 y_2$

$CD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1-2)^2+(2-2)^2}$

$= \sqrt{(-1)^2+(0)^2}$

$= \sqrt{(-1)^2}$

CD = 1 units

A(1,1) D(1,2)

$x_1 y_1$ $x_2 y_2$

$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1-1)^2+(2-1)^2}$

$= \sqrt{(0)^2+(1)^2}$

$= \sqrt{\cancel{(1)^2}}$

AD = 1 units

$\Rightarrow$ $\boxed{AB = BC = CD = AD}$

$\therefore$ It is a Rhombus

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7) If A(-1,1) B(1,3) and C(3,a) are points and if $\boxed{AB = BC}$, then find 'a'.

A(-1,1) B(1,3)

$x_1 y_1$ $x_2 y_2$

$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

B(1,3) C(3,a)

$x_1 y_1$ $x_2 y_2$

$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(1+1)^2+(3-1)^2}$$
$$= \sqrt{(2)^2+(2)^2}$$
$$= \sqrt{4+4}$$
$$AB = \sqrt{8} \text{ units}$$

$$= \sqrt{(3-1)^2+(a-3)^2}$$
$$= \sqrt{(2)^2+(a)^2+(3)^2-2(a)(3)}$$
$$= \sqrt{4+a^2+9-6a}$$
$$BC = \sqrt{a^2-6a+13} \text{ units}$$

Given:

$$\boxed{AB = BC}$$
$$\sqrt{8} = \sqrt{a^2-6a+13}$$
$$8 = a^2-6a+13$$
$$a^2-6a+13-8=0$$
$$a^2-6a+5=0$$
$$(a-1)(a-5)=0$$

$+5$: -1, -5

$$a-1=0 \quad\Big|\quad a-5=0$$
$$a=1 \quad\Big|\quad a=5$$

8) The abscissa of a point A is equal to its ordinate and its distance from the point B (1,3) is 10 units, what are the co-ordinates of A?

Let $A(a, a)$ and $B(1, 3)$
$\quad x_1, y_1 \qquad x_2\ y_2$

$$\boxed{\text{Distance} = 10 \text{ units}}$$
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=10$$
$$\sqrt{(1-a)^2+(3-a)^2}=10$$
$$\sqrt{(1)^2+(a)^2-2(1)(a)+(3)^2+(a)^2-2(3)(a)}=10$$

$$\sqrt{1+a^2-2a+9+a^2-6a} = 10$$

$$\sqrt{2a^2-8a+10} = 10$$

squaring on both sides

$$\left(\sqrt{2a^2-8a+10}\right)^{\cancel{2}} = (10)^2$$

$$2a^2-8a+10 = 100$$

$$(\div 2) \quad a^2-4a+5 = 50$$

$$a^2-4a+5-50 = 0$$

$$\boxed{a^2-4a-45=0}$$

-45

$+5 \quad -9$

$$(a+5)(a-9) = 0$$

$a+5=0$ | $a-9=0$

$a=-5$ | $a=9$

$$\Rightarrow \boxed{A(-5,-5) \text{ or } A(9,9)}$$

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9) The point (x, y) is equidistant from the points $(3,4)$ and $(-5,6)$. Find a relation between x and y.

$(x,y) \Rightarrow$ equidistant $\Rightarrow (3,4)$ and $(-5,6)$

$(x,y) \quad (3,4)$

$x_1 \, y_1 \quad x_2 \, y_2$

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(3-x)^2+(4-y)^2}$$

$(x, y) \quad (-5, 6)$

$x_1 \, y_1 \qquad x_2 \, y_2$

$$\text{Distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-5-x)^2 + (6-y)^2}$$

Since it is equidistant,

$$\sqrt{(3-x)^2 + (4-y)^2} = \sqrt{(-5-x)^2 + (6-y)^2}$$

$$(3)^2 + (x)^2 - 2(3)(x) + (4)^2 + (y)^2 - 2(4)(y) =$$

$$(-5)^2 + (x)^2 - 2(-5)(x) + (6)^2 + (y)^2 - 2(6)(y)$$

$$9 + \cancel{x^2} - 6x + 16 + \cancel{y^2} - 8y = 25 + \cancel{x^2} + 10x + 36 + \cancel{y^2} - 12y$$

$$25 - 6x - 8y = 61 + 10x - 12y$$

$$10x + 6x - 12y + 8y + 61 - 25 = 0$$

$$16x - 4y + 36 = 0$$

$$(\div 4) \qquad 4x - y + 9 = 0$$

$$4x + 9 = y$$

$$\Rightarrow y = 4x + 9$$

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10) Let $A(2, 3)$ and $B(2, -4)$ be two points. If P lies on the x-axis, such that $AP = \frac{3}{7} AB$, Find the co-ordinates of P.

$\Rightarrow$ P lies on X-axis $\Rightarrow \boxed{P(x, 0)}$

Given: $AP = \frac{3}{7} AB$

$A(2,3) \quad P(x,0)$

$x_1\, y_1 \qquad x_2\, y_2$

$$AP = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(x-2)^2+(0-3)^2}$$

$$= \sqrt{(x)^2+(2)^2-2(x)(2)+(-3)^2}$$

$$= \sqrt{x^2+4-4x+9}$$

$$AP = \sqrt{x^2-4x+13}$$

$A(2,3) \quad B(2,-4)$

$x_1\, y_1 \qquad x_2\, y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(2-2)^2+(-4-3)^2}$$

$$= \sqrt{(0)^2+(-7)^2}$$

$$= \sqrt{(-7)^2}$$

$$AB = 7$$

$$\boxed{AP = \frac{3}{7} AB}$$

$$\sqrt{x^2-4x+13} = \frac{3}{\cancel{7}}(\cancel{7})$$

$$\sqrt{x^2-4x+13} = 3$$

Squaring on both sides,

$$\left(\sqrt{x^2-4x+13}\right)^{\cancel{2}} = (3)^2$$

$$x^2-4x+13 = 9$$

$$x^2-4x+13-9=0$$

$$\boxed{x^2-4x+4=0}$$

$$+4$$
$$-2 \quad -2$$

$$(x-2)(x-2)=0$$

$$x-2=0 \quad \Big\| \quad x-2=0$$

$$\boxed{x=2} \quad \Big\| \quad \boxed{x=2}$$

$\Rightarrow P(x,0) \Rightarrow \boxed{P(2,0)}$

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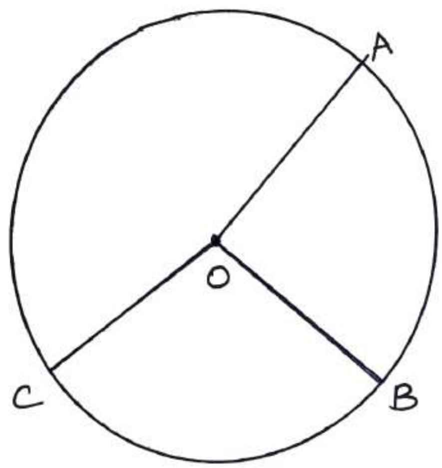
11) Show that the point (11,2) is the centre of the circle passing through the points (1,2) (3,-4) and (5,-6)

Centre 'O' $\Rightarrow$ O(11,2)

A(1,2) B(3,-4) C(5,-6)



O(11,2) A(1,2)
$x_1 y_1$ $\quad x_2 y_2$

$$OA = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(1-11)^2+(2-2)^2}$$

$$= \sqrt{(-10)^2+(0)^2}$$

$$= \sqrt{(-10)^2}$$

OA = 10 units ✓

$O(11,2)$ $B(3,-4)$
$x_1\ y_1$ $x_2\ y_2$

$$OB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(3-11)^2+(-4-2)^2}$$
$$= \sqrt{(-8)^2+(-6)^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100}$$
$$= \sqrt{(10)^{\cancel{2}}}$$

$OB = 10$ units ✓

$O(11,2)$ $C(5,-6)$
$x_1\ y_1$ $x_2\ y_2$

$$OC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(5-11)^2+(-6-2)^2}$$
$$= \sqrt{(-6)^2+(-8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$= \sqrt{(10)^{\cancel{2}}}$$

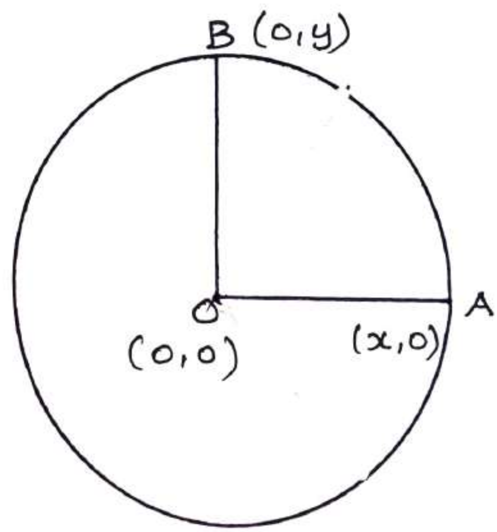
$OC = 10$ units ✓

$\Rightarrow$ $O(11,2)$ is the centre of the circle.

---

12) The radius of the circle with centre at origin is 30 units. Write the co-ordinates of the points where the circle Intersects the axes. Find the distance between any two such points.

Radius = 30 units



Centre $\Rightarrow$ $O(0,0)$ $\Leftarrow$ Origin

$O(0,0)$ $A(x,0)$
$x_1\ y_1$ $x_2\ y_2$

$OA = 30$ [Radius]

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = 30$$
$$\sqrt{(x-0)^2+(0-0)^2} = 30$$

$$\sqrt{(x)^2+(0)^2}=30$$

$$\sqrt{\cancel{(x)^2}}=30$$

$$\boxed{x=30} \Rightarrow A(30,0)$$

$O(0,0)$ $B(0,y)$
$x_1\ y_1$ $x_2\ y_2$

$OB=30$ [Radius]

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=30$$

$$\sqrt{(0-0)^2+(y-0)^2}=30$$

$$\sqrt{(0)^2+(y)^2}=30$$

$$\sqrt{\cancel{(y)^2}}=30$$

$$\boxed{y=30} \Rightarrow B(0,30)$$

Distance $\Rightarrow$ $A(30,0)$ $B(0,30)$
$x_1\ y_1$ $x_2\ y_2$

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(0-30)^2+(30-0)^2}$$

$$=\sqrt{(-30)^2+(30)^2}$$

$$=\sqrt{900+900}$$

$$=\sqrt{1800}$$

$$=\sqrt{2\times\underbrace{3\times 3}\times\underbrace{10\times 10}}$$

$$=3\times 10\sqrt{2}$$

$AB=30\sqrt{2}$ units

2	1800
3	900
3	300
10	100
	10

$\Rightarrow$ Distance between two points $AB=30\sqrt{2}$ units

EXERCISE 5.3

1) Find the midpoint of the line segment joining the points

(i) $(-2, 3)$ and $(-6, -5)$

$x_1\ y_1 \qquad x_2\ y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2-6}{2}, \frac{3-5}{2}\right)$$

$$= \left(\frac{-\cancel{8}^{4}}{\cancel{2}}, \frac{-\cancel{2}^{1}}{\cancel{2}}\right)$$

Midpoint $= (-4, -1)$

(ii) $(8, -2)$ and $(-8, 0)$

$x_1\ y_1 \qquad x_2\ y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{8-8}{2}, \frac{-2+0}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{-\cancel{2}^{1}}{\cancel{2}}\right)$$

Midpoint $= (0, -1)$

(iii) (a, b) and $(a+2b, 2a-b)$

$x_1\ y_1 \qquad x_2 \quad y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{a+a+2b}{2}, \frac{\cancel{b}+2a-\cancel{b}}{2}\right)$$

$$= \left(\frac{2a+2b}{2}, \frac{2a}{2}\right)$$

$$= \left(\frac{\cancel{2}(a+b)}{\cancel{2}}, \frac{\cancel{2}a}{\cancel{2}}\right)$$

Midpoint $= (a+b, a)$

(iv) $\left(\frac{1}{2}, -\frac{3}{7}\right)$ and $\left(\frac{3}{2}, \frac{-11}{7}\right)$

$x_1 \quad y_1 \qquad x_2 \quad y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{\frac{1}{2}+\frac{3}{2}}{2}, \frac{-\frac{3}{7}-\frac{11}{7}}{2}\right)$$

$$= \left(\frac{\frac{1+3}{2}}{2}, \frac{\frac{-3-11}{7}}{2}\right)$$

$$= \left(\frac{\cancel{4}^{1}}{\cancel{4}}, -\frac{\cancel{14}^{1}}{\cancel{14}}\right)$$

Midpoint $= (1, -1)$

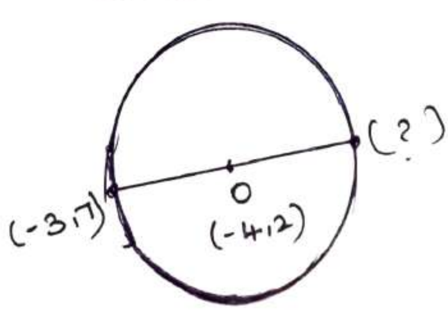
---

2) The centre of a circle is $(-4, 2)$. If one end of the diameter of the circle is $(-3, 7)$, then find the other end.

Centre $\Rightarrow O(-4, 2) \Leftarrow$ midpt

One end $\Rightarrow (-3, 7)$

Other end $\Rightarrow (\;?\;)$



Midpoint $= (-4, 2)$ $\quad\quad$ $(-3, 7)$ $(?)$

$x_1\, y_1 \quad x_2\, y_2$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (-4, 2)$$

$$\left(\frac{-3+x_2}{2}, \frac{7+y_2}{2}\right) = (-4, 2)$$

$\frac{-3+x_2}{2} = -4$	$\frac{7+y_2}{2} = 2$
$-3+x_2 = -8$	$7+y_2 = 4$
$x_2 = -8+3$	$y_2 = 4-7$
$\boxed{x_2 = -5}$	$\boxed{y_2 = -3}$

$\Rightarrow$ Other end is $(-5, -3)$

3) If the midpoint (x, y) of the line joining $(3, 4)$ and $(p, 7)$ lies on $2x+2y+1=0$, then what will be the value of p?

$(3, 4)$ $(p, 7)$

$x_1\, y_1 \quad x_2\, y_2$

$\boxed{\text{Given!}}$

$2x+2y+1=0$

Midpoint $= (x, y)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (x, y)$$

$$\left(\frac{3+p}{2}, \frac{4+7}{2}\right) = (x, y)$$

$\frac{3+p}{2} = x$	$\frac{4+7}{2} = y$
$\boxed{3+p = 2x} \rightarrow (1)$	$\boxed{11 = 2y} \rightarrow (2)$

Adding (1) & (2)

$3+p+11 = 2x+2y$

$$P + 14 = 2x + 2y$$

$$\Rightarrow \boxed{2x + 2y = P + 14}$$

Given: $\boxed{2x + 2y + 1 = 0}$

$$\cancel{2x} + \cancel{2y} = P + 14$$

$$\cancel{2x} + \cancel{2y} = -1$$

$$(-) \quad (-) \quad (+)$$

$$0 = P + 14 + 1$$

$$0 = P + 15$$

$$P + 15 = 0$$

$$P = -15$$

---

4) If the midpoint of the sides of the triangle are (2,4) (-2,3) and (5,2). Find the co-ordinates of the vertices of the triangle.

$$\frac{x_1 + x_2}{2} = 2$$

$$\boxed{x_1 + x_2 = 4} \rightarrow (1)$$

$$\frac{x_2 + x_3}{2} = -2$$

$$\boxed{x_2 + x_3 = -4} \rightarrow (3)$$

$$\frac{x_1 + x_3}{2} = 5$$

$$\boxed{x_1 + x_3 = 10} \rightarrow (5)$$

$$\frac{y_1 + y_2}{2} = 4$$

$$\boxed{y_1 + y_2 = 8} \rightarrow (2)$$

$$\frac{y_2 + y_3}{2} = 3$$

$$\boxed{y_2 + y_3 = 6} \rightarrow (4)$$

$$\frac{y_1 + y_3}{2} = 2$$

$$\boxed{y_1 + y_3 = 4} \rightarrow (6)$$

Adding ①, ③, ⑤

$2x_1+2x_2+2x_3=10$

$2(x_1+x_2+x_3)=10$

$x_1+x_2+x_3=\frac{10}{2}$

$\boxed{x_1+x_2+x_3=5}$

$4+x_3=5$

$x_3=5-4$

$x_3=1$

$\boxed{x_1+x_2+x_3=5}$

$x_1+(-4)=5$

$x_1=5+4$

$x_1=9$

$\boxed{x_1+x_2+x_3=5}$

$x_2+10=5$

$x_2=5-10$

$x_2=-5$

Adding ②, ④, ⑥

$2y_1+2y_2+2y_3=18$

$2(y_1+y_2+y_3)=18$

$y_1+y_2+y_3=\frac{18}{2}$

$\boxed{y_1+y_2+y_3=9}$

$8+y_3=9$

$y_3=9-8$

$y_3=1$

$\boxed{y_1+y_2+y_3=9}$

$y_1+6=9$

$y_1=9-6$

$y_1=3$

$\boxed{y_1+y_2+y_3=9}$

$y_2+4=9$

$y_2=9-4$

$y_2=5$

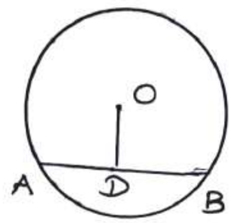
$\Rightarrow$ A (9, 3) B (-5, 5) C (1, 1)

5) O(0,0) is the centre of the circle whose one chord is AB, where the points A and B are (8,6) and (10,0) respectively. OD is the perpendicular

from the centre to the chord AB. Find the co-ordinates of the midpoint of OD.

$A(8,6)$ $B(10,0)$

$x_1\ y_1$ $\quad x_2\ y_2$



$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{8+10}{2}, \frac{6+0}{2}\right)$$

$$= \left(\frac{\cancel{18}^{9}}{\cancel{2}}, \frac{\cancel{6}^{3}}{\cancel{2}}\right)$$

$$\boxed{D = (9,3)}$$

$O(0,0)$ $D(9,3)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{Midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{0+9}{2}, \frac{0+3}{2}\right)$$

$$= \left(\frac{9}{2}, \frac{3}{2}\right)$$

$\Rightarrow$ midpoint of OD $= \left(\frac{9}{2}, \frac{3}{2}\right)$

---

6) The points $A(-5,4)$ $B(-1,-2)$ $C(5,2)$ are the vertices of an Isosceles right-angled triangle where the right angle is at B, Find the co-ordinates of D so that ABCD is a square.

$A(-5,4)$ $C(5,2)$

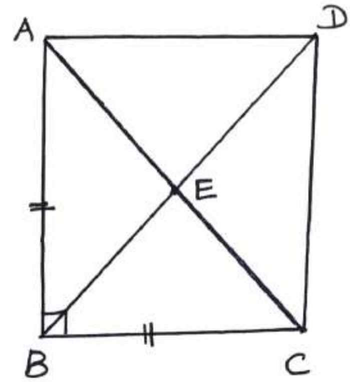
$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-5+5}{2}, \frac{4+2}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{\cancel{6}^{\,3}}{\cancel{2}}\right)$$

M.P of AC $= (0,3)$



$\Downarrow$

ABCD square

$B(-1,-2)$ $D(\ ?\ )$

$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{M.P. of } BD = \left(\frac{-1+x_2}{2}, \frac{-2+y_2}{2}\right)$$

Midpt of AC = Midpt of BD

$$(0,3) = \left(\frac{-1+x_2}{2}, \frac{-2+y_2}{2}\right)$$

$0 = \dfrac{-1+x_2}{2}$

$0 = -1+x_2$

$1 = x_2$

$x_2 = 1$

$3 = \dfrac{-2+y_2}{2}$

$6 = -2+y_2$

$6+2 = y_2$

$8 = y_2$

$y_2 = 8$

$\Rightarrow \boxed{D(1,8)}$

7) The points A(-3,6) B(0,7) C(1,9) are the midpoints of the sides DE, EF and FD of the triangle DEF. Show that the quadrilateral ABCD is a parallelogram.

A(-3,6) C(1,9)   B(0,7) D(?)

$x_1\ y_1 \quad x_2\ y_2 \qquad x_1\ y_1 \quad x_2\ y_2$

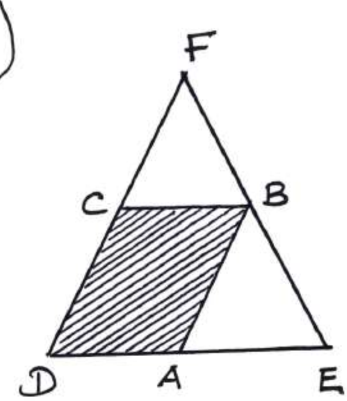
Midpoint of AC = Midpoint of BD

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{-3+1}{2}, \frac{6+9}{2}\right) = \left(\frac{0+x_2}{2}, \frac{7+y_2}{2}\right)$$

$$\left(\frac{-\cancel{2}^{1}}{\cancel{2}}, \frac{15}{2}\right) = \left(\frac{x_2}{2}, \frac{7+y_2}{2}\right)$$

$$\left(-1, \frac{15}{2}\right) = \left(\frac{x_2}{2}, \frac{7+y_2}{2}\right)$$



$-1 = \frac{x_2}{2}$

$-2 = x_2$

$x_2 = -2$

$\frac{15}{\cancel{2}} = \frac{7+y_2}{\cancel{2}}$

$15 = 7+y_2$

$15-7 = y_2$

$8 = y_2$

$y_2 = 8$

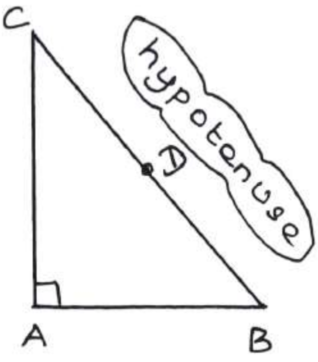
$\Rightarrow \boxed{D(-2,8)}$

---

8) A(-3,2) B(3,2) and C(-3,-2) are the vertices of the right triangle, right angled at A. Show that the mid-point of the hypotenuse is equidistant

from the vertices.

D ⇒ Midpt of BC



$B(3,2)\quad C(-3,-2)$

$x_1\ y_1\qquad x_2\ y_2$

$$\text{Midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{3-3}{2}, \frac{2-2}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{0}{2}\right)$$

$$D = (0,0)$$

$A(-3,2)\quad D(0,0)$

$x_1\ y_1\qquad x_2\ y_2$

$$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(0+3)^2+(0-2)^2}$$

$$= \sqrt{(3)^2+(-2)^2}$$

$$= \sqrt{9+4}$$

$$AD = \sqrt{13}\text{ units}$$

$B(3,2)\quad D(0,0)$

$x_1\ y_1\qquad x_2\ y_2$

$$BD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(0-3)^2+(0-2)^2}$$

$$= \sqrt{(-3)^2+(-2)^2}$$

$$= \sqrt{9+4}$$

$$BD = \sqrt{13}\text{ units}$$

$C(-3,-2)\quad D(0,0)$

$x_1\ y_1\qquad x_2\ y_2$

$$CD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(0+3)^2+(0+2)^2} = \sqrt{(3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{13}\text{ units}$$

⇒ The midpoint of the hypotenuse is equidistant from the vertices.

EXERCISE 5.4

1) Find the co-ordinates of the point which divides the line segment joining the points A (4, -3) and B (9, 7) in the ratio 3 : 2

$A(4,-3)\ B(9,7)$

$x_1\ y_1 \quad x_2\ y_2$

$\boxed{\begin{matrix} 3 : 2 \\ l \quad m \end{matrix}}$

$$P(x,y) = P\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$$

$$= P\left(\frac{3(9)+2(4)}{3+2}, \frac{3(7)+2(-3)}{3+2}\right)$$

$$= P\left(\frac{27+8}{5}, \frac{21-6}{5}\right)$$

$$= P\left(\frac{\cancel{35}^{7}}{\cancel{5}}, \frac{\cancel{15}^{3}}{\cancel{5}}\right)$$

$$= P(7,3)$$

---

2) In what ratio does the point P(2, -5) divide the line segment joining A (-3, 5) and B (4, -9)

$P(2,-5)\quad A(-3,5)\ B(4,-9)$ $\quad l:m?$

$x_1\ y_1 \quad x_2\ y_2$

$$P(x,y) = P\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$$

$$P(2,-5) = P\left(\frac{l(4)+m(-3)}{l+m}, \frac{l(-9)+m(5)}{l+m}\right)$$

$$P(2,-5) = P\left(\frac{4l-3m}{l+m}, \frac{-9l+5m}{l+m}\right)$$

$$2 = \frac{4l-3m}{l+m}$$

$$2(l+m) = 4l-3m$$

$$2l+2m = 4l-3m$$

$$2l-4l = -3m-2m$$

$$\cancel{-}2l = \cancel{-}5m$$

$$\frac{l}{m} = \frac{5}{2}$$

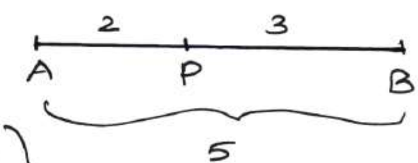
$\Rightarrow$ Ratio $\Rightarrow$ 5:2

---

3) Find the Co-ordinates of a Point P on the line segment joining A(1,2) and B(6,7) in such a way that $AP = \frac{2}{5}AB$

$A(1,2)$ $B(6,7)$
$x_1\ y_1$ $\quad x_2\ y_2$

$$\boxed{\begin{matrix} 2:3 \\ l\ \ m \end{matrix}}$$



$$P(x,y) = P\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$$

$$= P\left(\frac{2(6)+3(1)}{2+3}, \frac{2(7)+3(2)}{2+3}\right)$$

$$= P\left(\frac{12+3}{5}, \frac{14+6}{5}\right)$$

$$= P\left(\frac{\cancel{15}^{3}}{\cancel{5}}, \frac{\cancel{20}^{4}}{\cancel{5}}\right)$$

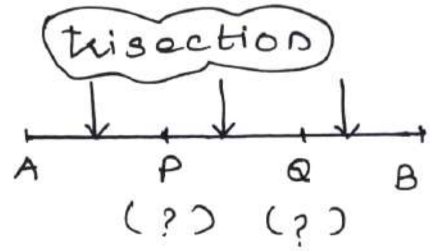
$$= P(3,4)$$

---

4) Find the co-ordinates of the points of trisection of line segment joining the points $A(-5,6)$ and $B(4,-3)$

$A(-5,6) \quad B(4,-3) \qquad \boxed{\begin{matrix} 1:2 \\ l:m \end{matrix}}$

$\quad x_1 \, y_1 \qquad\quad x_2 \, y_2$



$$P(x,y) = P\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$$

$$= P\left(\frac{1(4)+2(-5)}{1+2}, \frac{1(-3)+2(6)}{1+2}\right)$$

$$= P\left(\frac{4-10}{3}, \frac{-3+12}{3}\right)$$

$$= P\left(\frac{-\cancel{6}^{\,2}}{\cancel{3}}, \frac{\cancel{9}^{\,3}}{\cancel{3}}\right)$$

$$= P(-2,3)$$

$A(-5,6) \quad B(4,-3) \qquad \boxed{\begin{matrix} 2:1 \\ l \;\; m \end{matrix}}$

$\quad x_1 \, y_1 \qquad\quad x_2 \, y_2$

$$Q(x,y) = Q\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$$

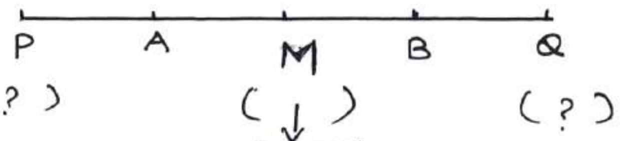
$$= Q\left(\frac{2(4)+1(-5)}{2+1}, \frac{2(-3)+1(6)}{2+1}\right)$$

$$= Q\left(\frac{8-5}{3}, \frac{-6+6}{3}\right)$$

$$= Q\left(\frac{\cancel{3}^{\,1}}{\cancel{3}}, \frac{0}{3}\right)$$

$$= Q(1,0)$$

5) The line segment joining A(6, 3) and B(-1, -4) is doubled in length by adding half of AB to each end. Find the co-ordinates of the new end point.



$\boxed{M.P}$

$A(6,3)$ $B(-1,-4)$

x_1, y_1 $x_2\ y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{6-1}{2}, \frac{3-4}{2}\right)$$

$$\boxed{M = \left(\frac{5}{2}, \frac{-1}{2}\right)}$$

$P(\ ?\ )$ $M\left(\frac{5}{2}, \frac{-1}{2}\right)$

x_1, y_1 $x_2\ y_2$

Midpoint $A = (6, 3)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (6,3)$$

$$\left(\frac{x_1+\frac{5}{2}}{2}, \frac{y_1-\frac{1}{2}}{2}\right) = (6,3)$$

$$\left(\frac{\frac{2x_1+5}{2}}{2}, \frac{\frac{2y_1-1}{2}}{2}\right) = (6,3)$$

$$\left(\frac{2x_1+5}{4}, \frac{2y_1-1}{4}\right) = (6,3)$$

$$\frac{2x_1+5}{4} = 6 \qquad \frac{2y_1-1}{4} = 3$$

$$2x_1+5 = 24 \qquad 2y_1-1 = 12$$

$$2x_1 = 24-5 \qquad 2y_1 = 12+1$$

$$2x_1 = 19 \qquad 2y_1 = 13$$

$$x_1 = \frac{19}{2} \qquad y_1 = \frac{13}{2}$$

$$\Rightarrow \boxed{P\left(\frac{19}{2}, \frac{13}{2}\right)}$$

$$M\left(\frac{5}{2}, -\frac{1}{2}\right) \qquad Q(\;?\;)$$

$$x_1 \; y_1 \qquad\qquad x_2 \; y_2$$

Midpoint $B = (-1, -4)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (-1,-4)$$

$$\left(\frac{\frac{5}{2}+x_2}{2}, \frac{-\frac{1}{2}+y_2}{2}\right) = (-1,-4)$$

$$\left(\frac{\frac{5+2x_2}{2}}{2}, \frac{\frac{-1+2y_2}{2}}{2}\right) = (-1,-4)$$

$$\left(\frac{5+2x_2}{4}, \frac{-1+2y_2}{4}\right) = (-1,-4)$$

$$\frac{5+2x_2}{4} = -1 \qquad \frac{-1+2y_2}{4} = -4$$

$$5+2x_2 = -4 \qquad -1+2y_2 = -16$$

$$2x_2 = -4-5 \qquad 2y_2 = -16+1$$

$2x_2 = -9$

$x_2 = \frac{-9}{2}$

$2y_2 = -15$

$y_2 = \frac{-15}{2}$

$\Rightarrow \boxed{Q\left(\frac{-9}{2}, \frac{-15}{2}\right)}$

---

6) Using section formula, show that the points $A(7,-5)$ $B(9,-3)$ and $C(13,1)$ are Collinear.

$A(7,-5)$ $B(9,-3)$

$x_1\ y_1 \quad x_2\ y_2$

$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(9-7)^2 + (-3+5)^2}$

$= \sqrt{(2)^2 + (2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= \sqrt{2 \times 2 \times 2}$

$AB = 2\sqrt{2}$ units

$B(9,-3)$ $C(13,1)$

$x_1\ y_1 \quad x_2\ y_2$

$BC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(13-9)^2 + (1+3)^2}$

$= \sqrt{(4)^2 + (4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 2}$

$BC = 4\sqrt{2}$ units

$A(7,-5)$ $C(13,1)$

$x_1\ y_1 \quad x_2\ y_2$

$AC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(13-7)^2 + (1+5)^2}$

$= \sqrt{(6)^2 + (6)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$= \sqrt{\underbrace{2\times 2}\times 2\times \underbrace{3\times 3}}$

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$AC = 6\sqrt{2}$ units

$\Rightarrow AC = AB + BC$

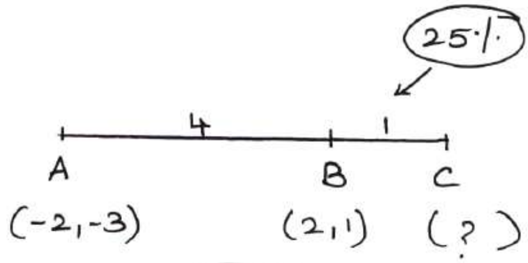
$6\sqrt{2} = 2\sqrt{2} + 4\sqrt{2}$

$\boxed{6\sqrt{2} = 6\sqrt{2}}$

$\therefore$ It is Collinear

---

7) A line segment AB is increased along its length by 25% by producing it to 'c' on the side B. If A and B have the co-ordinates (-2,-3) (2,1) respectively, then find the co-ordinates of 'c'?



4 : 1
l m

$BC = 25\%\ AB$

$BC = \frac{\cancel{25}^{1}}{\cancel{100}_{4}} AB$

$BC = \frac{1}{4} AB$

$$\boxed{\frac{BC}{AB} = \frac{1}{4}}$$

A(-2, -3)   C(?)   B(2,1) $\boxed{\begin{array}{c} 4:1 \\ l:m \end{array}}$

$x_1\ y_1$ $x_2\ y_2$

$$P(x,y) = P\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$$

$$P(2,1) = P\left(\frac{4(x_2)+1(-2)}{4+1}, \frac{4(y_2)+1(-3)}{4+1}\right)$$

$$P(2,1) = P\left(\frac{4x_2-2}{5}, \frac{4y_2-3}{5}\right)$$

$$2 = \frac{4x_2-2}{5}$$

$$10 = 4x_2-2$$

$$10+2 = 4x_2$$

$$12 = 4x_2$$

$$4x_2 = 12$$

$$x_2 = \frac{\cancel{12}^{3}}{\cancel{4}}$$

$$x_2 = 3$$

$$1 = \frac{4y_2-3}{5}$$

$$5 = 4y_2-3$$

$$5+3 = 4y_2$$

$$8 = 4y_2$$

$$4y_2 = 8$$

$$y_2 = \frac{\cancel{8}^{2}}{\cancel{4}}$$

$$y_2 = 2$$

$$\Rightarrow \boxed{C(3,2)}$$

EXERCISE 5.5

1) Find the centroid of the triangle whose vertices are

(i) $(2,-4)$ $(-3,-7)$ $(7,2)$

$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$

$$\text{Centroid} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{2-3+7}{3}, \frac{-4-7+2}{3}\right)$$

$$= \left(\frac{9-3}{3}, \frac{-11+2}{3}\right)$$

$$= \left(\frac{\cancel{6}^{2}}{\cancel{3}}, -\frac{\cancel{9}^{3}}{\cancel{3}}\right)$$

Centroid, $G = (2,-3)$

(ii) $(-5,-5)$ $(1,-4)$ $(-4,-2)$

$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$

$$\text{Centroid} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{-5+1-4}{3}, \frac{-5-4-2}{3}\right)$$

$$= \left(\frac{-9+1}{3}, \frac{-11}{3}\right)$$

Centroid, $G = \left(\frac{-8}{3}, \frac{-11}{3}\right)$

2) If the centroid of a triangle is at $(4,-2)$ and two of its vertices are

(3, -2) and (5, 2) then find the third vertex of the triangle.

$(3,-2)$ $(5,2)$ $(\,?\,)$ → Third vertex
$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$

$$\boxed{\text{Centroid} = (4, -2)}$$

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = (4,-2)$$

$$\left(\frac{3+5+x_3}{3}, \frac{-2+2+y_3}{3}\right) = (4,-2)$$

$$\left(\frac{8+x_3}{3}, \frac{0+y_3}{3}\right) = (4,-2)$$

$$\frac{8+x_3}{3} = 4$$

$$8+x_3 = 12$$

$$x_3 = 12-8$$

$$x_3 = 4$$

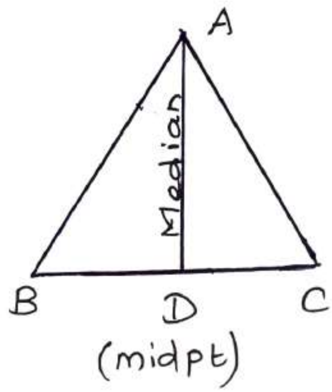
$$\frac{y_3}{3} = -2$$

$$y_3 = -6$$

⇒ Third vertex is $(4,-6)$

---

3) Find the length of median through A of a triangle whose vertices are A (-1, 3) B (1, -1) and C (5, 1)



$B(1,-1)$ $C(5,1)$
$x_1\ y_1$ $\quad x_2\ y_2$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{1+5}{2}, \frac{-1+1}{2}\right)$$

$$= \left(\frac{\cancel{6}^{3}}{\cancel{2}}, \frac{0}{3}\right)$$

$$\boxed{D = (3,0)}$$

$A(-1,3)$ $D(3,0)$

$x_1\ y_1$ $\quad x_2\ y_2$

$$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(3+1)^2+(0-3)^2}$$

$$= \sqrt{(4)^2+(-3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25} = \sqrt{(5)^{\cancel{2}}}$$

$= 5$ units

$\Rightarrow$ The length of median $AD = 5$ units

---

4) The vertices of a triangle are $(1,2)$ $(h,-3)$ and $(-4,k)$. If the centroid of the triangle is at the point $(5,-1)$ then find the value of $\sqrt{(h+k)^2+(h+3k)^2}$

$(1,2)$ $(h,-3)$ $(-4,k)$

$x_1\ y_1$ $\quad x_2\ y_2$ $\quad x_3\ y_3$

Centroid $= (5,-1)$

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = (5,-1)$$

$$\left(\frac{1+h-4}{3}, \frac{2-3+k}{3}\right) = (5,-1)$$

$$\left(\frac{h-3}{3}, \frac{-1+k}{3}\right) = (5,-1)$$

$\frac{h-3}{3} = 5$

$h-3 = 15$

$h = 15+3$

$\boxed{h=18}$

$\frac{-1+k}{3} = -1$

$-1+k = -3$

$k = -3+1$

$\boxed{k=-2}$

find the value of:

$$\sqrt{(h+k)^2+(h+3k)^2}$$

$$= \sqrt{(18-2)^2+(18+3(-2))^2}$$

$$= \sqrt{(16)^2+(18-6)^2}$$

$$= \sqrt{256+144}$$

$$= \sqrt{400}$$

$$= \sqrt{(20)^{\cancel{2}}}$$

$= 20$ units

---

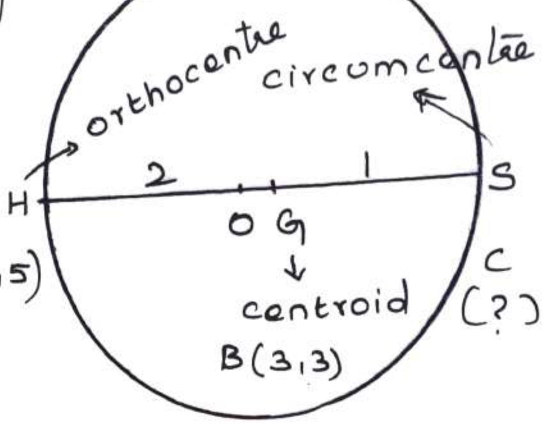
5) Orthocentre and Centroid of a triangle are $A(-3,5)$ and $B(3,3)$ respectively. If 'c' is the circumcentre and AC is the diameter of the circle, then find the radius of the circle.

$A(-3,5)$ $C(?)$
$x_1\ y_1$ $x_2\ y_2$

$$P(x,y) = P\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$$

$$P(3,3) = P\left(\frac{2(x_2)+1(-3)}{2+1}, \frac{2(y_2)+1(5)}{2+1}\right)$$

A(-3,5)



$$\boxed{\begin{matrix}2:1\\ l\ \ m\end{matrix}}$$

$$P(3,3) = P\left(\frac{2x_2-3}{3}, \frac{2y_2+5}{3}\right)$$

$$3 = \frac{2x_2-3}{3}$$

$$9 = 2x_2-3$$

$$9+3 = 2x_2$$

$$12 = 2x_2$$

$$2x_2 = 12$$

$$x_2 = \frac{\cancel{12}\ 6}{\cancel{2}}$$

$$x_2 = 6$$

$$3 = \frac{2y_2+5}{3}$$

$$9 = 2y_2+5$$

$$9-5 = 2y_2$$

$$4 = 2y_2$$

$$2y_2 = 4$$

$$y_2 = \frac{\cancel{4}\ 2}{\cancel{2}}$$

$$y_2 = 2$$

$\therefore$ circumcentre, S $\Rightarrow$ $\boxed{C(6,2)}$

AC → Diameter

A(-3, 5) C(6, 2)
x_1 y_1 x_2 y_2

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(6+3)^2+(2-5)^2}$$

$$= \sqrt{(9)^2+(-3)^2}$$

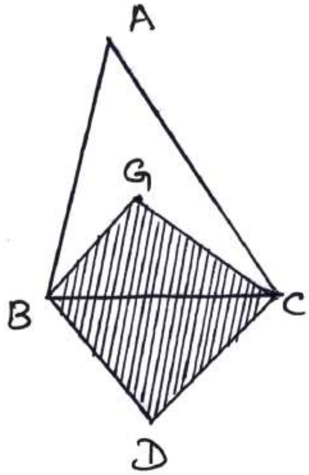
$$= \sqrt{81+9} = \sqrt{90} = \sqrt{2\times3\times3\times5} = 3\sqrt{10} \text{ units}$$

$\therefore$ Radius of the circle $= \frac{3\sqrt{10}}{2}$ units

6) ABC is a triangle whose vertices are $A(3,4)$ $B(-2,-1)$ $C(5,3)$. If G is the centroid and BDCG is a parallelogram then find the coordinates of the vertex D.

$A(3,4)$ $B(-2,-1)$ $C(5,3)$

$x_1\ y_1 \qquad x_2\ y_2 \qquad x_3\ y_3$



$$\text{Centroid, } G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{3-2+5}{3}, \frac{4-1+3}{3}\right)$$

$$= \left(\frac{8-2}{3}, \frac{7-1}{3}\right)$$

$$= \left(\frac{\cancel{6}^{2}}{\cancel{3}}, \frac{\cancel{6}^{2}}{\cancel{3}}\right)$$

$$\boxed{G = (2,2)}$$

BDCG is a parallelogram

$\Downarrow$

(Diagonals bisect each other)

$B(-2,-1)$ $C(5,3)$ $\qquad$ $D(\ ?\ )$ $G(2,2)$

$x_1\ y_1 \quad x_2\ y_2 \qquad x_1\ y_1 \quad x_2\ y_2$

Midpoint of BC = Midpoint of DG

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{-2+5}{2}, \frac{-1+3}{2}\right) = \left(\frac{x_1+2}{2}, \frac{y_1+2}{2}\right)$$

$$\left(\frac{3}{2}, \frac{\cancel{2}^{1}}{\cancel{2}}\right) = \left(\frac{x_1+2}{2}, \frac{y_1+2}{2}\right)$$

$$\left(\frac{3}{2}, 1\right) = \left(\frac{x_1+2}{2}, \frac{y_1+2}{2}\right)$$

$$\frac{3}{\cancel{2}} = \frac{x_1+2}{\cancel{2}}$$

$$3 = x_1+2$$

$$3-2 = x_1$$

$$1 = x_1$$

$$x_1 = 1$$

$$1 = \frac{y_1+2}{2}$$

$$2 = y_1+2$$

$$2-2 = y_1$$

$$0 = y_1$$

$$y_1 = 0$$

$$\Rightarrow D(1,0)$$

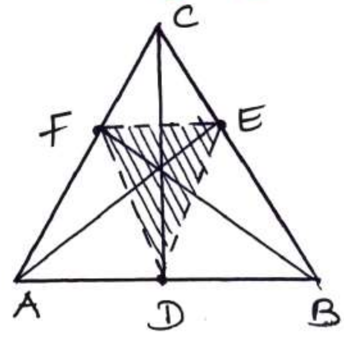
---

7) If $\left(\frac{3}{2}, 5\right)$ $\left(7, -\frac{9}{2}\right)$ and $\left(\frac{13}{2}, -\frac{13}{2}\right)$ are midpoints of the sides of a triangle, then find the centroid of the triangle.

[The centroid of the triangle obtained by joining the mid-points of the sides of a triangle is the same as the centroid of the original triangle].

$\left(\frac{3}{2}, 5\right)$ $\left(7, -\frac{9}{2}\right)$ $\left(\frac{13}{2}, -\frac{13}{2}\right)$

$x_1 \; y_1 \quad x_2 \; y_2 \quad x_3 \; y_3$



$$\text{Centroid} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{\frac{3}{2}+7+\frac{13}{2}}{3}, \frac{5-\frac{9}{2}-\frac{13}{2}}{3}\right)$$

$$= \left(\frac{\frac{3+14+13}{2}}{3}, \frac{\frac{10-9-13}{2}}{3}\right)$$

$$= \left(\frac{30}{6}, \frac{10-22}{6} \right)$$

$$= \left(\frac{\cancel{30}^{5}}{\cancel{6}}, \frac{-1\cancel{2}^{2}}{\cancel{6}} \right)$$

$$= (5, -2)$$

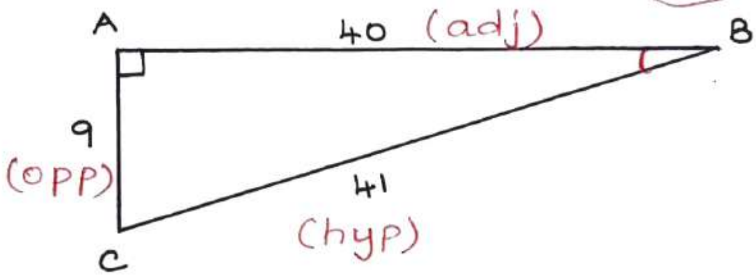
$\Rightarrow$ Centroid $G = (5, -2)$

CHAPTER-6

TRIGONOMETRY

Ex 6.1

1) From the given figure, find all the trigonometric ratios of angle B



$\sin B = \frac{opp}{hyp} = \frac{9}{41}$

$\cos B = \frac{adj}{hyp} = \frac{40}{41}$

$\tan B = \frac{opp}{adj} = \frac{9}{40}$

$\text{Cosec} B = \frac{hyp}{opp} = \frac{41}{9}$

$\sec B = \frac{hyp}{adj} = \frac{41}{40}$

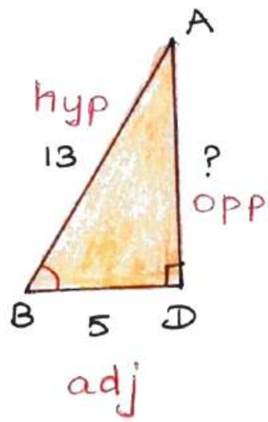
$\cot B = \frac{adj}{opp} = \frac{40}{9}$

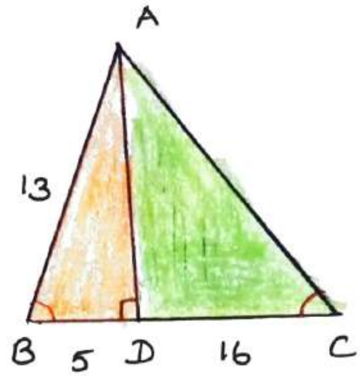
---

2) From the given figure, find the values of (i) $\sin B$ (ii) $\sec B$ (iii) $\cot B$ (iv) $\cos C$ (v) $\tan C$ (vi) $\text{Cosec} C$

In Δ ABD,

By Pythagoras Theorem,





$$AB^2 = AD^2 + BD^2$$

$$13^2 = AD^2 + 5^2$$

$$169 = AD^2 + 25$$

$$AD^2 + 25 = 169$$

$$AD^2 = 169 - 25$$

$$AD^2 = 144$$

$$AD^{\cancel{2}} = 12^{\cancel{2}}$$

$$\boxed{AD = 12} \Leftarrow (\text{opp})$$

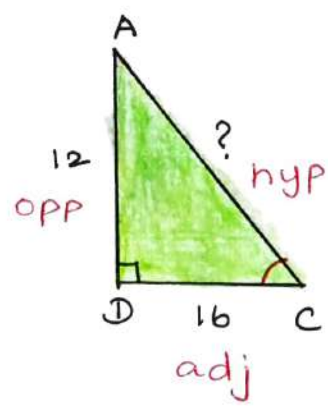
(i) $\sin B = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{12}{13}$

(ii) $\sec B = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{13}{5}$

(iii) $\cot B = \dfrac{\text{adj}}{\text{opp}} = \dfrac{5}{12}$

In Δ ADC,

By Pythagoras Theorem,



$$AC^2 = AD^2 + DC^2$$

$$AC^2 = 12^2 + 16^2$$

$$AC^2 = 144 + 256$$

$$AC^2 = 400$$

$$AC^{\cancel{2}} = 20^{\cancel{2}}$$

$$\boxed{AC = 20} \Leftarrow (\text{hyp})$$

(iv) $\cos C = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{16}{20}$

(v) $\tan C = \dfrac{\text{opp}}{\text{adj}} = \dfrac{12}{16}$

(vi) $\operatorname{cosec} C = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{20}{12}$

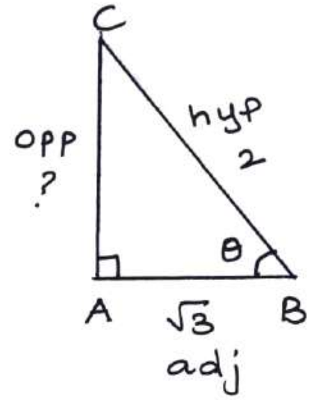
---

3) If $2\cos\theta = \sqrt{3}$, then find all the trigonometric ratios of angle θ

Given:

$$2\cos\theta = \sqrt{3}$$

$$\boxed{\cos\theta = \frac{\sqrt{3}}{2}} \quad \frac{\text{adj}}{\text{hyp}}$$



By Pythagoras Theorem,

$$BC^2 = AB^2 + AC^2$$

$$2^2 = (\sqrt{3})^2 + AC^2$$

$$4 = 3 + AC^2$$

$$3 + AC^2 = 4$$

$$AC^2 = 4 - 3$$

$$AC^2 = 1$$

$$AC^{\cancel{2}} = 1^{\cancel{2}}$$

$$\boxed{AC = 1} \Leftarrow (\text{opp})$$

TRIGONOMETRIC RATIOS:

$$\sin\theta = \frac{opp}{hyp} = \frac{1}{2}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{opp}{adj} = \frac{1}{\sqrt{3}}$$

$$\operatorname{Cosec}\theta = \frac{hyp}{opp} = \frac{2}{1}$$

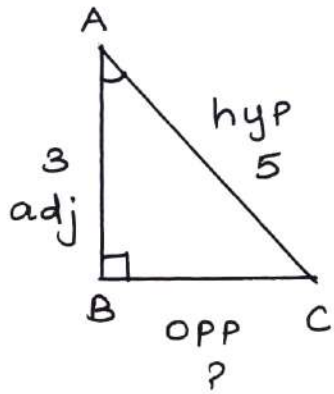
$$\operatorname{Sec}\theta = \frac{hyp}{adj} = \frac{\sqrt{3}}{2}$$

$$\operatorname{Cot}\theta = \frac{adj}{opp} = \frac{\sqrt{3}}{1}$$

---

4) If $\cos A = \frac{3}{5}$, then find the value of

$$\frac{\sin A - \cos A}{2\tan A}$$



Given

$$\boxed{\cos A = \frac{3}{5}} \quad \boxed{\frac{adj}{hyp}}$$

By Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$

$$5^2 = 3^2 + BC^2$$

$$25 = 9 + BC^2$$

$$9 + BC^2 = 25$$

$$BC^2 = 25 - 9$$

$$BC^2 = 16$$

$$BC^2 = 4^2$$

$$\boxed{BC = 4} \Leftarrow (\text{opp})$$

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$

$$\Rightarrow \frac{\sin A - \cos A}{2\tan A}$$

$$= \frac{\frac{4}{5} - \frac{3}{5}}{2\left(\frac{4}{3}\right)}$$

$$= \frac{\frac{4-3}{5}}{\frac{8}{3}}$$

$$= \frac{\frac{1}{5}}{\frac{8}{3}}$$

$$= \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$$

$$\Rightarrow \boxed{\frac{\sin A - \cos A}{2\tan A} = \frac{3}{40}}$$

---

5. If $\cos A = \frac{2x}{1+x^2}$, then find the value of $\sin A$ and $\tan A$ in terms of x.

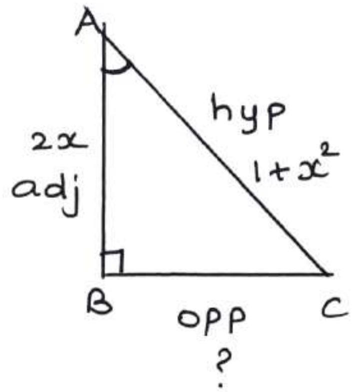
$\boxed{5}$

Given:

$$\cos A = \frac{2x}{1+x^2} \rightarrow \boxed{\frac{adj}{hyp}}$$

By Pythagoras Theorem,

$$(AC)^2 = AB^2 + BC^2$$



$$(1+x^2)^2 = (2x)^2 + BC^2$$

$$(1)^2 + (x^2)^2 + 2(1)(x^2) = 4x^2 + BC^2$$

$$1 + x^4 + 2x^2 = 4x^2 + BC^2$$

$$1 + x^4 + 2x^2 - 4x^2 = BC^2$$

$$1 + x^4 - 2x^2 = BC^2$$

$$(1-x^2)^{\cancel{2}} = BC^{\cancel{2}}$$

$$1 - x^2 = BC$$

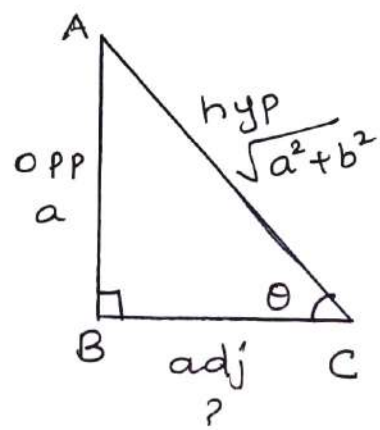
$$\boxed{BC = 1 - x^2} \Leftarrow (opp)$$

$$\Rightarrow \sin A = \frac{opp}{hyp} = \frac{1-x^2}{1+x^2}$$

$$\Rightarrow \tan A = \frac{opp}{adj} = \frac{1-x^2}{2x}$$

---

6) If $\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$, then show that $b\sin\theta = a\cos\theta$



Given

$$\boxed{\sin\theta = \frac{a}{\sqrt{a^2+b^2}}} \rightarrow \boxed{\frac{opp}{hyp}}$$

By Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$

$$\left(\sqrt{a^2+b^2}\right)^{\cancel{2}} = a^2 + BC^2$$

$$\cancel{a^2} + b^2 = \cancel{a^2} + BC^2$$

$$b^{\cancel{2}} = BC^{\cancel{2}}$$

$$b = BC$$

$$\boxed{BC = b} \Leftarrow (\text{adj})$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{b}{\sqrt{a^2+b^2}}$$

Show that,

$$b\sin\theta = a\cos\theta$$

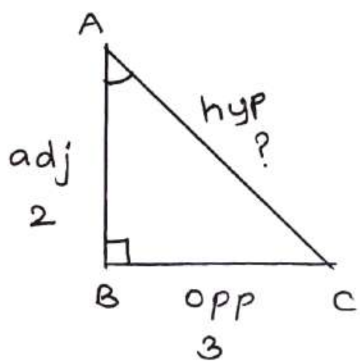
$$b\left(\frac{a}{\sqrt{a^2+b^2}}\right) = a\left(\frac{b}{\sqrt{a^2+b^2}}\right)$$

$$\frac{ab}{\sqrt{a^2+b^2}} = \frac{ab}{\sqrt{a^2+b^2}}$$

Hence Verified.

---

7) If $3\cot A = 2$, then find the value of $\dfrac{4\sin A - 3\cos A}{2\sin A + 3\cos A}$



Given

$$3\cot A = 2$$

$$\boxed{\cot A = \frac{2}{3}} \quad \boxed{\frac{\text{adj}}{\text{opp}}}$$

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 2^2 + 3^2$$

$$AC^2 = 4 + 9$$

$$AC^2 = 13$$

$$\boxed{AC = \sqrt{13}} \Leftarrow \text{hyp}$$

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{13}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{13}}$$

$$\Rightarrow \boxed{\frac{4\sin A - 3\cos A}{2\sin A + 3\cos A}}$$

$$= \frac{4\left(\frac{3}{\sqrt{13}}\right) - 3\left(\frac{2}{\sqrt{13}}\right)}{2\left(\frac{3}{\sqrt{13}}\right) + 3\left(\frac{2}{\sqrt{13}}\right)}$$

$$= \frac{\frac{12}{\sqrt{13}} - \frac{6}{\sqrt{13}}}{\frac{6}{\sqrt{13}} + \frac{6}{\sqrt{13}}} = \frac{\frac{12-6}{\cancel{\sqrt{13}}}}{\frac{6+6}{\cancel{\sqrt{13}}}} = \frac{\cancel{6}^{1}}{\cancel{12}_{2}} = \boxed{\frac{1}{2}}$$

---

8) If $\cos\theta : \sin\theta = 1:2$, then find the value of $\frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$.

Given

$$\cos\theta : \sin\theta = 1:2$$

$$\boxed{\frac{\cos\theta}{\sin\theta} = \frac{1}{2}}$$

To find:

$$\frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$$

$$= \frac{8(1) - 2(2)}{4(1) + 2(2)}$$

$$= \frac{8-4}{4+4}$$

$$= \frac{\cancel{4}^{1}}{\cancel{8}_{2}}$$

$$\Rightarrow \boxed{\frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta} = \frac{1}{2}}$$

9) From the given figure, prove that $\theta + \phi = 90°$. Also prove that there are two right angled triangles. Find $\sin\alpha$, $\cos\beta$ and $\tan\phi$.

(i) To prove: $\boxed{\theta + \phi = 90°}$

By pythagoras Theorem,

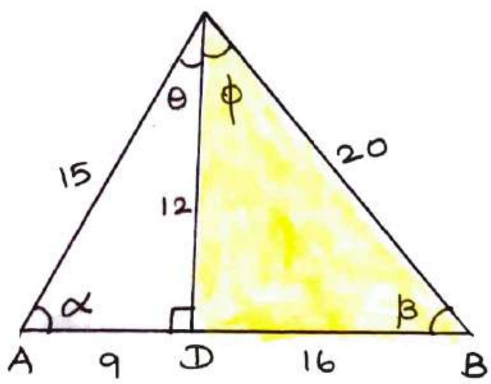
In ΔABC,

$AB^2 = AC^2 + BC^2$

$25^2 = 15^2 + 20^2$

$625 = 225 + 400$

$\boxed{625 = 625}$



$AB = 9 + 16 = 25$

$\therefore \theta + \phi = 90°$

$\Rightarrow \angle C = 90°$

(ii) To prove:

There are two right angled triangle.

In Δ ADC,

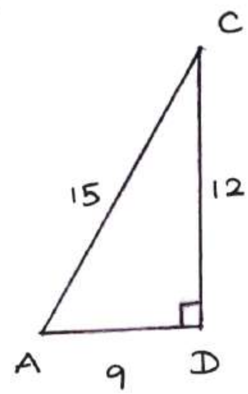
By pythagoras Theorem,

$$AC^2 = CD^2 + AD^2$$

$$15^2 = 12^2 + 9^2$$

$$225 = 144 + 81$$

$$\boxed{225 = 225}$$

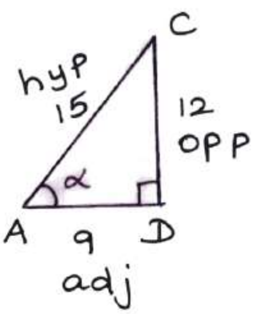


$\Rightarrow \angle D = 90^\circ$

$\Rightarrow$ There are two right angled triangle.

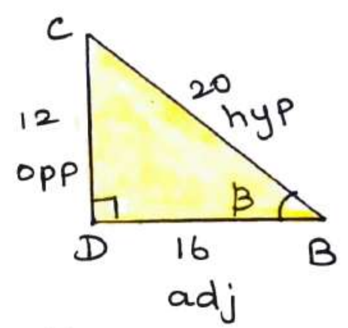
(i) $\boxed{\sin\alpha}$

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{12}{15}}$$



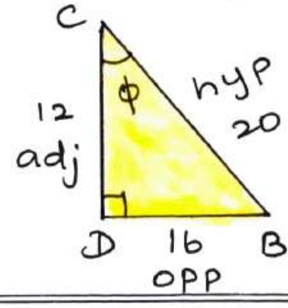
(ii) $\boxed{\cos\beta}$

$$\cos\beta = \frac{\text{adj}}{\text{hyp}} = \boxed{\frac{16}{20}}$$



(iii) $\boxed{\tan\phi}$

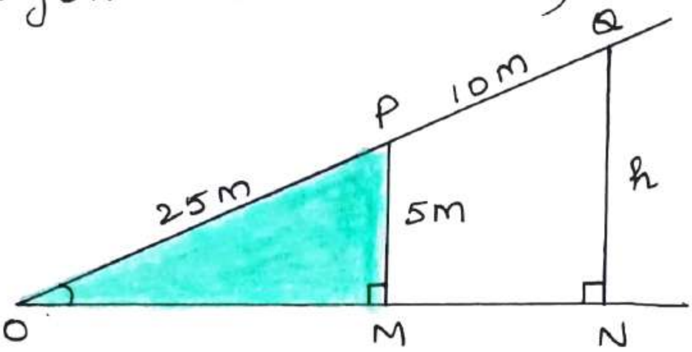
$$\tan\phi = \frac{\text{opp}}{\text{adj}} = \boxed{\frac{16}{12}}$$



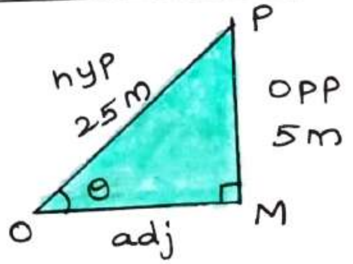
---

10) A boy standing at a point O finds his kite flying at a point P with distance OP = 25 m. It is at a height of 5 m from the ground. When the

thread is extended by 10m from P, it reaches a point Q. What will be the height QN of the kite from the ground? (use trigonometric ratios)



In Δ OPM

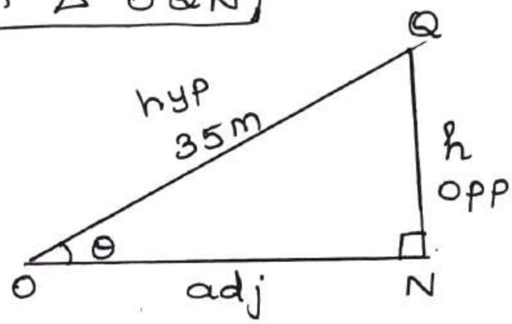


$\sin\theta = \dfrac{opp}{hyp}$

$\sin\theta = \dfrac{\cancel{5}\,1}{\cancel{25}\,5}$

$\boxed{\sin\theta = \dfrac{1}{5}} \rightarrow (1)$

In Δ OQN



$\sin\theta = \dfrac{opp}{hyp}$

$\boxed{\sin\theta = \dfrac{h}{35}} \rightarrow (2)$

From (1) & (2)

$\dfrac{1}{\cancel{5}_1} = \dfrac{h}{\cancel{35}_7}$

$7 = h$

$h = 7m$

EXERCISE 6.2

1) Verify the following equalities:

(i) $\sin^2 60^\circ + \cos^2 60^\circ = 1$

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$\frac{3}{4} + \frac{1}{4} = 1$$

$$\frac{3+1}{4} = 1$$

$$\frac{\cancel{4}^{1}}{\cancel{4}} = 1$$

$$1 = 1$$

Hence Verified.

(ii) $\cos 90^\circ = 1 - 2\sin^2 45^\circ = 2\cos^2 45^\circ - 1$

$$0 = 1 - 2\left(\frac{1}{\sqrt{2}}\right)^2 = 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1$$

$$0 = 1 - \cancel{2}\left(\frac{1}{\cancel{2}}\right) = \cancel{2}\left(\frac{1}{\cancel{2}}\right) - 1$$

$$0 = 1 - 1 = 1 - 1$$

$$\boxed{0 = 0 = 0}$$

Hence Verified.

(iii) $1 + \tan^2 30^\circ = \sec^2 30^\circ$

$$1 + \left(\frac{1}{\sqrt{3}}\right)^2 = \left(\frac{2}{\sqrt{3}}\right)^2$$

$$1 + \frac{1}{3} = \frac{4}{3}$$

$$\frac{3+1}{3} = \frac{4}{3}$$

$$\boxed{\frac{4}{3} = \frac{4}{3}}$$

Hence Verified.

(iv) $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \sin 90^\circ$

$$\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) = 1$$

$$\frac{1}{4} + \frac{3}{4} = 1$$

$$\frac{1+3}{4} = 1$$

$$\frac{\cancel{4}^{1}}{\cancel{4}} = 1$$

$$1 = 1$$

Hence Verified.

2) Find the value of the following:

(i) $\dfrac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \dfrac{\sec 60^\circ}{\cot 45^\circ} - \dfrac{5 \sin 90^\circ}{2 \cos 0^\circ}$

$$= \frac{1}{2} + \frac{2}{1} - \frac{5(1)}{2(1)}$$

$$= \frac{1}{2} + \frac{2\times2}{1\times2} - \frac{5}{2}$$

$$= \frac{1+4-5}{2} = \frac{5-5}{2} = \frac{0}{2} = \boxed{0}$$

(ii) $(\sin 90^\circ + \cos 60^\circ + \cos 45^\circ) \times (\sin 30^\circ + \cos 0^\circ - \cos 45^\circ)$

$$= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2} + 1 - \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{2+1}{2} + \frac{1}{\sqrt{2}}\right) \times \left(\frac{1+2}{2} - \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right) \times \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$$

$(a+b) \times (a-b)$

$$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \qquad \left[(a+b)(a-b) = a^2 - b^2\right]$$

$$= \frac{9}{4} - \frac{1 \times 2}{2 \times 2}$$

$$= \frac{9-2}{4} = \boxed{\frac{7}{4}}$$

(iii) $\sin^2 30^\circ - 2\cos^3 60^\circ + 3\tan^4 45^\circ$

$$= \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + 3(1)^4$$

$$= \frac{1}{4} - \cancel{2}\left(\frac{1}{\cancel{8}_4}\right) + 3(1)$$

$$= \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} + 3$$

$$= \boxed{3}$$

---

3) Verify $\cos 3A = 4\cos^3 A - 3\cos A$ when $A = 30^\circ$

Given $\Rightarrow$ $\boxed{A = 30^\circ}$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\cos 3(30^\circ) = 4\cos^3 30^\circ - 3\cos 30^\circ$$

$$\cos 90^\circ = 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$$

$$0 = \cancel{4}\left(\frac{3\sqrt{3}}{\cancel{8}_2}\right) - \frac{3\sqrt{3}}{2}$$

$$0 = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$\boxed{\begin{aligned} &\underbrace{\sqrt{3} \times \sqrt{3}} \times \sqrt{3} \\ &= 3 \times \sqrt{3} \\ &= 3\sqrt{3} \end{aligned}}$$

$\boxed{0 = 0}$

Hence Verified.

---

4) Find the value of $8 \sin 2x \cos 4x \sin 6x$, when $x = 15^\circ$

Given $\Rightarrow$ $\boxed{x = 15^\circ}$

$8 \sin 2x \cos 4x \sin 6x$

$= 8 \sin 2(15^\circ) \cos 4(15^\circ) \sin 6(15^\circ)$

$= 8 \sin 30^\circ \cos 60^\circ \sin 90^\circ$

$= \overset{4}{\cancel{8}} \times \frac{1}{\cancel{2}} \times \frac{1}{2} \times 1$

$= \overset{2}{\cancel{4}} \times \frac{1}{\cancel{2}} \times 1$

$= 2 \times 1$

$= \boxed{2}$

EXERCISE 6.3

I Find the value of the following:

(i) $\left(\frac{\cos 47^\circ}{\sin 43^\circ}\right)^2 + \left(\frac{\sin 72^\circ}{\cos 18^\circ}\right)^2 - 2\cos^2 45^\circ$

$= \left[\frac{\cos(90^\circ - 43^\circ)}{\sin 43^\circ}\right]^2 + \left[\frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ}\right]^2 - 2\left(\frac{1}{\sqrt{2}}\right)^2$

$= \left[\frac{\cancel{\sin 43^\circ}}{\cancel{\sin 43^\circ}}\right]^2 + \left[\frac{\cancel{\cos 18^\circ}}{\cancel{\cos 18^\circ}}\right]^2 - \cancel{2}\left(\frac{1}{\cancel{2}}\right)$

$= (1)^2 + (1)^2 - 1$

$= 1 + \cancel{1} - \cancel{1}$

$= \boxed{1}$

$\cos(90^\circ - \theta) = \sin\theta$
$\sin(90^\circ - \theta) = \cos\theta$

(ii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} + \frac{\cos\theta}{\sin(90^\circ - \theta)} - 8\cos^2 60^\circ$

$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} + \frac{\cos\theta}{\cos\theta} - 8\left(\frac{1}{2}\right)^2$

$= \frac{\cancel{\sin 20^\circ}}{\cancel{\sin 20^\circ}} + \frac{\cancel{\sin 31^\circ}}{\cancel{\sin 31^\circ}} + \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}} - \overset{2}{\cancel{8}}\left(\frac{1}{\cancel{4}}\right)$

$= 1 + 1 + 1 - 2 \quad = 3 - 2 \quad = \boxed{1}$

(iii) $\tan 15^\circ \tan 30^\circ \tan 45^\circ \tan 60^\circ \tan 75^\circ$

$= \tan 15^\circ \times \frac{1}{\cancel{\sqrt{3}}} \times 1 \times \cancel{\sqrt{3}} \times \tan 75^\circ$

$= \tan(90^\circ - 75^\circ) \times \tan 75^\circ$

$\tan(90^\circ - \theta) = \cot\theta$

$= \cot 75^\circ \times \tan 75^\circ$

$= \cancel{\cot 15^\circ} \times \frac{1}{\cancel{\cot 15^\circ}}$

$= \boxed{1}$

(iv) $\frac{\cot\theta}{\tan(90^\circ-\theta)} + \frac{\cos(90^\circ-\theta)\tan\theta\sec(90^\circ-\theta)}{\sin(90^\circ-\theta)\cot(90^\circ-\theta)\operatorname{cosec}(90^\circ-\theta)}$

$= \frac{\cancel{\cot\theta}}{\cancel{\cot\theta}} + \left[\frac{\sin\theta \times \cancel{\tan\theta} \times \operatorname{cosec}\theta}{\cos\theta \times \cancel{\tan\theta} \times \sec\theta}\right]$

$= 1 + \left[\frac{\sin\theta \times \operatorname{cosec}\theta}{\cos\theta \times \sec\theta}\right]$

$= 1 + \left[\frac{\cancel{\sin\theta} \times \frac{1}{\cancel{\sin\theta}}}{\cancel{\cos\theta} \times \frac{1}{\cancel{\cos\theta}}}\right]$

$= 1 + \left(\frac{1}{1}\right)$

$= 1 + 1$

$= \boxed{2}$

$\sec(90^\circ-\theta) = \operatorname{cosec}\theta$
$\operatorname{cosec}(90^\circ-\theta) = \sec\theta$
$\cot(90^\circ-\theta) = \tan\theta$

EXERCISE 6.4

1) Find the value of the following:

(i) $\sin 49^\circ = 0.7547$

(ii) $\cos 74^\circ 39'$

$$\begin{aligned} \cos 74^\circ 36' &= 0.26\overset{4}{\cancel{5}}\overset{16}{\cancel{6}} \\ \text{(meandif) } 3' &= \underline{8} \ (-) \\ & \underline{0.2648} \end{aligned}$$

$\Rightarrow \cos 74^\circ 39' = 0.2648$

(iii) $\tan 54^\circ 26'$

$$\begin{aligned} \tan 54^\circ 24' &= 1.3968 \\ \text{(M.D) } 2' &= \underline{17} \ (+) \\ & \underline{1.3985} \end{aligned}$$

$\Rightarrow \tan 54^\circ 26' = 1.3985$

(iv) $\sin 21^\circ 21'$

$$\begin{aligned} \sin 21^\circ 18' &= 0.3633 \\ \text{(M.D) } 3' &= \underline{8} \ (+) \\ & \underline{0.3641} \end{aligned}$$

$\Rightarrow \sin 21^\circ 21' = 0.3641$

(v) $\cos 33^\circ 53'$

$$\begin{aligned} \cos 33^\circ 48' &= 0.8310 \\ \text{(M.D) } 5' &= \underline{8} \ (-) \\ & \underline{0.8302} \end{aligned}$$

$\Rightarrow$ $\cos 33^\circ 53' = 0.8302$

(vi) $\tan 70^\circ 17'$

$\tan 70^\circ 12' = 2.7776$

(M.D) $5' = \quad 131 \; (+)$

$\underline{2.7907}$

$\Rightarrow$ $\tan 70^\circ 17' = 2.7907$

2) Find the value of θ

(i) $\sin\theta = 0.9975$

$\sin 85^\circ 54' = 0.9974$

(M.D) $3' = \quad 1 \; (+)$

$\Rightarrow$ $\sin 85^\circ 57' = \underline{0.9975}$

$\therefore$ $\theta = 85^\circ 57'$

(ii) $\cos\theta = 0.6763$

$\cos 47^\circ 24' = 0.6769$

(M.D) $3' = \quad 6 \; (-)$

$\Rightarrow$ $\cos 47^\circ 27' = \underline{0.6763}$

$\therefore$ $\theta = 47^\circ 27'$

(iii) $\tan\theta = 0.0720$

$\tan 4^\circ 6' = 0.0717$

(M.D) $1' = \quad 3 \; (+)$

$\Rightarrow$ $\tan 4^\circ 7' = \underline{0.0720}$

$\therefore \theta = 4^\circ 7'$

(iv) $\cos\theta = 0.0410$

$\cos 87^\circ 36' = 0.0419$

(M.D) $3' = \underline{\quad 9}$ (−)

$\Rightarrow \cos 87^\circ 39' = \underline{0.0410}$

$\therefore \theta = 87^\circ 39'$

(v) $\tan\theta = 7.5958$

$\tan 82^\circ 30' = 7.5958$

$\therefore \theta = 82^\circ 30'$

---

3) Find the value of the following:

(i) $\sin 65^\circ 39' + \cos 24^\circ 57' + \tan 10^\circ 10'$

$\sin 65^\circ 36' = 0.9107$

$3' = \underline{\quad 4}$ (+)

$\sin 65^\circ 39' = \underline{0.9111}$ ✓

$\cos 24^\circ 54' = 0.9070$

$3' = \underline{\quad 4}$ (−)

$\cos 24^\circ 57' = \underline{0.9066}$ ✓

$\tan 10^\circ 6' = 0.1781$

$4' = \underline{\quad 12}$ (+)

$\tan 10^\circ 10' = \underline{0.1793}$ ✓

$\Rightarrow \sin 65^\circ 39' + \cos 24^\circ 57' + \tan 10^\circ 10'$

$= 0.9111 + 0.9066 + 0.1793$

$= 1.9970.$

(ii) $\tan 70^\circ 58' + \cos 15^\circ 26' - \sin 84^\circ 59'$

$\tan 70^\circ 54' = 2.8878$

$4' = \underline{\quad 104} \;(+)$

$\tan 70^\circ 58' = \underline{2.8982}$ ✓

$\cos 15^\circ 24' = 0.9641$

$2' = \underline{\quad 2} \;(-)$

$\cos 15^\circ 26' = \underline{0.9639}$ ✓

$\sin 84^\circ 54' = 0.9960$

$5' = \underline{\quad 2} \;(+)$

$\sin 54^\circ 59' = \underline{0.9962}$ ✓

$\Rightarrow \tan 70^\circ 58' + \cos 15^\circ 26' - \sin 84^\circ 59'$

$= 2.8982 + 0.9639 - 0.9962$

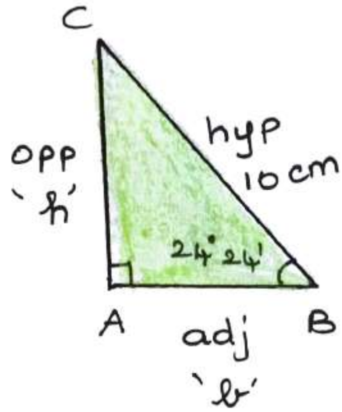
$= 3.8621 - 0.9962$

$= 2.8659$

2.8982 0.9639 (+) 3.8621	2 17 15 11 11 $\cancel{3}.\cancel{8}\cancel{6}\cancel{2}\cancel{1}$ 0.9962 (−) 2.8659	

4) Find the area of a right triangle whose hypotenuse is 10 cm and one of the acute angle is $24^\circ 24'$.

$$\sin\theta = \frac{opp}{hyp}$$



$$\sin 24^\circ 24' = \frac{h}{10}$$

$$0.4131 = \frac{h}{10}$$

$$0.4131 \times 10 = h$$

$$4.131 = h$$

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 24^\circ 24' = \frac{b}{10}$$

$$0.9107 = \frac{b}{10}$$

$$0.9107 \times 10 = b$$

$$9.107 = b$$

$$
\begin{array}{r}
9.107 \times 2.065 \\
\hline
45535 \\
54642 \\
0000 \\
18214 \\
\hline
18.805955
\end{array}
$$

$\Rightarrow$ Area of a triangle $= \boxed{\frac{1}{2} \times b \times h}$

$$= \frac{1}{\cancel{2}} \times 9.107 \times \overset{2.065}{\cancel{4.131}}$$

$$= 9.107 \times 2.065$$

$$= 18.805955$$

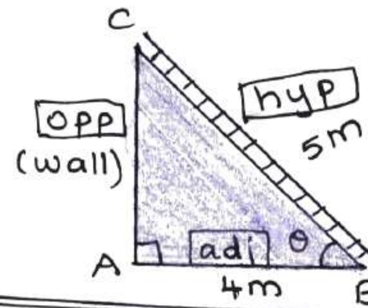
$$= 18.81\,cm^2$$

5) Find the angle made by a ladder of length 5m with the ground. If one of its end is 4m away from the wall and the other end is on the wall.

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos\theta = \frac{4}{5}$$

$$\begin{array}{r} 0.8 \\ 5\overline{)\,40} \\ \underline{40} \\ \underline{0} \end{array}$$



$$\cos\theta = 0.8$$

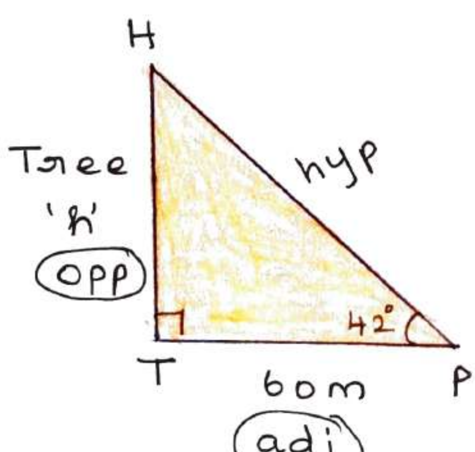
$$\cos\theta = \cos 36^\circ 52'$$

$$\theta = 36^\circ 52'$$

$\cos 36^\circ 48'$	=	0.8007
$4'$	=	7 (-)
$\cos 36^\circ 52'$	=	0.8000

6) In the given figure, HT shows the height of a tree standing vertically. From a point P, the angle of elevation of the top of the tree (that is $\angle P$) measures 42° and the distance to the tree is 60 metres. Find the height of the tree.

$$\tan\theta = \frac{opp}{adj}$$



$$\tan 42^\circ = \frac{h}{60}$$

$$0.9004 = \frac{h}{60}$$

$0.9004 \times 60 = h$

$54.0240 = h$

$\boxed{h = 54.0240\,m}$

$\Rightarrow$ The height of the tree is

54.02 m

Chapter - 7

Mensuration

1. Heron's Formula

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ sq.units

$S = \frac{a+b+c}{2}$ [s is Semi-perimeter]

Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$ sq.units

Area of triangle (b & h) $= \frac{1}{2} b \times h$ sq.units

Exercise - 7.1

1. Using Heron's Formula, find the area of a triangle whose sides are

(i) 10cm, 24cm, 26cm.

Sol:- $a = 10cm$
$b = 24\ cm$
$c = 26\ cm$

$$S = \frac{a+b+c}{2}$$

$$= \frac{10+24+26}{2}$$

$$= \frac{60}{2}$$

$$\boxed{S = 30cm}$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{30(30-10)(30-24)(30-26)}$$

$$= \sqrt{30(20)(6)(4)}$$

$$= \sqrt{6\times5\times4\times5\times6\times4}$$

$$= 6\times5\times4$$

$$\boxed{A = 120\ cm^2}$$

(ii) 1.8m , 8m, 8.2m

Sol: $a = 1.8m$

$b = 8m$

$c = 8.2m$

$$S = \frac{a+b+c}{2}$$

$$= \frac{1.8 + 8 + 8.2}{2}$$

$$= \frac{18.0}{2}$$

$$\boxed{S = 9m}$$

$$\begin{array}{r} 1.8 \\ +\ 8.0 \\ 8.2 \\ \hline 18.0 \end{array}$$

$$\begin{array}{r} 9.0 \\ -\ 1.8 \\ \hline 7.2 \end{array}$$

$$\begin{array}{r} 9.0 \\ 8.2 \\ \hline 0.8 \end{array}$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{9(9-1.8)(9-8)(9-8.2)}$$

$$= \sqrt{9 \times 7.2 \times 1 \times 0.8}$$

$$= \sqrt{7.2 \times 7.2}$$

$$\boxed{A = 7.2\ m^2}$$

---

(2) The sides of the triangular ground are 22m, 120m, and 122m. Find the area and cost of levelling the ground at the rate of ₹20 per m^2.

Sol:- $a = 22m$
$b = 120m$
$c = 122m$

$$S = \frac{a+b+c}{2}$$

$$= \frac{22+120+122}{2}$$

$$= \frac{\cancel{264}^{132}}{\cancel{2}}$$

$$\boxed{S = 132\,m}$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{132(132-22)(132-120)(132-122)}$$

$$= \sqrt{132 \times 110 \times 12 \times 10}$$

$132 = 12 \times 11$

$110 = 11 \times 10$

$$= \sqrt{12 \times 11 \times 11 \times 10 \quad 12 \times 10}$$

$$\begin{array}{r} 1320 \\ \times\ 20 \\ \hline 26400 \end{array}$$

$$= 12 \times 11 \times 10$$

$$\boxed{A = 1320\,m^2}$$

Cost of levelling ground per m^2 = ₹ 20

∴ Cost of levelling $1320 m^2 = 1320 \times 20$

= ₹26,400

③ The perimeter of a triangular plot is 600m. If the sides are in the ratio 5:12:13, then find the area of the plot.

Sol:-

$$a = 5x$$
$$b = 12x$$
$$c = 13x$$

$$\text{Perimeter} = 600\text{m}$$
$$5x + 12x + 13x = 600$$
$$30x = 600$$
$$x = \frac{\cancel{600}^{20}}{\cancel{30}}$$
$$\boxed{x = 20}$$

$$a = 5x = 5(20) = 100\text{m}$$
$$b = 12x = 12(20) = 240\text{m}$$
$$c = 13x = 13(20) = 260\text{m}$$

$$S = \frac{a+b+c}{2}$$
$$= \frac{100 + 240 + 260}{2}$$

$$= \frac{600}{2}$$

$$\boxed{S = 300\,m}$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{300(300-100)(300-240)(300-260)}$$

$$= \sqrt{300 \times 200 \times 60 \times 40}$$

$$= \sqrt{3 \times 100 \times 2 \times 100 \times \underline{2 \times 3 \times 10} \times \underline{2 \times 2 \times 10}}$$

$$= 3 \times 2 \times 2 \times 100 \times 10$$

$$\boxed{A = 12000\,m^2}$$

---

(4) Find the area of an equilateral triangle whose perimeter is 180cm.

Sol :- Equilateral triangle

$\boxed{\text{3 sides equal}}$

$$\text{Perimeter} = 180\,cm$$

$$3a = 180$$

$$a = \frac{180}{3}$$

$$\boxed{a = 60\,cm}$$

$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \times (60)^2$$

$$= \frac{\sqrt{3}}{\cancel{4}_{1}} \times 60 \times \cancel{60}^{15}$$

$$\boxed{A = 900\sqrt{3}\ \text{cm}^2}$$

$$\begin{array}{r} 60 \\ \times 15 \\ \hline 300 \\ 60 \\ \hline 900 \end{array}$$

$\sqrt{3} = 1.732$

(or)

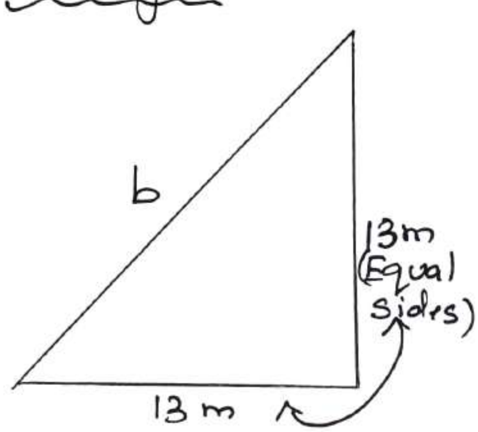
$$= 900 \times 1.732$$

$$\boxed{A = 1558.800\ \text{cm}^2}$$

---

(5) An advertisement board is in the form of an isosceles triangle with perimeter 36m and each of the equal sides are 13m. Find the cost of painting it at ₹ 17.50 per square metre.

Sol:- Isosceles Triangle.



$$\text{Perimeter} = 36\text{m}$$

$$13 + 13 + b = 36$$

$$26 + b = 36$$

$$b = 36 - 26$$

$$\boxed{b = 10\text{m}}$$

$$S = a+b+c$$

$$= \frac{13+13+10}{2}$$

$$= \frac{\cancel{36}^{18}}{\cancel{2}_{1}}$$

$$\boxed{S = 18\ m}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-13)(18-13)(18-10)}$$

$$= \sqrt{18 \times 5 \times 5 \times 8}$$

$$144 = 12 \times 12$$

$$= \sqrt{144 \times 5 \times 5}$$

$$= \sqrt{12 \times 12 \times 5 \times 5}$$

$$\begin{array}{r} 17.50 \\ \times\ 60 \\ \hline 1050.00 \end{array}$$

$$= 12 \times 5$$

$$\boxed{A = 60\ m^2}$$

Cost of painting per sq m = ₹17.50

$\therefore$ Cost of painting $60\ m^2$ = 60×17.50

= ₹1050.0

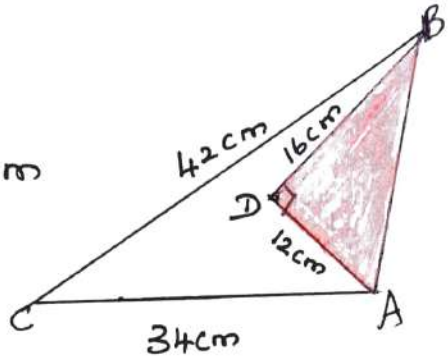
---

(6) Find the area of the unshaded region.

Sol:-

In Δ^{le} ABD

Using Pythagoras theorem



$$AB^2 = AD^2 + BD^2$$

$$= 12^2 + 16^2$$

$$= 144 + 256$$

$$AB^2 = 400$$

$$AB = \sqrt{400}$$

$$\boxed{AB = 20\,cm}$$

b = 12cm
h = 16cm

Area of Δ^{le} ABD $= \frac{1}{2} \times b \times h$

$$= \frac{1}{\cancel{2}_1} \times 12 \times \cancel{16}^{8}$$

$$\boxed{A = 96\,cm^2}$$

In Δ^{le} ABC

$a = 34\,cm$, $b = 42\,cm$, $c = 20\,cm$

$$S = \frac{a+b+c}{2}$$

$$= \frac{34+42+20}{2}$$

$$= \frac{\cancel{96}^{48}}{\cancel{2}}$$

$$\boxed{S = 48\ cm}$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{48(48-34)(48-42)(48-20)}$$

$$= \sqrt{48 \times 14 \times 6 \times 28}$$

$$= \sqrt{6 \times 8 \times 7 \times 2 \times 6 \times 7 \times 4}$$

$$= \sqrt{6 \times 6 \times 7 \times 7 \times 8 \times 8}$$

$$= 6 \times 7 \times 8$$

$$\boxed{A = 336\ cm^2}$$

$$\begin{array}{r} 336 \\ 96 \\ \hline 0 \end{array}$$

$\therefore$ Area of Unshaded region =

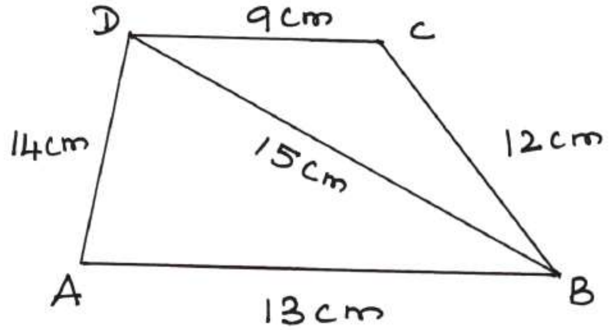
Area of Δ^{le} ABC − Area of Δ^{le} ABD

$= 336 - 96$

Area of Unshaded Region $\}$ $= 240\ cm^2$

(7) Find the area of a quadrilateral ABCD whose sides are AB = 13cm, BC = 12cm, CD = 9cm, AD = 14cm and diagonal BD = 15cm.

Sol:-



Δ^{le} ABD

$a = 13cm, b = 14cm, c = 15cm$

$S = \frac{a+b+c}{2}$

$= \frac{13+14+15}{2}$

$= \frac{42}{2}$

$\boxed{S = 21cm}$

$A = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

$= \sqrt{7 \times 3 \times 4 \times 2 \times 7 \times 3 \times 2}$

$= 7 \times 3 \times 2 \times 2$

$\boxed{A = 84cm^2}$

Δ^{le} BDC

$a = 12cm, b = 9cm, c = 15cm$

$S = \frac{a+b+c}{2}$

$= \frac{12+9+15}{2}$

$= \frac{\cancel{36}^{18}}{\cancel{2}_1}$

$\boxed{S = 18cm}$

$A = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{18(18-12)(18-9)(18-15)}$

$= \sqrt{18 \times 6 \times 9 \times 3}$

$= \sqrt{18 \times 18 \times 3 \times 3}$

$= 18 \times 3$

$\boxed{A = 54cm^2}$

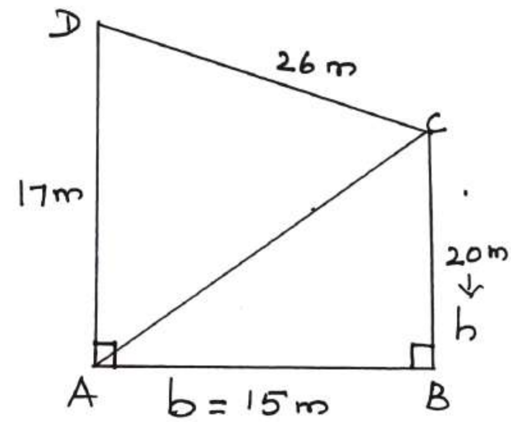
$\therefore$ Area of quadrilateral = Area of Δ^{le} ABD + Area of Δ^{le} BDC

$= 84 + 54$

$\boxed{\text{Area of Quadrilateral} = 138\ cm^2}$

---

(8) A park is in the shape of a quadrilateral. The sides of the park are 15m, 20m, 26m and 17m and the angle between the first two sides is a right angle. Find the area of the park.



Sol:- In Δ^{le} ABC

Using Pythagoras theo

$AC^2 = AB^2 + BC^2$

$= 15^2 + 20^2$

$= 225 + 400$

$625 = 25 \times 25$

$AC^2 = 625$

$AC = \sqrt{625}$

$\boxed{AC = 25\ m}$

$\therefore$ Area of $\Delta^{le} ABC = \frac{1}{2} b \times h$

$= \frac{1}{\cancel{2}_1} \times 15 \times \cancel{20}^{10}$

$\boxed{A = 150 m^2}$

In Δ^{le} ACD

$a = 17m \quad b = 26m \quad c = 25m$

$A = \sqrt{S(S-a)(S-b)(S-c)}$

$S = \frac{17 + 26 + 25}{2}$

$= \frac{\cancel{68}\, 34}{\cancel{2}_1}$

$\boxed{S = 34 m}$

$\therefore A = \sqrt{34(34-17)(34-26)(34-25)}$

$= \sqrt{34 \times 17 \times 8 \times 9}$

$= \sqrt{\underline{17 \times 2} \times 17 \times \underline{2 \times 4} \times \underline{3 \times 3}}$

$= 17 \times 2 \times 3 \times 2$

$\boxed{A = 204 m^2}$

$\therefore$ Area of quadrilateral = Area of Δ^{le} ABC + Area of Δ^{le} ACD

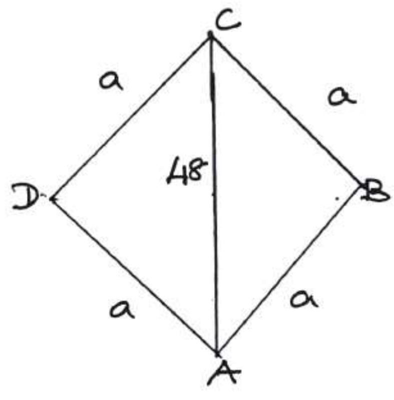
$= 150 + 204$

$$\boxed{\text{Area of quadrilateral} = 354\ m^2}$$

(9) A land is in the shape of rhombus. The perimeter of the land is 160m and one of the diagonal is 48m. Find the area of the land.

Sol:-

Rhombus



Perimeter = 160m

$4a = 160$

$a = \frac{160}{4}$

$$\boxed{a = 40m}$$

Area of rhombus = 2 Area of Δ^{le} ABC

In Δ^{le} ABC

$a = 40m$, $b = 40m$ $c = 48m$

$$S = \frac{a+b+c}{2}$$

$$= \frac{40+40+48}{2}$$

$$= \frac{\cancel{128}\,64}{\cancel{2}\,1}$$

$$\boxed{S = 64\,m}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{64(64-40)(64-40)(64-48)}$$

$$= \sqrt{64 \times 24 \times 24 \times 16}$$

$$= \sqrt{\underline{8 \times 8} \times 24 \times 24 \times \underline{4 \times 4}}$$

$$= 8 \times 24 \times 4$$

$$\boxed{A = 768\,m^2}$$

$\therefore$ Area of rhombus $= 2 \times 768$

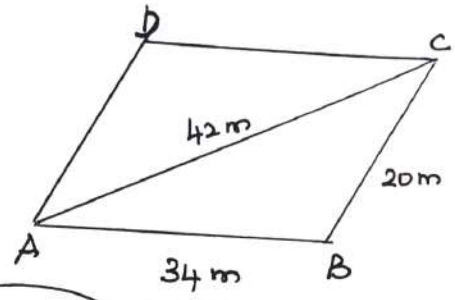
$= 1536\,m^2$

---

(10) The adjacent sides of a parallelogram measures 34m, 20m and the measure of one of the diagonal is 42m. Find the area of parallelogram.

Sol: -

Parallelogram



Area of Parallelogram = 2 (Area of Δ^{le}ABC)

Δ^{le} ABC

$a = 34m, \quad b = 20m, \quad c = 42m$

$$S = \frac{a+b+c}{2}$$

$$= \frac{34+20+42}{2}$$

$$= \frac{\cancel{96}^{48}}{\cancel{2}_{1}}$$

$$\boxed{S = 48m}$$

$$\text{Area of } \Delta^{le} ABC = \sqrt{S(S-a)(S-b)(S-c)}$$

$$A = \sqrt{48(48-34)(48-20)(48-42)}$$

$$= \sqrt{48 \times 14 \times 28 \times 6}$$

$$= \sqrt{\underline{6 \times 4 \times 2} \times 7 \times 2 \times 7 \times 4 \times 6}$$

$$A = 6 \times 7 \times 4 \times 2$$

$$\boxed{A = 336\ m^2}$$

$$\begin{array}{r} 42 \\ \times 8 \\ \hline 336 \end{array}$$

$\therefore$ Area of Parallelogram $= 2(336)$

$= 672\ m^2$

Surface Area of Cuboid and Cube

Shape	Lateral Surface area LSA	Total Surface area TSA
Cuboid	$2h(l+b)$	$2(lb+bh+hl)$
Cube	$4a^2$	$6a^2$

Exercise - 7.2

(1) Find the Total Surface Area and the Lateral Surface Area of a cuboid whose dimensions are length = 20cm, breadth = 15cm and height = 8cm

Sol:- $l = 20cm$

$b = 15cm$

$h = 8cm$

Sol:

$$TSA = 2(lb+bh+hl)$$

$$= 2[(20\times15)+(15\times8)+(8\times20)]$$

$$= 2[300+120+160]$$

$$= 2[580]$$

$$\begin{array}{r} 580 \\ \times\ 2 \\ \hline 1160 \\ \hline \end{array}$$

$$\boxed{TSA = 1160\ cm^2}$$

$$LSA = 2h(l+b)$$

$$= 2\times8(20+15)$$

$$= 16\times35$$

$$\begin{array}{r} 35 \\ \times16 \\ \hline 210 \\ 35\ \\ \hline 560 \\ \hline \end{array}$$

$$\boxed{LSA = 560\ cm^2}$$

---

(2) The dimensions of a cuboidal box are 6m × 400cm × 1.5m. Find the cost of painting its entire outer surface at the rate of ₹22 per m^2.

Sol:- Cuboid

$l = 6m$

$b = 400cm = \frac{400}{100} = 4m$

$h = 1.5m$

$$TSA = 2(lb + bh + hl)$$

$$= 2[(6\times4) + (4\times1.5) + (1.5\times6)]$$

$$= 2[24 + 6.0 + 9.0]$$

$$= 2(39)$$

$$\boxed{TSA = 78\,m^2}$$

Cost of painting entire outer surface per sq m = ₹ 22

$\therefore$ Cost of painting $78\,m^2 = 78\times22$

$= ₹\,1716$

$$\begin{array}{r} 78 \\ \times 22 \\ \hline 156 \\ 156 \\ \hline 1716 \end{array}$$

---

(3) The dimension of a hall is $10m \times 9m \times 8m$. Find the cost of white washing the walls and ceiling at the rate of ₹ 8.50 per m^2.

Sol

Walls = LSA
Ceiling = Area of Rectangle

$l = 10m$
$b = 9m$
$h = 8m$

Area to be White Washed = LSA + Area of Rectangle

$= 2h(l+b) + (l \times b)$

$= 2\times 8(10+9) + (10\times 9)$

$= 16(19) + 90$

$= 304 + 90$

$\boxed{\text{Area} = 394\,m^2}$

$$\begin{array}{r} 16 \\ \underline{19} \\ 144 \\ \underline{16} \\ \underline{304} \end{array}$$

$$\begin{array}{r} 394 \\ \underline{8.50} \\ 19700 \\ \underline{3152} \\ \underline{3349.00} \end{array}$$

Cost of white washing walls and ceiling per m^2 $\}$ = ₹ 8.50

∴ cost of white washing $394\,m^2 = 394 \times 8.50$

$= ₹\,3349.0$

---

(4) Find the TSA and LSA of the cube whose side is .

(i) 8 m (ii) 21 cm (iii) 7.5 cm

Sol Cube

(i) $a = 8\,m$

$TSA = 6a^2$

$= 6(8)^2$

$= 6(64)$

$\boxed{TSA = 384\,m^2}$

$LSA = 4a^2$

$= 4(64)$

$\boxed{LSA = 256\,m^2}$

(ii) $a = 21\,cm$

$TSA = 6a^2$

$= 6(21)^2$

$= 6(441)$

$\boxed{TSA = 2646\,cm^2}$

$LSA = 4a^2$

$= 4(441)$

$\boxed{LSA = 1764\,cm^2}$

(iii) $a = 7.5\,cm$

$TSA = 6a^2$

$= 6(7.5)^2$

$= 6(56.25)$

$\boxed{TSA = 337.50\,cm^2}$

$LSA = 4a^2$

$= 4(56.25)$

$\boxed{LSA = 225.0\,cm^2}$

(5) If the total Surface area of the Cube is 2400 cm^2. then find its lateral Surface area.

Sol:

Cube

$$TSA = 2400\ cm^2$$

$$6a^2 = 2400$$

$$a^2 = \frac{\cancel{2400}\ 400}{\cancel{6}}$$

$$\boxed{a^2 = 400}$$

$$\therefore LSA = 4a^2$$

$$= 4(400)$$

$$\boxed{LSA = 1600\ cm^2}$$

(6) A Cubical Container Of Side 6.5 m is to be painted on the entire Outer Surface. Find the area to be painted and the total cost of painting it at the rate of ₹ 24 per m^2

Sol:-

Cube

$$a = 6.5m$$

$$TSA = 6a^2$$
$$= 6(6.5)^2$$
$$= 6 \times 42.25$$
$$\boxed{TSA = 253.50\ m^2}$$

$$\begin{array}{r} 6.5 \\ \times 6.5 \\ \hline 325 \\ 390 \\ \hline 42.25 \\ \times\ 6 \\ \hline 253.50 \end{array}$$

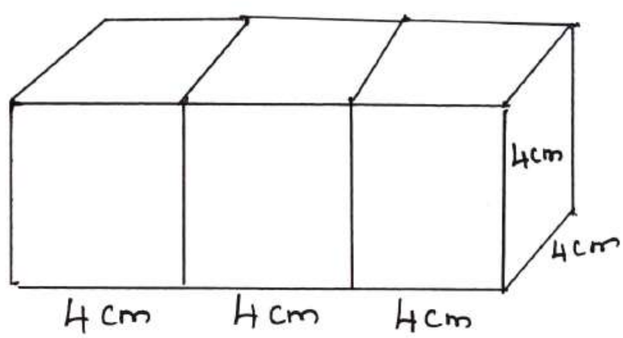
Cost of painting per sq m = ₹ 24

∴ cost of painting $253.50\ m^2 = 253.50 \times 24$

$= ₹\ 6084.0$

(7) Three identical cubes of side 4cm are joined end to end. Find the total surface area and lateral surface area of the new resulting cuboid.

Sol: -



$l = 4+4+4 = 12\,cm$

$b = 4\,cm$

$h = 4\,cm$

$$TSA = 2(lb+bh+hl)$$

$$= 2\left[(12\times4)+(4\times4)+(4\times12)\right]$$

$$= 2\left[48+16+48\right]$$

$$= 2\times 112$$

$$\boxed{TSA = 224\ cm^2}$$

$$LSA = 2h(l+b)$$

$$= 2\times4\ (12+4)$$

$$= 8\times16$$

$$\boxed{LSA = 128\ cm^2}$$

$$\begin{array}{r} 16 \\ \times 8 \\ \hline 128 \end{array}$$

Volume

Shape	Volume
Cuboid	lbh
Cube	a^3

Exercise - 7.3

(1) Find the Volume of a cuboid whose dimensions are.

(i) length = 12 cm, breadth = 8 cm, height = 6 cm
(ii) length = 60 m, breadth = 25 m, height = 1.5 m

Sol:- Cuboid

(i) $l = 12\,cm$

$b = 8\,cm$

$h = 6\,cm$

$V = lbh$

$= 12 \times 8 \times 6$

$\boxed{V = 576\,cm^3}$

$$\begin{array}{r} \overset{3}{96} \\ \times 6 \\ \hline 576 \end{array}$$

(ii) $l = 60\,m$

$b = 25\,m$

$h = 1.5\,m$

$V = lbh$

$= 60 \times 25 \times 1.5$

$\boxed{V = 2250.\,m^3}$

$$\begin{array}{r} 60 \\ 25 \\ \hline 300 \\ 12 \\ \hline 1500 \end{array}$$

$$\begin{array}{r} 15 \\ 1.5 \\ \hline 75 \\ 15 \\ \hline 22500 \end{array}$$

(2) The dimensions of a match box are 6 cm × 3.5 cm × 2.5 cm. Find the Volume of packet containing 12 Such match boxes.

Sol:- Cuboid

$l = 6\,cm$

$b = 3.5\,cm$

$h = 2.5\,cm$

Volume of 1 match box = lbh

$= 6 \times 3.5 \times 2.5$

$= 52.50 \, cm^3$

$$\begin{array}{r} 2.5 \\ \times 3.5 \\ \hline 125 \\ 75 \\ \hline 8.75 \\ \times \ 6 \\ \hline 52.50 \end{array}$$

$\therefore$ Volume of 12 match box $= 12 \times 52.50$

$= 630 \, cm^3$

$$\begin{array}{r} 525 \\ 12 \\ \hline 1050 \\ 525 \\ \hline 630.00 \end{array}$$

---

(3) The length, breadth and height of a chocolate box are in the ratio 5:4:3. If its Volume is $7500 \, cm^3$, then find its dimensions.

Sol

Cuboid

$l = 5x$

$b = 4x$

$h = 3x$

$V = 7500 \, cm^3$

$lbh = 7500$

$(5x)(4x)(3x) = 7500$

$60x^3 = 7500$

$x^3 = \dfrac{\overset{125}{\cancel{7500}}}{\underset{1}{\cancel{60}}}$

$$x^3 = 125$$

$$x^3 = 5^3$$

$$\therefore x = 5$$

$\therefore$ $l = 5x = 5(5) = 25$ cm

$b = 4x = 4(5) = 20$ cm

$h = 3x = 3(5) = 15$ cm

---

(4) The length, breadth and depth of a pond are 20.5 m, 16m, 8m respectively. Find the capacity of the pond in litres.

Sol :-

Cuboid

$l = 20.5$ m

$b = 16$ m

$h = 8$ m

$$\begin{array}{r} 20.5 \\ \times\ 16 \\ \hline 12.30 \\ 205 \\ \hline 328.0 \\ \times\ \ 8 \\ \hline 2624.0 \\ \hline \end{array}$$

Capacity = Volume = lbh

$= 20.5 \times 16 \times 8$

Volume $= 2624\ m^3$

$$\boxed{1 m^3 = 1000\ l}$$

$2624\ m^3 = 2624 \times 1000$

$= 2624000$ litres

(5) The dimensions of a brick are 24cm × 12cm × 8cm. How many such bricks will be required to build a wall of 20m length, 48cm breadth, and 6m height?

Sol

Brick

$l = 24\,cm$

$b = 12\,cm$

$h = 8\,cm$

$V = lbh$

$V = 24 \times 12 \times 8$

Wall

$l = 20m \Rightarrow 20 \times 100 = 2000\,cm$

$b = 48\,cm$

$h = 6m \Rightarrow 6 \times 100 = 600\,cm$

$V = lbh$

$V = 2000 \times 48 \times 600$

$$\therefore \text{No of bricks} = \frac{\text{Vol of Wall}}{\text{Vol of Brick}}$$

$$= \frac{\overset{1000}{\cancel{2000}} \times \overset{6}{\cancel{48}} \times \overset{\overset{\overset{25}{\cancel{75}}}{\cancel{150}}}{\overset{300}{\cancel{600}}}}{\underset{\underset{\cancel{6}\,2}{\cancel{12}}}{\cancel{24}} \times \underset{\cancel{2}}{\cancel{12}} \times \cancel{8}}$$

$= 1000 \times 25$

$$\boxed{\text{No of bricks} = 25000}$$

(6) The Volume of a Container is $1440 m^3$. The length and breadth of the Container are 15m and 8m respectively. Find its height.

Sol:

Cuboid

$l = 15m$

$b = 8m$

$h = ?$

$V = 1440 m^3$

$lbh = 1440$

$15 \times 8 \times h = 1440$

$h = \dfrac{\cancel{1440}^{\cancel{180}^{12}}}{\cancel{15}_1 \times \cancel{8}_1}$

$\boxed{h = 12m}$

---

(7) Find the Volume of a Cube each of whose side is (i) 5cm, (ii) 3.5m (iii) 21cm

Sol:-   Cube

(i) $a = 5\,cm$

$V = a^3$

$= 5^3$

$\boxed{V = 125\,cm^3}$

(ii) $a = 3.5\,m$

$V = a^3$

$= (3.5)^3$

$\boxed{V = 42.875\,m^3}$

$$\begin{array}{r} 3.5 \\ \times 3.5 \\ \hline 175 \\ 105 \\ \hline 12.25 \\ \times 35 \\ \hline 6125 \\ 3675 \\ \hline 42.875 \end{array}$$

(iii) $a = 21\,cm$

$V = a^3$

$= (21)^3$

$\boxed{V = 9261\,cm^3}$

$$\begin{array}{r} 441 \\ \times 21 \\ \hline 441 \\ 882 \\ \hline 9261 \end{array}$$

---

(8) A cubical milk tank can hold 125000 litres of milk. Find the length of its side in metres.

Sol:-

Cube

Capacity = Volume = 125000 litres

$V = 125000$ litres

$125000 \text{ litres} = \frac{125\cancel{000}}{1\cancel{000}}$

$125000 \text{ litres} = 125 m^3$

$\therefore V = 125 m^3$

$a^3 = 125$

$a^3 = 5^3$

$\therefore a = 5m$

$\therefore \boxed{\text{Side} = 5m}$

---

(9) A metallic cube with side 15 cm is melted and formed into a cuboid. If the length and height of the cuboid is 25 cm and 9 cm respectively then find the breadth of the cuboid.

Sol:-

Cube	Cuboid
$a = 15 cm$	$l = 25 cm$
$V = a^3$	$h = 9 cm$
$V = (15)^3$	$b = ?$
	$V = l \times b \times h$
	$V = 25 \times 9 \times b$

Vol of Cuboid = Vol of cube

$25 \times 9 \times b = (15)^3$

$$b = \frac{15 \times \cancel{15}^{\cancel{5}^{1}} \times \cancel{15}^{\cancel{5}^{1}}}{\cancel{25}_{\cancel{5}_{1}} \times \cancel{9}_{\cancel{3}_{1}}}$$

$$\boxed{b = 15 \text{ cm}}$$

$\therefore$ breath of Cuboid = 15 cm.

Chapter -8

Statistics

Arithmetic Mean

★ Raw Data

$$\bar{X} = \frac{\text{Sum of all Observation}}{\text{Number of Observation}}$$

ie, $\bar{X} = \frac{\Sigma x}{n}$

★ Assumed Mean Method

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\boxed{d = x - A}$$

A is Assumed mean.

---

* Step deviation

$$\bar{x} = A + \left(\frac{\Sigma fd}{\Sigma f}\right) \times c$$

$\boxed{C = \text{class Width}}$

Exercise - 8.1

(1) In a Week, temperature of a Certain place is measured during winter are as follows 26°c, 24°c, 28°c, 31°c, 30°c, 26°c, 24°c. Find the mean temperature of the Week.

Sol:-

26°c, 24°c, 28°c, 31°c, 30°c, 26°c, 24°c

$\boxed{n = 7}$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{26+24+28+31+30+26+24}{7}$$

$$= \frac{\cancel{189}^{27}}{\cancel{7}}$$

$$\therefore \boxed{\bar{x} = 27°c}$$

(2) The mean weight of 4 members of a family is 60kg. Three of them have the weight 56kg, 68kg, and 72kg respectively. Find the weight of the fourth member.

Sol:-

Weight of 3 members = 56kg, 68kg & 72kg.

$$\left.\begin{array}{l}\text{Let the weight of} \\ \text{4}^{\text{th}}\text{ member}\end{array}\right\} = A$$

$$\left.\begin{array}{l}\text{Mean Weight of} \\ \text{4 members}\end{array}\right\} = 60\text{kg}$$

ie, $\bar{x} = 60$

$$\frac{\Sigma x}{n} = 60$$

$$\frac{56+68+72+A}{4} = 60$$

$$\frac{196 + A}{4} = 60$$

$$196 + A = 240$$

$$A = 240 - 196$$

$$\boxed{A = 44\,kg}$$

$\therefore$ Weight of 4^{th} member $= 44kg$.

---

(3) In a class test in mathematics, 10 students scored 75 marks, 12 students scored 60 marks, 8 students scored 40 marks and 3 students scored 30 marks. Find the mean of their score.

<u>Sol</u> :-

Number of students $\Rightarrow n = 10 + 12 + 8 + 3$

$\therefore \boxed{n = 33}$

Mark scored by 10 students

$= 10 \times 75 = 750$

Mark Scored by 12 Students

$= 12 \times 60 = 720$

Mark Scored by 8 Students

$= 8 \times 40 = 320$

Mark Scored by 3 Students

$= 3 \times 30 = 90$

$\therefore$ Total marks Scored by 33 Students $\Big\} = 750 + 720 + 320 + 90$

$$\therefore \boxed{\Sigma x = 1880}$$

$$\therefore \text{Mean} = \frac{\Sigma x}{n}$$

$$\bar{X} = \frac{1880}{33}$$

$$\begin{array}{r} 56.96 \\ 33\overline{)1880} \\ -165 \\ \hline 230 \\ -198 \\ \hline 320 \\ -297 \\ \hline 230 \end{array}$$

$$\bar{X} = 56.96$$

(or)

$$\boxed{\bar{X} = 57}$$

(4) In a research Laboratory scientists treated 6 mice with lung cancer using natural medicine. Ten days

later, they measured the Volume of the tumor in each mouse and given the results in the table.

Mouse Marking	1	2	3	4	5	6
Tumor Volume (mm^3)	145	148	142	141	139	140

Find the mean.

Sol: $\boxed{n = 6}$

$$\text{Mean} = \frac{\Sigma x}{n}$$

$$= \frac{145 + 148 + 142 + 141 + 139 + 140}{6}$$

$$= \frac{855}{6}$$

$$\begin{array}{r} 142.5 \\ 6\overline{)855} \\ -6 \\ \hline 25 \\ -24 \\ \hline 15 \\ -12 \\ \hline 30 \\ -30 \\ \hline 0 \end{array}$$

$$\boxed{\bar{x} = 142.5}$$

(5) If the mean of the following data is 20.2, then find the Value of P.

Marks	10	15	20	25	30
No. of Students	6	8	P	10	6

Sol:-

Mean = 20.2

Marks x	No. of students f	fx
10	6	60
15	8	120
20	P	20P
25	10	250
30	6	180
TOTAL	30+P	610+20P

$$\bar{x} = 20.2$$

$$\frac{\Sigma fx}{\Sigma f} = 20.2$$

$$\frac{610+20P}{30+P} = 20.2$$

$$610 + 20p = 20.2(30+p)$$

$$610 + 20p = 606.0 + 20.2p$$

$$610 - 606 = 20.2p - 20p$$

$$4 = 0.2p$$

$$\begin{array}{r} 20.2 \\ -20.0 \\ \hline 0.2 \end{array}$$

$$\Rightarrow 0.2p = 4$$

$$p = \frac{4}{0.2} \times \frac{10}{10}$$

$$= \frac{40}{2}$$

$$\therefore \boxed{p = 20}$$

---

(6) In the class, weight of students is measured for class records. Calculate mean weight of the class students using direct method.

Weight (kg)	15-25	25-35	35-45	45-55	55-65	65-75
No. of students	4	11	19	14	0	2

Sol:-

C.I	x	f	fx
15-25	20	4	80
25-35	30	11	330
35-45	40	19	760
45-55	50	14	700
55-65	60	0	0
65-75	70	2	140
Total		50	2010

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{201\cancel{0}}{5\cancel{0}}$$

$$\boxed{\bar{X} = 40.2}$$

$$\begin{array}{r} 40.2 \\ 5\overline{)201} \\ -20 \\ \hline 010 \\ -10 \\ \hline 0 \end{array}$$

(7) Calculate the mean of the following distribution using Assumed Mean Method.

Sol :-

C.I	x	f	$A = 25$, $d = x - A$	fd
0-10	5	5	$5 - 25 = -20$	-100
10-20	15	7	$15 - 25 = -10$	-70
20-30	$\overset{A}{\boxed{25}}$	15	$25 - 25 = 0$	0
30-40	35	28	$35 - 25 = 10$	280
40-50	45	8	$45 - 25 = 20$	160
Total.		63		270

Assumed Mean

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 25 + \frac{270}{63}$$

$$= 25 + 4.28$$

$$\boxed{\bar{X} = 29.28}$$

$$\begin{array}{r} 4.28 \\ 63\,)\,270 \\ -252 \\ \hline 180 \\ -126 \\ \hline 540 \\ -504 \\ \hline \end{array}$$

$$\begin{array}{r} 25.00 \\ 4.28 \\ \hline 29.28 \end{array}$$

(8) Find the Arithmetic Mean of the following data using Step Deviation Method :

Age	15-19	20-24	25-29	30-34	35-39	40-44
No of Persons	4	20	38	24	10	9

Sol :-

C.I	x	f	$A = 32$ $c = 5$ $d = \frac{x-A}{c}$	fd
14.5 - 19.5	17	4	$\frac{17-32}{5} = \frac{-15}{5} = -3$	-12
19.5 - 24.5	22	20	$\frac{22-32}{5} = \frac{-10}{5} = -2$	-40
24.5 - 29.5	27	38	$\frac{27-32}{5} = \frac{-5}{5} = -1$	-38
29.5 - 34.5	$\overset{A}{\boxed{32}}$	24	$\frac{32-32}{5} = 0$	0
34.5 - 39.5	37	10	$\frac{37-32}{5} = \frac{5}{5} = 1$	10
39.5 - 44.5	42	9	$\frac{42-32}{5} = \frac{10}{5} = 2$	18
Total.		105		-62

Step Deviation

$$\bar{X} = A + \left(\frac{\Sigma fd}{\Sigma f}\right) \times C$$

$$= 32 + \left(\frac{-62}{\cancel{105}_{21}}\right) \times \cancel{5}^{1}$$

$$= 32 - \frac{62}{21}$$

$$= 32 - 2.95$$

$$\boxed{\bar{X} = 29.05}$$

$$\begin{array}{r} 32.00 \\ (-)\ 2.95 \\ \hline 29.05 \\ \hline \end{array}$$

$$\begin{array}{r} 2.95 \\ 21\overline{)62} \\ -42 \\ \hline 200 \\ -189 \\ \hline 110 \\ -105 \\ \hline \end{array}$$

Median

* When N is odd

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{observation}$$

* when N is even

$$\text{Median} = \left[\frac{\left(\frac{N}{2}\right)^{th} \text{observation} + \left(\frac{N}{2}+1\right)^{th} \text{observation}}{2}\right]$$

* Grouped Frequency Distribution

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

l = lower limit of the median class

N = Total frequency (Σf)

m = Cumulative frequency of the class, preceeding the median class.

c = Width of the median class

f = Highest frequency of median class.

Exercise - 8.2

(1) Find the median of the given values :-

47, 58, 62, 71, 83, 21, 43, 47, 41

Sol: -

Arranging in ascending order.

21, 41, 43, 47, 47, 53, 62, 71, 83

$\boxed{N = 9} \Rightarrow$ odd.

$\therefore$ Median $= \left(\frac{N+1}{2}\right)^{th}$ term

$= \left(\frac{9+1}{2}\right)^{th}$ term

$= \left(\frac{10}{2}\right)^{th}$ term

$= 5^{th}$ term

$\therefore \boxed{\text{Median} = 47}$

---

(2) Find the Median of the given data

36, 44, 86, 31, 37, 44, 86, 35, 60, 51

Sol: - Arranging is Ascending Order.

31, 35, 36, 37, $\underline{44}$, $\underline{44}$, 51, 60, 86, 86

$\boxed{N = 10} \Rightarrow$ even

$$\text{Median} = \frac{\left[\left(\frac{N}{2}\right)^{th} \text{term} + \left(\frac{N}{2}+1\right)^{th} \text{term}\right]}{2}$$

$$= \frac{\left[\left(\frac{10}{2}\right)^{th} \text{term} + \left(\frac{\cancel{10}^{5}}{\cancel{2}}+1\right)^{th} \text{term}\right]}{2}$$

$$= \frac{(5^{th} \text{term}) + (6^{th} \text{term})}{2}$$

$$= \frac{44+44}{2}$$

$$= \frac{88}{2}$$

$$\boxed{\text{Median} = 44}$$

---

(3) The median of observation 11, 12, 14, 18, $x+2$, $x+4$, 30, 32, 35, 41 arranged in ascending order is 24. Find the values of x.

Sol

11, 12, 14, 18, $\underline{x+2}$, $\underline{x+4}$, 30, 32, 35, 41

$\boxed{N = 10} \Rightarrow$ even.

Median $= 24$

$$\frac{\left(\frac{N}{2}\right)^{th}\text{term} + \left(\frac{N}{2}+1\right)^{th}\text{term}}{2} = 24$$

$$\left(\frac{\cancel{10}^{5}}{\cancel{2}}\right)^{th}\text{term} + \left(\frac{\cancel{10}^{5}}{\cancel{2}}+1\right)^{th}\text{term} = 24 \times 2$$

$$5^{th}\text{ term} + 6^{th}\text{ term} = 48$$

$$x+2 + x+4 = 48$$

$$2x+6 = 48$$

$$2x = 48-6$$

$$2x = 42$$

$$x = \frac{42}{2}$$

$$\boxed{x = 21}$$

(4) A researcher studying the behaviour of mice has recorded the time (in seconds) taken by each mouse to locate its food by considering 13 different mice as 31, 33, 63, 33, 28, 29, 33, 27, 27, 34, 35, 28, 32. Find the median time that mice spent in searching its food.

Sol: -

Arrange in Ascending Order.

27, 27, 28, 28, 29, 31, $\underline{\underline{32}}$, 33, 33, 33, 34, 35, 63

$$\boxed{N = 13}$$

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{13+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{\cancel{14}}{\cancel{2}}\right)^{th} \text{term}$$

$$= 7^{th} \text{ term}$$

$\therefore$ Median = 32

(5) The following are the marks Scored by the Students in the Summative Assessment exam.

Class	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	2	7	15	10	11	5

Calculate the Median.

Sol

C.I	f	cf
0 - 10	2	2
10 - 20	7	9
20 - 30	15	24
30 - 40	10	34
40 - 50	11	45
50 - 60	5	50
	50 = N	

Median class $= \left(\frac{N}{2}\right)^{th}$ Value

$= \left(\frac{50}{2}\right)^{th}$ Value

$= 25^{th}$ Value

$\therefore$ Median class $= 30 - 40$

$l = 30$

$\frac{N}{2} = 25$

$m = 24$

$c = 10 \Rightarrow$ ie, $(0-10 \Rightarrow 10-0 = 10$ etc$)$

$f = 10$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

$$= 30 + \left(\frac{25 - 24}{\cancel{10}}\right) \times \cancel{10}$$

$$= 30 + 1$$

$$\boxed{\text{Median} = 31}$$

---

(6) The mean of five positive integers is twice their median. If four of the integers are 3, 4, 6, 9 and median is 6. then find the fifth integer.

Sol :-

Four integers ⇒ 3, 4, 6, 9

$\bar{x} = \frac{\Sigma x}{n}$

let fifth integer ⇒ x

Median = 6

Given :- Mean of 5 integer = twice Median

$$\frac{3+4+6+9+x}{5} = 2 \times 6$$

$$\frac{22+x}{5} = 12$$

$$22+x = 12 \times 5$$

$$22+x = 60$$

$$x = 60 - 22$$

$$\boxed{x = 38}$$

∴ Fifth integer = 38

Mode

* Raw Data

Most frequently occuring data

* Grouped frequency

$$\text{Mode} = l + \left[\frac{f - f_1}{2f - f_1 - f_2}\right] \times c$$

⇒ Class interval with maximum frequency is modal class.

⇒ l = lower limit

⇒ f = frequency of modal class

⇒ f_1 = frequency of Preceding the modal class.

⇒ f_2 = frequency of Succeeding the modal class.

⇒ c = Width of class interval

Exercise - 8.3

(1) The monthly Salary of 10 employees in a factory are given below:

₹5000, ₹7000, ₹5000, ₹7000, ₹8000, ₹7000, ₹7000, ₹8000, ₹7000, ₹5000

Find the mean, median, and mode.

Sol:-

(i) Mean $= \frac{\Sigma x}{n}$

$$= \frac{5000 + 7000 + 5000 + 7000 + 8000 + 7000 + 7000 + 8000 + 7000 + 5000}{10}$$

$$= \frac{6600\cancel{0}}{1\cancel{0}}$$

$$\boxed{\text{Mean} = 6600}$$

(ii) Median

Arrange in Ascending Order.

5000, 5000, 5000, 7000, 7000, 7000, 7000,
7000, 8000, 8000

$\boxed{n = 10} \Rightarrow$ even.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\text{term} + \left(\frac{n}{2}+1\right)^{th}\text{term}}{2}$$

$$= \frac{\left(\frac{\cancel{10}^{5}}{\cancel{2}_{1}}\right)^{th}\text{term} + \left(\frac{\cancel{10}^{5}}{\cancel{2}_{1}}+1\right)^{th}\text{term}}{2}$$

$$= \frac{5^{th}\text{term} + 6^{th}\text{term}}{2}$$

$$= \frac{7000 + 7000}{2}$$

$$= \frac{\cancel{14000}^{7000}}{\cancel{2}}$$

$$\boxed{\text{Median} = 7000}$$

(iii) Mode

Mode = 7000 [Repeated 5 times]

(2) Find the mode of the given data

3.1, 3.2, 3.3, 2.1, 1.3, 3.3, 3.1

Sol:- Mode = 3.1 and 3.3

[Bimodal]

(3) For the data 11, 15, 17, $x+1$, 19, $x-2$, 3 if the mean is 14, find the value of x. Also find the mode of the data.

Sol

11, 15, 17, $x+1$, 19, $x-2$, 3

Mean = 14

$$\frac{\Sigma x}{n} = 14$$

$$\frac{11+15+17+x+1+19+x-2+3}{7} = 14$$

$$64+2x = 98$$

$$2x = 98-64$$

$$2x = 34$$

$\therefore \quad x = \frac{34}{2}$

$$\boxed{x = 17}$$

$x + 1 = 17 + 1 = 18$

$x - 2 = 17 - 2 = 15$

$\therefore$ Data = 11, 15, 17, 18, 19, 15, 3

$\therefore$ $\boxed{\text{Mode} = 15}$

(4) The demand of track suit of different sizes as obtained by a survey is given below:-

Size	38	39	40	41	42	43	44	45
No of Persons	36	15	37	13	26	8	6	2

Sol

Mode = 40 (37 persons demands)

(5) Find the mode of the following data:-

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	22	38	46	34	20

Sol

Marks	f
0 - 10	22
10 - 20	38
20-30	46
30- 40	34
40- 50	20

Modal Class = 20-30

$$l = 20$$

$$f = 46$$

$$f_1 = 38$$

$$f_2 = 34$$

$$c = 10$$

$$\text{Mode} = l + \left[\frac{f - f_1}{2f - f_1 - f_2}\right] \times c$$

$$= 20 + \left[\frac{46-38}{2(46)-38-34}\right] \times 10$$

$$= 20 + \left[\frac{8}{92-38-34}\right] \times 10$$

$$= 20 + \left[\frac{8}{92-72}\right] \times 10$$

$$= 20 + \left(\frac{\cancel{8}^{4}}{\cancel{20}_{\cancel{2}_{1}}}\right) \times \cancel{10}^{1}$$

$$= 20 + 4$$

$$\boxed{\text{Mode} = 24}$$

---

(6) Find the mode of the following distribution:

Weight (kg)	25-34	35-44	45-54	55-64	65-74	75-84
Number of Students	4	8	10	14	8	6

Sol:-

Weight C.I	f
24.5 - 34.5	4
34.5 - 44.5	8
44.5 - 54.5	10
54.5 - 64.5	14
64.5 - 74.5	8
74.5 - 84.5	6

Modal class = 54.5 - 64.5

$l = 54.5$

$f = 14$

$f_1 = 10$

$f_2 = 8$

$c = 10$

$$\text{Mode} = l + \left(\frac{f - f_1}{2f - f_1 - f_2}\right) \times c$$

$$= 54.5 + \left[\frac{14 - 10}{2(14) - 10 - 8}\right] \times 10$$

$$= 54.5 + \left[\frac{4}{28-18}\right] \times 10$$

$$= 54.5 + \left(\frac{4}{\cancel{10}_1}\right) \times \cancel{10}^1$$

$$= 54.5 + 4$$

$$\therefore \boxed{\text{Mode} = 58.5}$$

CHAPTER-9

PROBABILITY

EXERCISE 9.1

1) You are walking along a street. If you just choose a stranger crossing you, what is the probability that his next birthday will fall on a Sunday?

$\boxed{S \Rightarrow \text{No. of days in a week}}$

$S = \{\text{sun, mon, Tue, wed, Thur, fri, sat}\}$

$n(S) = 7$

Let 'A' be the probability that his next birthday will fall on a Sunday.

$A = \{\text{sun}\}$

$n(A) = 1$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$

---

2) What is the probability of drawing a King or a Queen or a Jack from a deck of cards?

Cards

$n(S) = 52$

King or Queen or Jack

$A \cup B \cup C$

Let A be the probability of drawing a King

♠	♣	♦	♥
K	K	K	K
Q	Q	Q	Q
J	J	J	J

$$n(A) = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the probability of drawing a Queen.

$$n(B) = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$$

Let C be the probability of getting a Jack.

$$n(C) = 4$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{52}$$

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$= \frac{4}{52} + \frac{4}{52} + \frac{4}{52}$$

$$= \frac{4+4+4}{52}$$

$$= \frac{12}{52}$$

---

3) What is the probability of throwing an even number with a single standard dice of six faces?

Single dice:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6$$

Let A be the probability of throwing an even number with a single standard dice of six faces.

$$A = \{2, 4, 6\}$$

$$n(A) = 3$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{6}$$

---

4) There are 24 balls in a pot. If 3 of them are Red, 5 of them are Blue and the remaining are Green then, what is the probability of picking out (i) a Blue ball (ii) a Red ball and (iii) a Green ball?

$$\left.\begin{array}{l}\text{Red balls} = 3 \\ \text{Blue balls} = 5\end{array}\right\} 8 \qquad n(S) = 24 \text{ (balls)}$$

Green balls $= 24 - 8 = 16$

(i) Let A be the probability of picking out a Blue ball.

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{24}$$

(ii) Let B be the probability of picking out a Red ball.

$n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{24}$$

(iii) Let C be the probability of picking out a Green ball.

$n(C) = 16$

$$P(C) = \frac{n(C)}{n(S)} = \frac{16}{24}$$

---

5) When two coins are tossed, what is the probability that two heads are obtained?

Two Coins

$S = \{HH, TT, HT, TH\}$

$n(S) = 4$

Let A be the probability that two heads are obtained.

$A = \{HH\}$

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

---

6) Two dice are rolled, Find the Probability that the sum is (i) equal to 1 (ii) equal to 4 (iii) less than 13.

TWO DICE:

$$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$$
$$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$$
$$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$$
$$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$$

$n(S) = 36$

(i) Let A be the probability that the sum is equal to 1

$n(A) = 0$

$$P(A) = \frac{n(A)}{n(S)} = \frac{0}{36}$$

(ii) Let B be the probability that the sum is equal to 4

$B = \{(1,3)\ (2,2)\ (3,1)\}$

$n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

(iii) Let C be the probability that the sum is less than 13

$$C = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$$

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

$n(C) = 36$

$P(C) = \frac{n(C)}{n(S)} = \frac{\cancel{36}}{\cancel{36}} = 1$

---

7) A manufacturer tested 7000 LED lights at random and found that 25 of them were defective. If a LED light is selected at random, what is the probability that the selected LED light is a defective one.

defective - 25

$n(S) = 7000$ (LED Lights)

Let A be the probability that the selected LED light is a defective one.

$n(A) = 25$

$P(A) = \frac{n(A)}{n(S)} = \frac{25}{7000}$

---

8) In a football match, a goalkeeper of a team can stop the goal, 32 times out of 40 attempts tried by a team. Find the probability that the oppenent team can convert the attempt into

a goal.

$n(S) = 40$ (attempts)

Let A be the probability that the opponent team can convert the attempt into a goal.

$n(A) = 40 - 32 = 8$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{40}$$

9) What is the probability that the spinner will not land on a multiple of 3?

$S = \{1,2,3,4,5,6,7,8\}$

$n(S) = 8$

Let A be the probability that the spinner will not land on a multiple of 3.

$A = \{1,2,4,5,7,8\}$

Multiple of 3

3, 6

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{8}$$

10) Frame two problems in calculating Probability based on the spinner shown here.

$S = \{1,2,3,4,5,6,7,8\}$

$n(S) = 8$

(i) Let A be the probability of not getting a multiple of 2

$A = \{1, 3, 5, 7\}$

Multiple of 2

2, 4, 6, 8

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8}$

(ii) Let B be the probability of getting a multiple of 3

$B = \{3, 6\}$

$n(B) = 2$

$P(B) = \frac{n(B)}{n(S)} = \frac{2}{8}$

EXERCISE 9.2

1) A company manufactures 10000 laptops in 6 months. Out of which 25 of them are found to be defective. When you choose one Laptop from the manufactured, what is the probability that selected Laptop is a good one.

$n(S) = 10{,}000$ (Laptops)

Defective Laptops = 25

$\therefore$ Good Laptops $= 10{,}000 - 25$

$= 9975$

Let A be the probability that the selected Laptop is a good one.

$n(A) = 9975$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{9975}{10000}$$

---

2) In a survey of 400 youngsters aged 16-20 years, it was found that 191 have their voter ID card. If a youngster is selected at random, find the probability that the youngster does not have their Voter ID-card.

$n(S) = 400$ (youngsters)

VOTER - ID = 191

Let A be the probability that the youngster does not have their voter ID-card.

$n(A) = 400 - 191$

$n(A) = 209$

$\therefore \quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{209}{400}$.

---

3) The probability of guessing the correct answer to a certain question is $\dfrac{x}{3}$. If the probability of not guessing the correct answer is $\dfrac{x}{5}$, then find the value of x.

P(guessing the correct answer) $= \dfrac{x}{3}$

$\Rightarrow \; P(A) = \dfrac{x}{3}$

P(not guessing the correct answer) $= \dfrac{x}{5}$

$\Rightarrow \; P(A') = \dfrac{x}{5}$

We know,

$$\boxed{P(A) + P(A') = 1}$$

$$\frac{x}{3} + \frac{x}{5} = 1$$

$$\frac{5x + 3x}{15} = 1$$

3	3, 5
5	1, 5
	1, 1

L.C.M
$= 3 \times 5$
$= 15$

$$\frac{8x}{15} = 1$$

$$8x = 15$$

$$x = \frac{15}{8}$$

4) If a probability of a player winning a particular tennis match is 0.72. What is the probability of the player loosing the match?

P(winning the match) = 0.72.

$\Rightarrow$ $P(A) = 0.72$

P (not winning the match) = ?

$\Rightarrow$ $P(A') = ?$

We know,

$$\boxed{P(A) + P(A') = 1}$$

$$0.72 + P(A') = 1$$

$$P(A') = 1 - 0.72$$

$$P(A') = 0.28$$

$$\begin{array}{r} \cancel{1}.\cancel{0}\cancel{0} \\ 0.72\ (-) \\ \hline 0.28 \end{array}$$

5) 1500 families were surveyed and following data was recorded about their maids at houses.

Type of maids	Only Part time	Only full time	both
No. of Families	860	370	250

A family is selected at random. Find the probability that the family selected has (i) Both types of maids. (ii) Part time maids (iii) No maids.

$\boxed{n(S) = 1500}$

(i) Let A be the probability of the family selected has both types of maids.

$$n(A) = 250$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \boxed{\frac{250}{1500}}$$

(ii) Let B be the probability that the family selected has part time maids.

$$n(B) = 860$$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \boxed{\frac{860}{1500}}$$

(iii) Let c be the probability that the family selected has no maids.

$$n(c) = 1500 - (860 + 370 + 250)$$

$$n(c) = 1500 - 1480$$

$$n(c) = 20$$

$$\therefore P(c) = \frac{n(c)}{n(S)} = \boxed{\frac{20}{1500}}$$